Yield Curve Modeling

Constructing the Curve with Futures & Swaps

Ingredients for Building the Zero Coupon Spot Curve

- Cash Rates
- FRA Rates (T-Bills)
- Futures Prices
- Swap Rates

Lab: Constructing the Short End

		Ask	Bid	Days
3 Month	$ LIBOR	4.25%	4.23%	91
$ FRAs	3-6	4.40%	4.35%	91
	6-9	4.55%	4.50%	92
	9-12	4.70%	4.65%	91

Lab: Constructing the Short End

- Excel Spreadsheet: Yield Curve Modeling.xls
- Worksheet: Short End
- Use discount factor method
- See cell notes for hints

Solution: Short End

Period	**DF**	**Spot Rate**
3 months	0.9894	4.2500%
6 months	0.9785	4.3486%
9 months	0.9672	4.4498%
12 months	0.9559	4.5519%

- Notes:
  - ✓ $DF_{6\text{-}3} = (1 + \text{Ask} \times \text{Days} / 360)$
  - ✓ $DF_6 = DF_{6\text{-}3} \times DF_3$
  - ✓ $R_6 = (-1 + 1/DF_6) \times 360 / (91 + 91)$

Futures Libor Rates

- Yield = 100 - Settlement Price
- Example:
  - ✓ Futures price = 93.0
  - ✓ Futures yield = 100 - 93.0 = 7.0%
- Note: this is not exactly the forward rate

Futures & Forwards

- Assumption is often that 100-F = forward rate
- Not exact for several reasons:
  - ✓ Interest differentials on margin surplus & funding
  - ✓ Futures are marked to market
  - ✓ "Convexity" - stochastic interest rates give rise to differences (Cox, Ingersoll, Ross JFE)

Convexity

Definition: *Positive Convexity*

Present value increases with rate decline
exceeds
Present value decline with rate increase

- What is the convexity of a Euro $ future and an interest rate swap?
- Are convexity differences priced?

Convexity: Future vs Swap

- Marking-to-market alters the convexity of a future
  - ✓Gains/Losses are Settled Daily
  - ✓PV Gain/Loss = Gain/Loss
- Convexity Future: Approximately Zero
- Convexity of long swap (pay fix/rec floating) is negative
  - ✓Fixed led +ve, Floating leg -ve

Pricing Convexity Differences

- If not priced
  - ✓ Short swap/short futures buys positive convexity for free
    - Significant for longer tenor securities 5+ years
    - Arbitrage gains with rate increases/declines
- If priced
  - ✓ Forward rates implied by FRA's or swaps differ from forward rates implied by futures

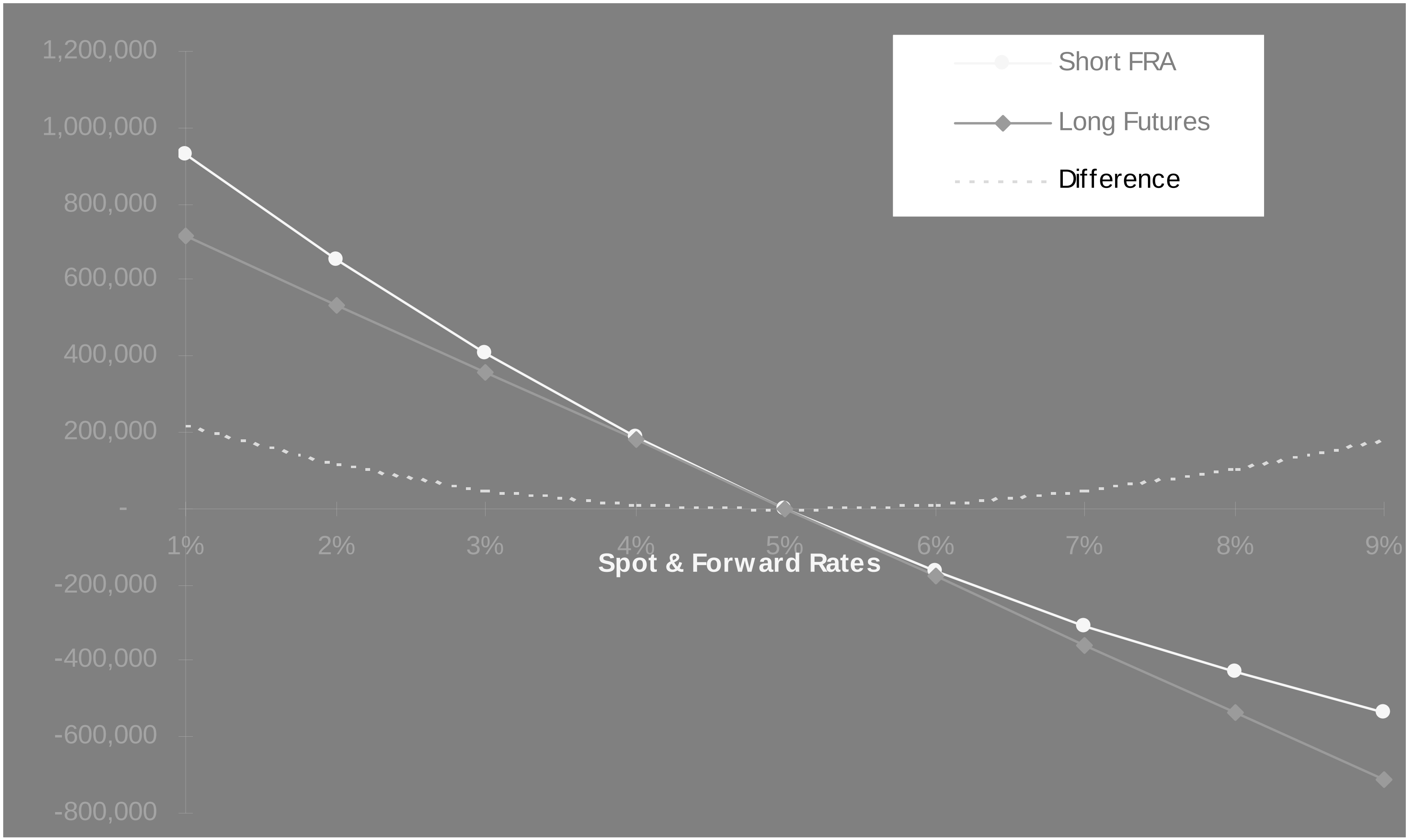
Lab: FRA-Futures Convexity

- Sell $100 81 v 84M IMM dated FRA @ 5.00%
- Hedge by Selling Futures @ 95.00
- Yield curve is flat at 5%
- Work out:
  - ✓ Equivalent futures position
  - ✓ Gain or loss on FRA and equivalent Futures position for parallel shifts +/- 4%
- Worksheet: FRA-Futures
  - ✓ See cell notes for help

Solution: FRA - Futures

- D81 = 0.7150
- FRA Value for 1bp change in YC = $1,788
- Therefore equivalent position is:
  - ✓ $1,788/ $25 = Long 72 Futures
  - ✓(or if hedging, sell 72 futures contracts)
- Examine changes in position value due to shifts in spot & forward rates

Chart: FRA-Futures Convexity



Convexity

- Short FRA has positive convexity
- Futures have zero convexity
- Difference must be paid for:
  - ✓ Forward rate is lower than implied by futures price
  - ✓ (100 - futures price) is greater than forward rate
  - ✓ Need adjustment factor to take account of the volatility of the two rates and their correlation

Convexity Adjustment Factor

- Depends on term structure dynamics
- Rule of thumb (Burghardt & Hoskins)
- Change in spread between forward rate and futures:

$$\Delta S = \sigma_f \times \sigma_{zcb} \times \sigma_{f,zcb}$$

σ_f = standard deviation of change in forward rate

σ_{zcb} = standard deviation of zero coupon bond return

$\sigma_{f,zcb}$ = correlation of forward rate change and zero coupon bond returns

Convexity Adjustment Factor

- Futures and forwards are the same at expiry
- Rule gives change in difference over time
- Calculate the change for each three month period
- Standard deviation of returns on ZCB:
  - ✓ Duration (Maturity) * SD of Yield (Spot Rate) changes
    - Standard deviations and correlations will be slightly different for each period
    - Derived from historical data or option prices

Adjustment Factor Example

- Assume:
  - Annualized SD of changes in all futures prices = 1%
  - Annualized SD of change in ZCB yields = 1%
  - Correlation = 0.99
- First contract has 3 months to expiry:

  $\Delta S_{0\text{-}3}$ = [0.01 x 0.01 x (4.5 / 12) x 0.99] / 4 = 0.09375 bp

  Average maturity of deposit period

- Second Contract:

  $\Delta S_{3\text{-}6}$ = [0.01 x 0.01 x (7.5 / 12) x 0.99] / 4 = 0.15625 bp
- Adjustment for 6 months = $\Delta S_{0\text{-}3} + \Delta S_{3\text{-}6}$ = 0.25bp

Adjustment Factors: Typical Values

✓Six Months:	0.25bp
✓One Year:	0.5bp
✓Two Years:	1.0bp
✓Three Years:	3.5 bp
✓Five Years:	17bp
✓Ten Years:	63bp

- Significant 3 years and beyond

Building the Curve

Today | F1 Start | 90 days | F2 Start | 90 days

10-May 18-Jun 16-Sep 17-Sep

CASH	FUTURES
1w: 5.50 - 5.38 (17-May)	
1m: 5.60 - 5.48 (10-Jun)	Jun: 94.00 (18-Jun)
2m: 5.71 - 5.59 (10-Jul)	Sep: 94.83 (16-Sep)

✓ STEP 1: Interpolate spot rate to expiry of F1:

- R_{36} = 5.6% + (5.71% - 5.60%) x 8 / 31) = 5.628%
- $D_{36} = 1 / (1 + R_{36} \times 36 / 360) = 0.9944$

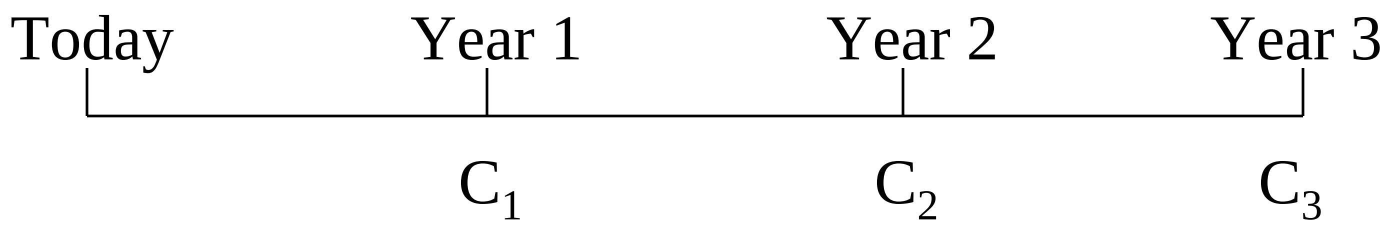
✓ STEP 2: Compute Forward Discount Factors:

- $D_{F1} = 1 / (1 + 6\% \times 90/360) = 0.9852$
- $D_{126} = D_{36} \times D_{F1} = 0.9797$
- $R_{126} = (-1 + 1 / 0.9797) \times 360 / 126 = 5.92\%$

Building the Curve Using Futures

- Find spot rate to expiry of first futures contract
  - ✓Interpolate from cash rates
- Calculate 90-day forward rate from expiry
  - ✓100-futures price less adjustment factor
- Combine to give spot rate to 90 days from expiry
- Extrapolate to expiry of next futures contract
- Repeat steps above for successive futures contracts

Extending the Curve Using Swaps



- A swap is an exchange of an FRN for a bond
  - ✓ The FRN trades at par on coupon payment dates
- FRN Price = 100 = $D_1C_1 + D_2C_2 + D_3C_3 + 100D_3$

  (Bond Price: $D_1C_1 + D_2C_2 + D_3C_3 + 100D_3$)
  - ✓ Cash flows are known from the swap coupon
  - ✓ The curve has already been built out to 2 years using futures
  - ✓ D_1 and D_2 are known, calculate D_3 by *bootstrapping*

Bootstrap Method with Swaps

- Example:
  - ✓Three year swap rate = 5%
  - ✓$D_1 = 0.9655$ $D_2 = 0.9259$
  - ✓$100 = 0.9655 \times 5 + 0.9259 \times 5 + 105 \times D_3$
  - ✓$D_3 = (100 - 4.8275 - 4.6295) / 105 = 0.8623$
- Repeat with 4, 5, . . . year swaps to complete the curve

Lab: Delmar Capital

- Excel Workbook: Yield Curve Modeling .xls
- Worksheet: Delmar Capital
- Build yield curve using:
  - ✓Cash & Futures
  - ✓Cash , Futures & Swaps
  - ✓Cash & Adjusted Futures
- See Notes & Solution

Delmar Capital: Yield Curves