Theories of the Yield Curve

Copyright © 1996-2006
Investment Analytics

The Yield Curve

- What is the yield curve?
- How is the curve constructed?
- Why is the yield curve shaped the way it is?
- Why does its shape change?
- How can a trader profit from this?

The Yield Curve

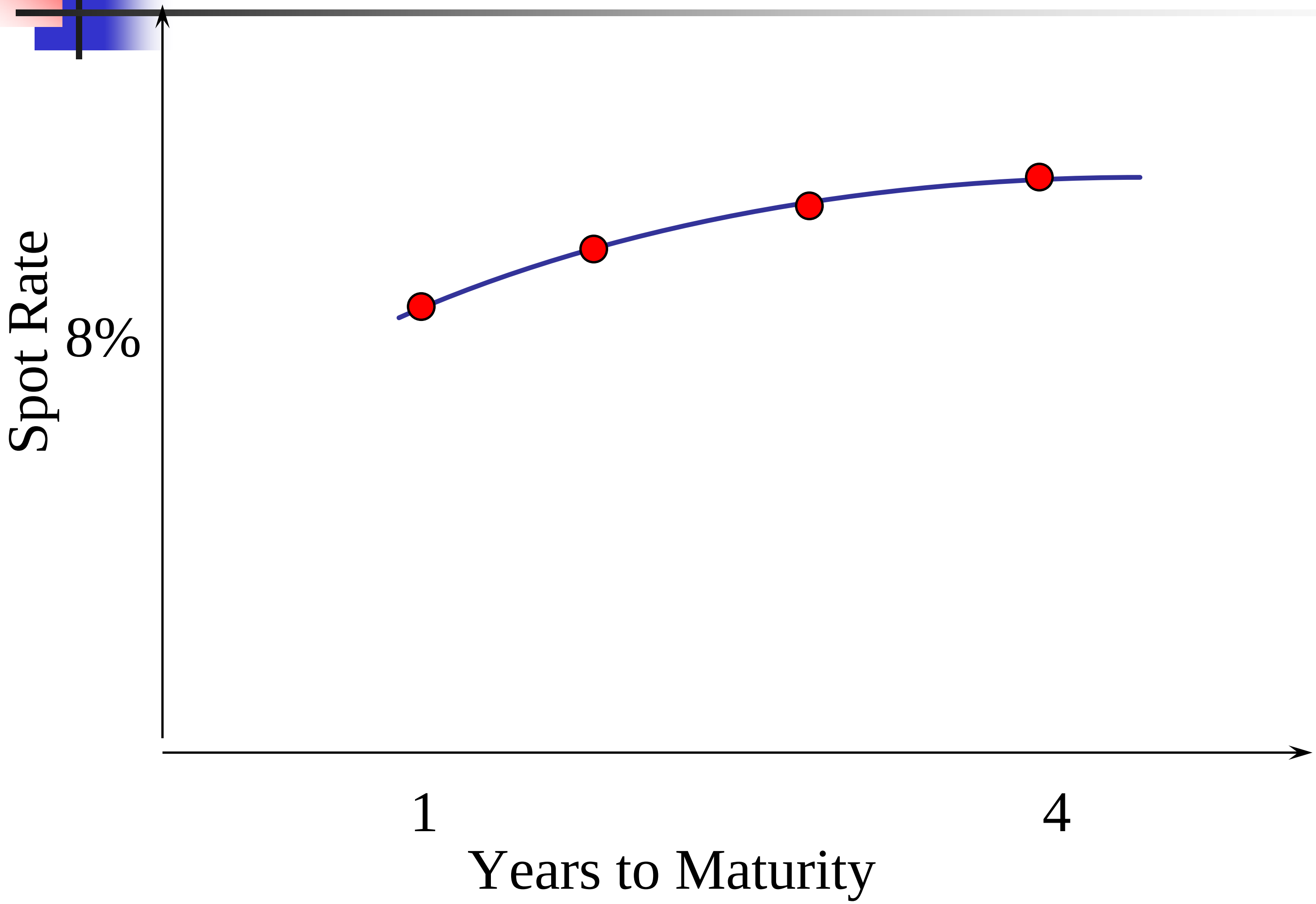
- Zero-Coupon Bonds, Face Value \$1,000:
- Term / Price / Discount / YTM

Term	Price	Discount	YTM
1	925.93	$1/(1+y_1)$	8.000%
2	841.75	$1/(1+y_2)^2$	8.995%
3	758.33	$1/(1+y_3)^3$	9.660%
4	683.18	$1/(1+y_4)^4$	9.993%

- Spot Yield (Zero Coupon Yield)
  - y_1 is called the one year spot rate
  - y_2 is called the two year spot rate

Copyright © 1996-2006 Investment Analytics

Yield Curve Example



Copyright © 1996-2006 Investment Analytics

Building a Yield Curve

- In practice we have coupon bonds, not just zeros
- Term / Price / Discount / YTM

Term	Price	Discount	YTM
1	925.93 Z	$1/(1+y_1)$	8.000%
2	841.75 Z	$1/(1+y_2)^2$	8.995%
3	952.40 C		

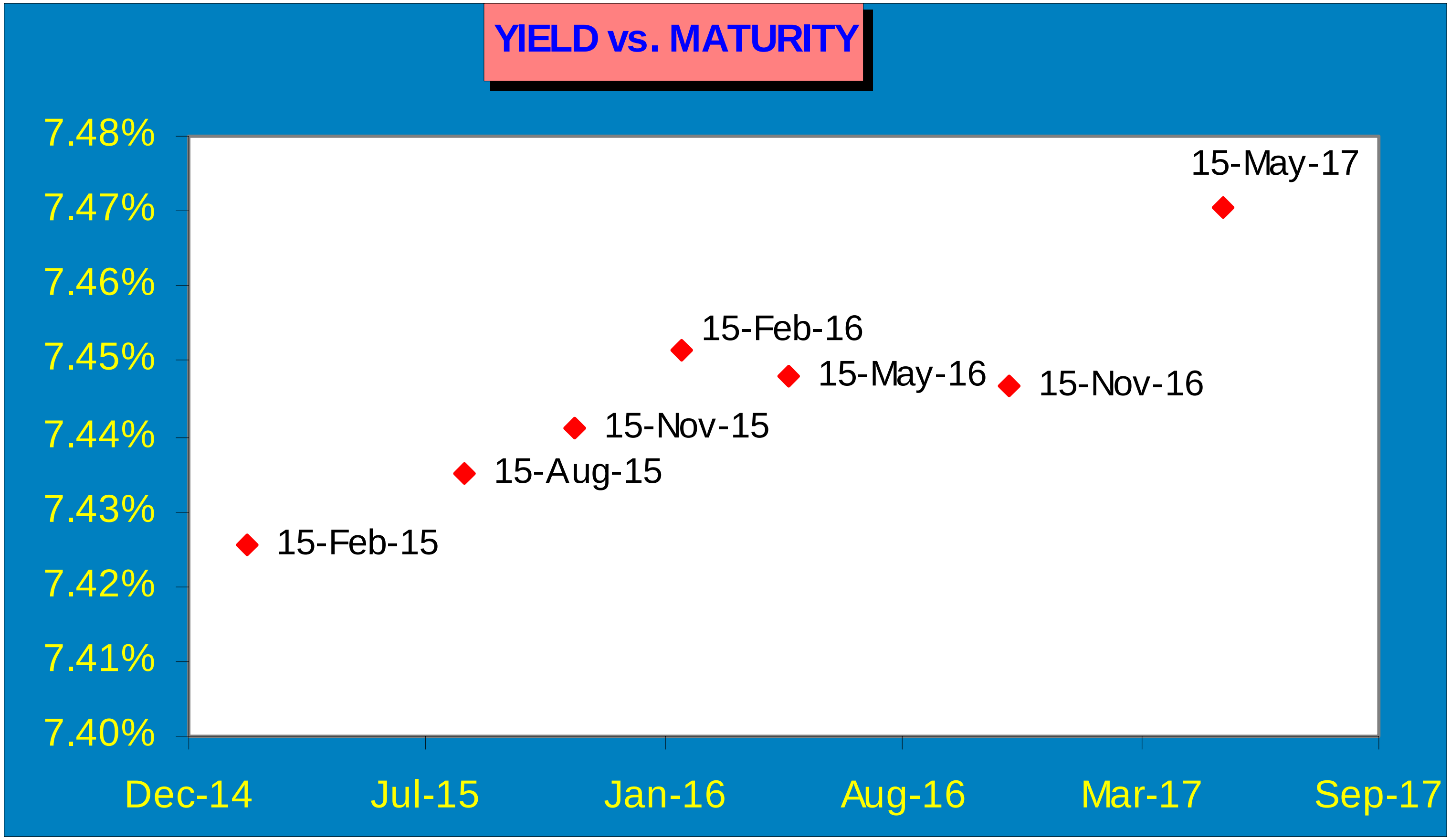
- Bond in year 3 is a coupon bond
  - Pays 8% coupon ($80 per year)
  - How do we proceed?

Bootstrapping

- Method: split into coupon and principal payments and treat each as a zero

- Then solve equation:
  - $952.40 = \$80/(1+y_1) + \$80/(1+y_2)^2 + \$1080/(1+y_3)^3$
  - y_1 & y_2 are known
  - $y_3 = 10.020\%$

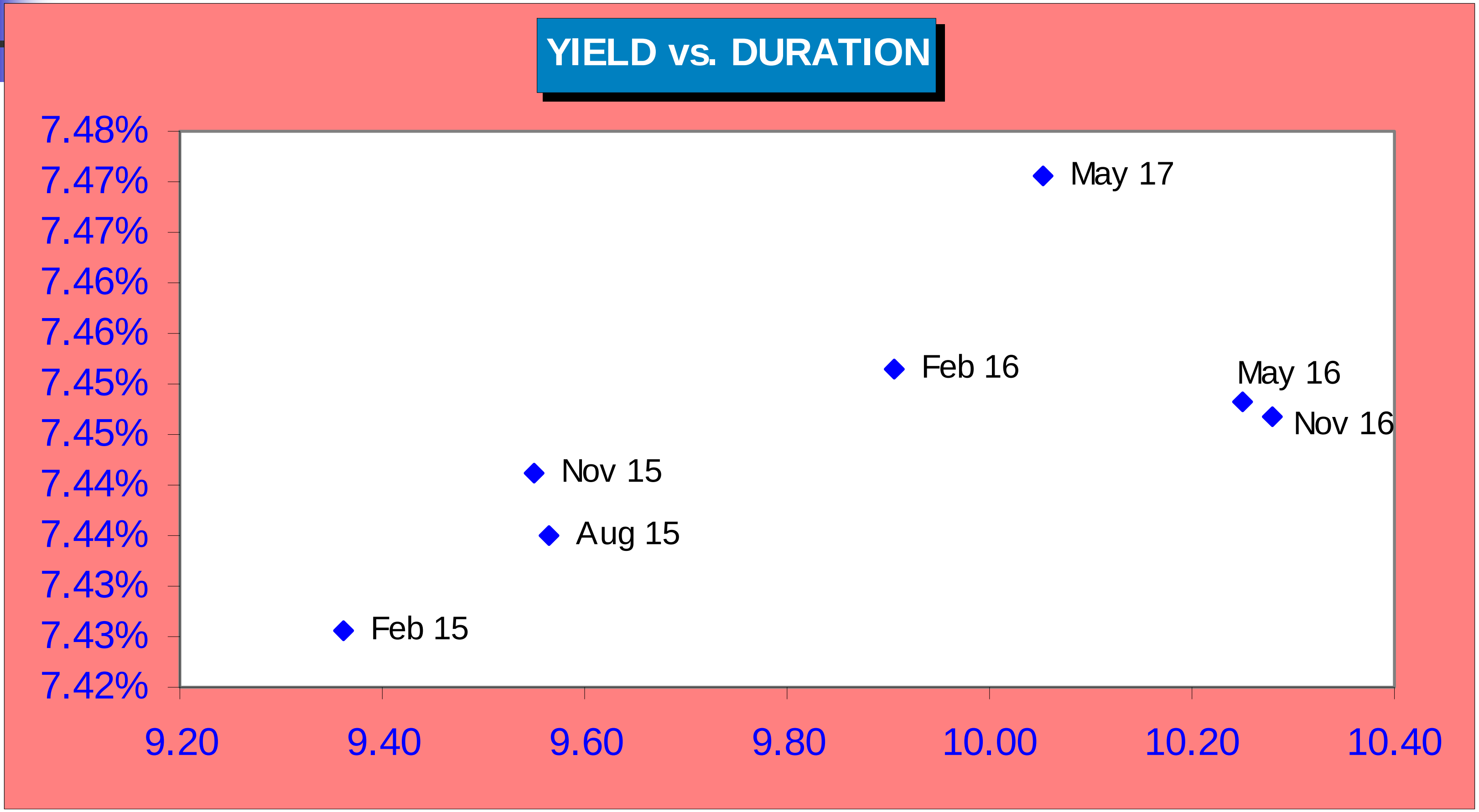
Example: US Treasury Yield Curve



Yield Curve Analysis

- Fairly normal yield curve
  - Yield on the 9 1/4 of Feb ‘16 looks to be a basis point too high
  - 2.4bp pickup on the 8 /4% of May ‘17 indicates value in this sector
- Clear relationship between yield and tenor
- What about relationship between yield and risk?
  - Use duration as a proxy for risk
  - Plot yield vs. duration
  - Makes relative values more distinct

Yield vs. Duration



Copyright © 1996-2006 Investment Analytics

Yield Enhancement Swap

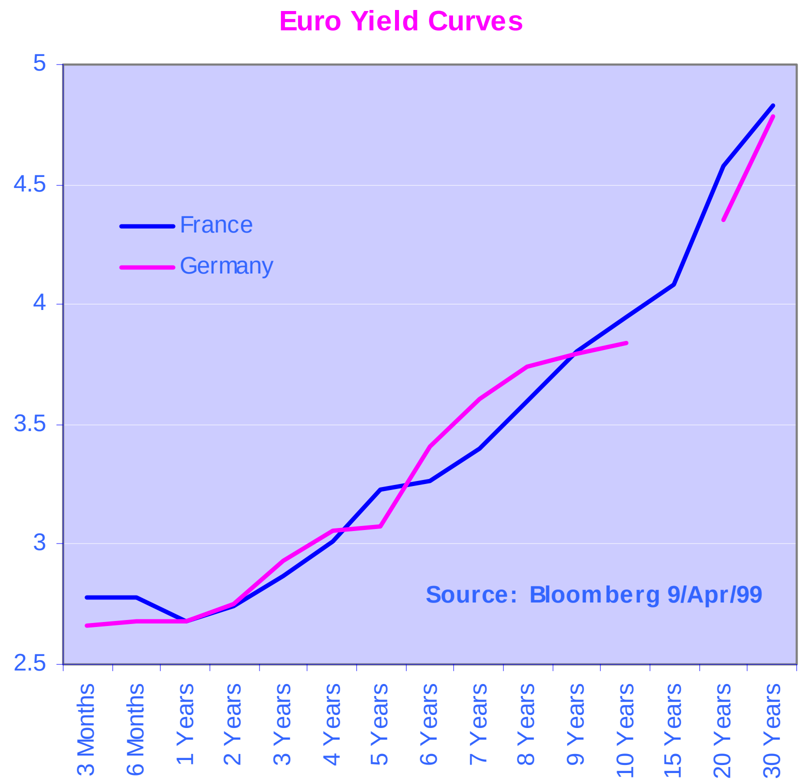
- Because it has higher coupon, the 8 3/4 of May '17 has lower duration than the 7 1/4 of May '16 or the 7 1/2 or Nov '16.
- By trading at slightly higher yield, the market would appear to be underpricing it slightly
- Bond Swap:

Action	Maturity	Coupon	Price	YTM	Duration
Sell	15-Nov-16	7 1/2%	100 18/32	7.4467%	10.278
Buy	15-May-17	8 3/4%	11323/32	7.4706%	10.054

Limitations to Traditional Yield Curve Analysis

- Yield curve:
  - A primitive expression of risk/return tradeoff
- Drawbacks
  - Maturity is poor indicator of bond price volatility
  - YTM is not a measure of potential return
    - For Buy and Hold investor, assumes coupons are reinvested at YTM
    - For Active investor, assumes that if bond is sold prior to maturity, it is sold at same yield as on purchase date

Example: Euro Yield Curves



Copyright © 1996-2006 Investment Analytics

Other Yield Curves

- Swaps curve
  - Swap rate (coupon) by tenor
  - Swap curve lies above treasury curve
    - Due to default risk
    - Swap rates quoted as spread over same maturity treasury yield
- Corporate bond yield curve
  - Trades at spread over treasury curve
    - Default risk
  - Many corporate bonds include option features
    - Callable, putable, convertible
    - Calculate option-adjusted spread

LIBOR Spot Rates

- Spots quoted as add-on interest
- Actual/360 daycount
- Example: 3 month deposit
  - Today is Jan 12 2001
  - Deposit matures April 12, 2001
  - Number of days: 91
  - Rate is r, P is principal
- Value at maturity: $P \times (1 + r \times 91 / 360)$

Daycounts

- How many days in a month and year
  - 30/360 (Money Market)
    - in one month, get 1+(30/360)r
  - Actual/360 (LIBOR)
    - in one month get 1 + (31/360)r if 31 days
  - Actual/365 (Treasury)
    - (or actual/actual: adjust for leap year)

Discount Factors and Compounding

- Notation:
  - R = % Interest rate, T = Time (days), D = Discount Factor
- R is simple:
  - $D = 1 / (1 + R \times T / 360)$
  - $R = (-1 + 1/D) * 360 / T$
- R is annually compounded (LIBOR):
  - $D = 1 / (1 + R)^{T/360}$
  - $R = -1 + (1 / D)^{360/T}$
- R is Semi-annually compounded (Treasury)
  - $D = 1 / (1 + R / 2)^{T/182.5}$
  - $R = 2 * (-1 + 1 / D)^{182.5/T}$
- R is continuously compounded:
  - $D = e^{-RT/360}$
  - $R = -Ln(D) \times 360 / T$

Yield Curve Theories

- Expectations Theory
- Liquidity Preference Theory
- Risk Theory

Copyright © 1996-2006 Investment Analytics

Expectations Theory

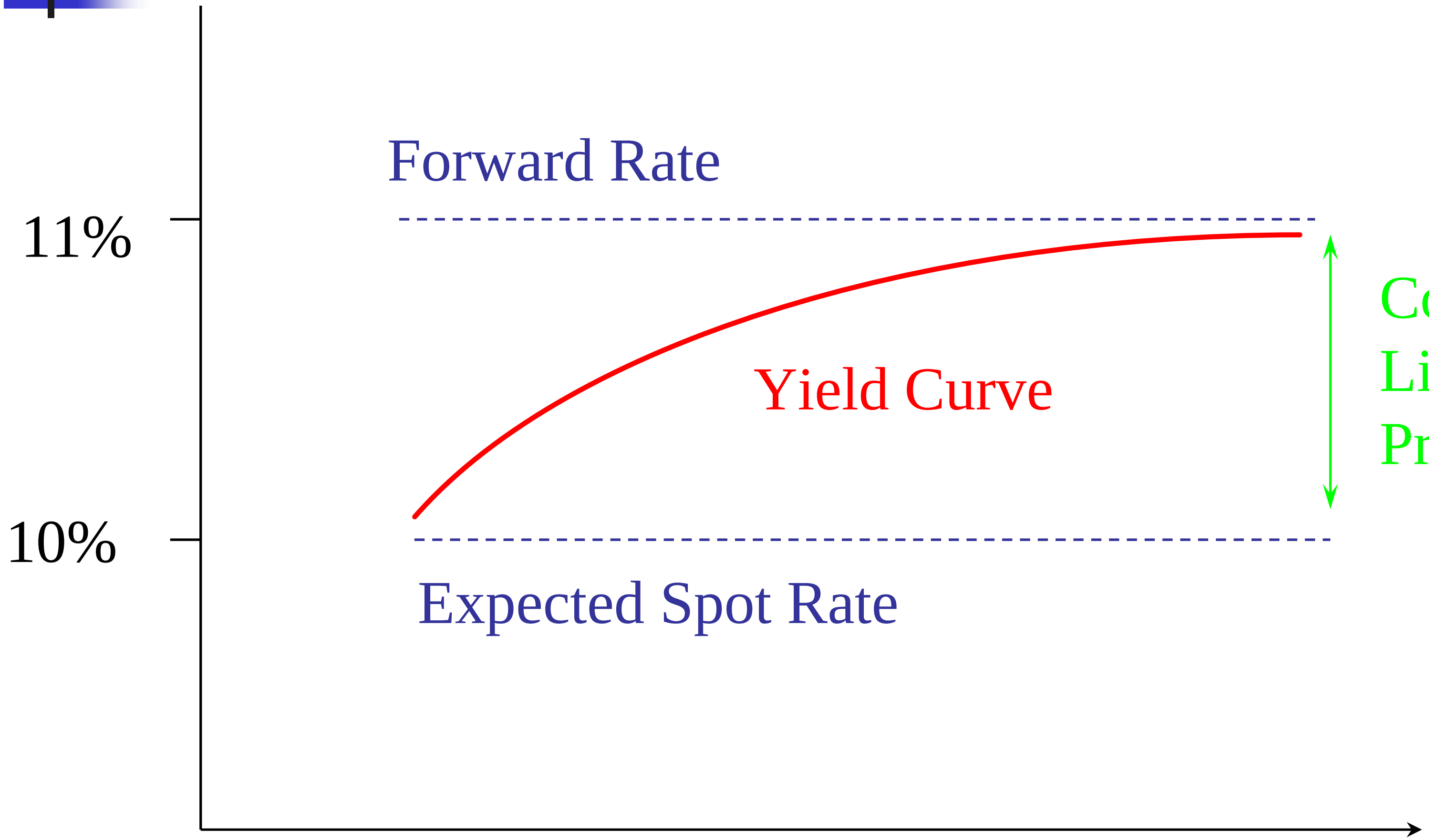
- Forward rate = Expected future spot rate
- $F_T = E(S_T)$
- Implications:
  - Bond yields relate to expected future spot rates
    - $(1 + y_2)^2 = (1 + S_1)\ (1 + f_2) = (1 + S_1)\ (1 + E[S_2])$
  - Upward sloping yield curve means investors anticipate higher interest rates

Copyright © 1996-2006 Investment Analytics

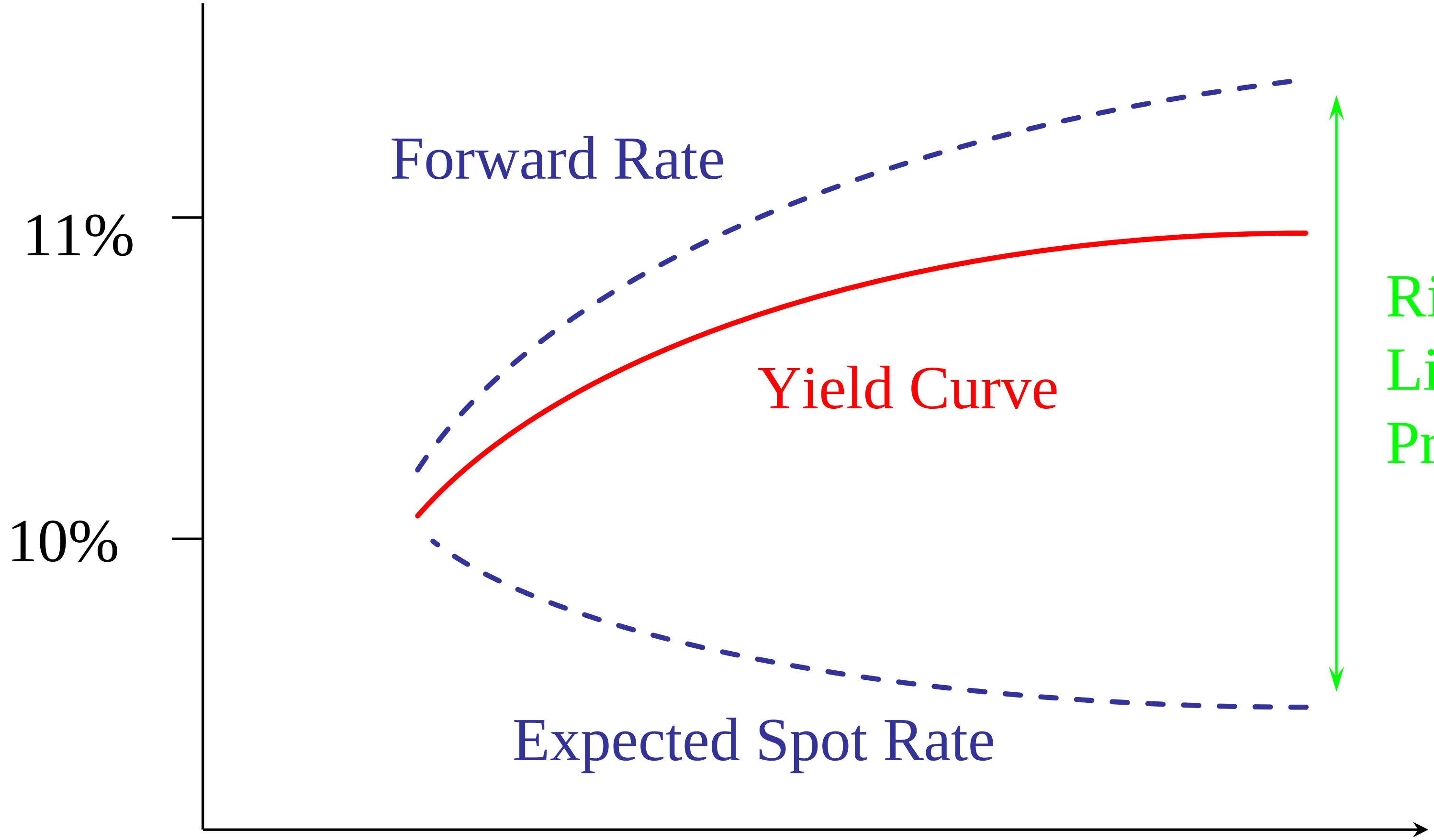
Liquidity Preference Theory

- Investors require a liquidity premium to hold long term securities
- $F_T > E[S_T]$
- Liquidity Premium: $L_T = F_T - E[S_T]$
- Example: $S_1 = E[S_2] = 10\%$
  - Expectations Hypothesis
    - $(1 + y_2)^2 = (1 + S_1)(1 + E[S_2]) \Rightarrow y_2 = 10\%$
  - Liquidity Preference
    - $F_2 = 11\% > E[S_2] = 10\% \; (L_2 = 1\%)$
    - $(1 + y_2)^2 = (1 + S_1)(1 + f_2) \Rightarrow y_2 = 10.5\%$

Constant Liquidity Premium



Rising Liquidity Premium



Risk Measures

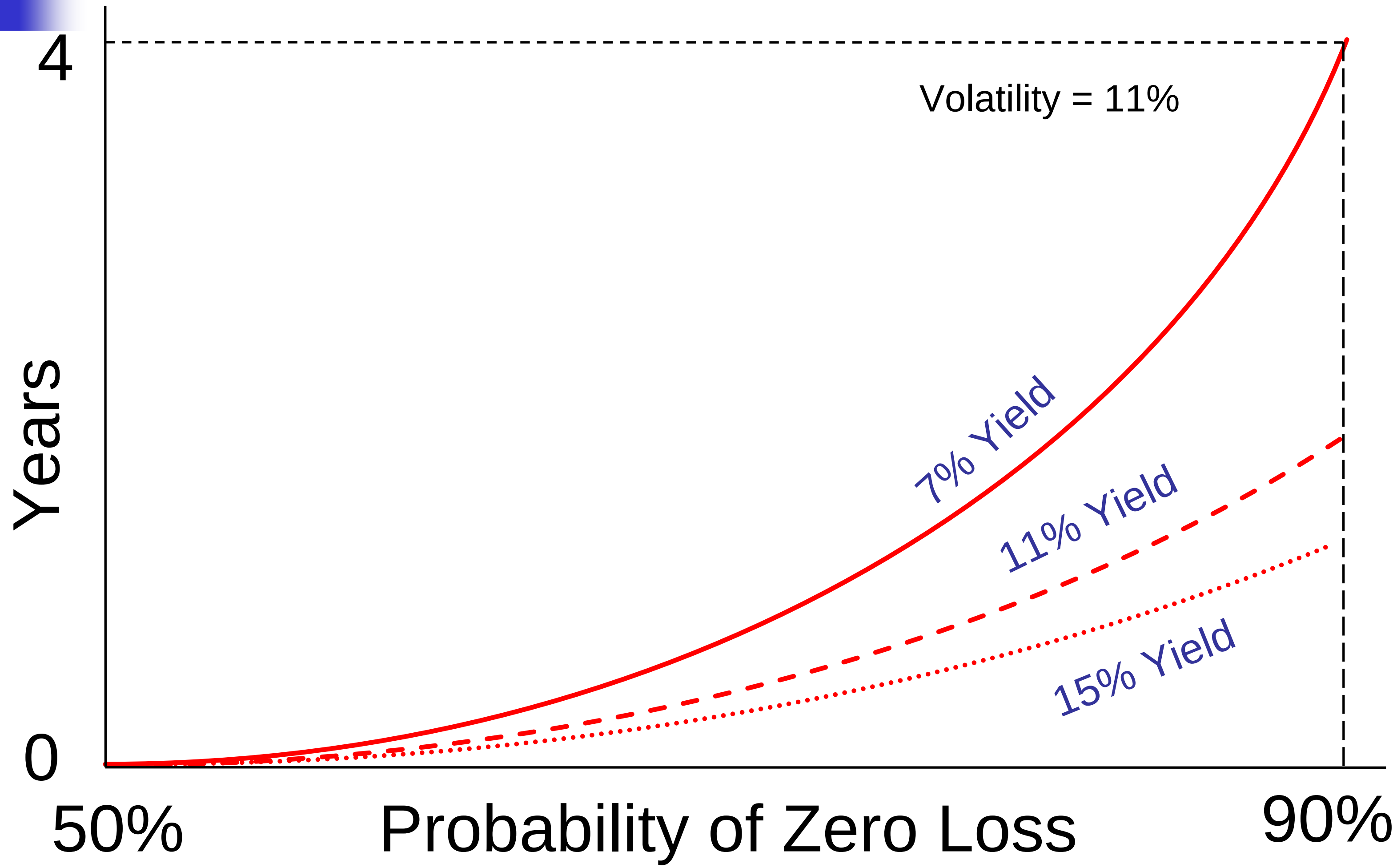
- Price Risk:
  - Change in price for 1% change in yield (dollar duration or "PV of an 01")
- Probability of Zero Loss (over 1 month):
  - Likelihood that price of an issues falls by no more than interest earned (over 1 month)
- Required Holding Period:
  - Period require to hold a security so that the probability of zero loss exceeds a specified level

Risk and Yield

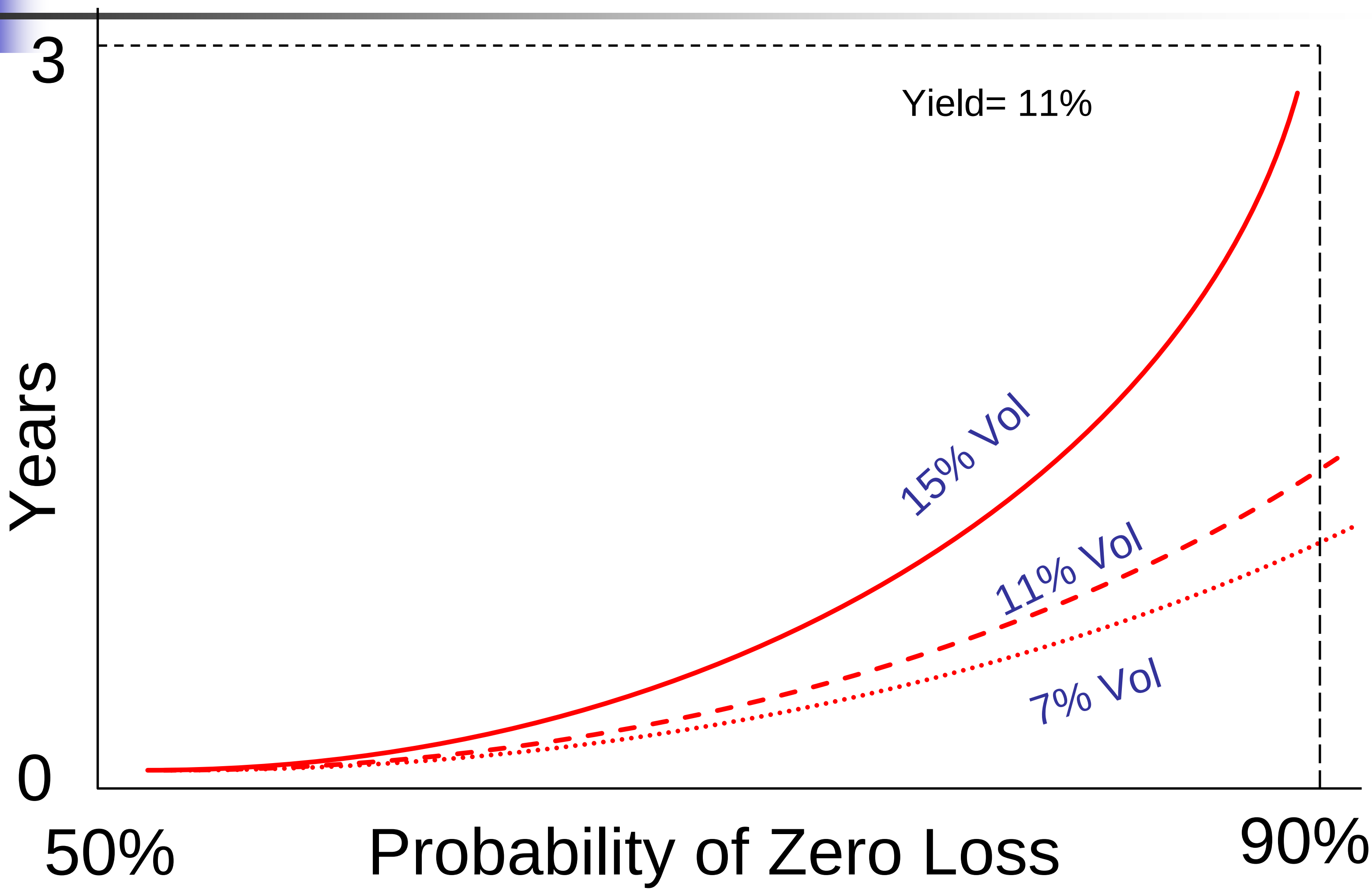
- Price risk is proportional to duration
  - 30 year bond has greater price risk than 2 year note
- Higher yield means lower price risk
  - A par bond at 15% yield has a price risk just over half that of a par bond at 7% yield

Copyright © 1996-2006 Investment Analytics

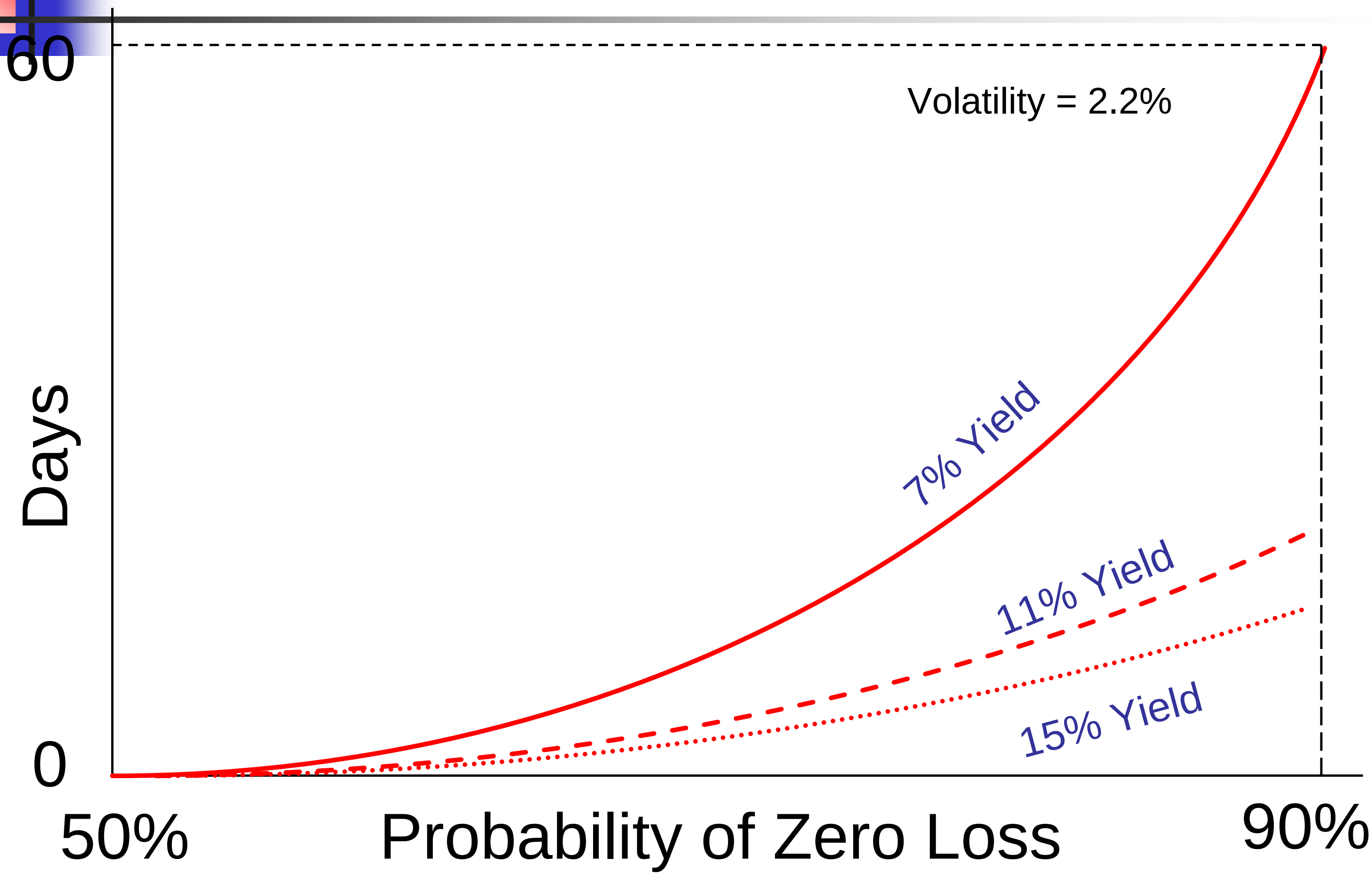
Holding Period for 30Y Bond x Yield



Holding Period for 30Y Bond x Vol



Holding Period for 2Y Note x Yield



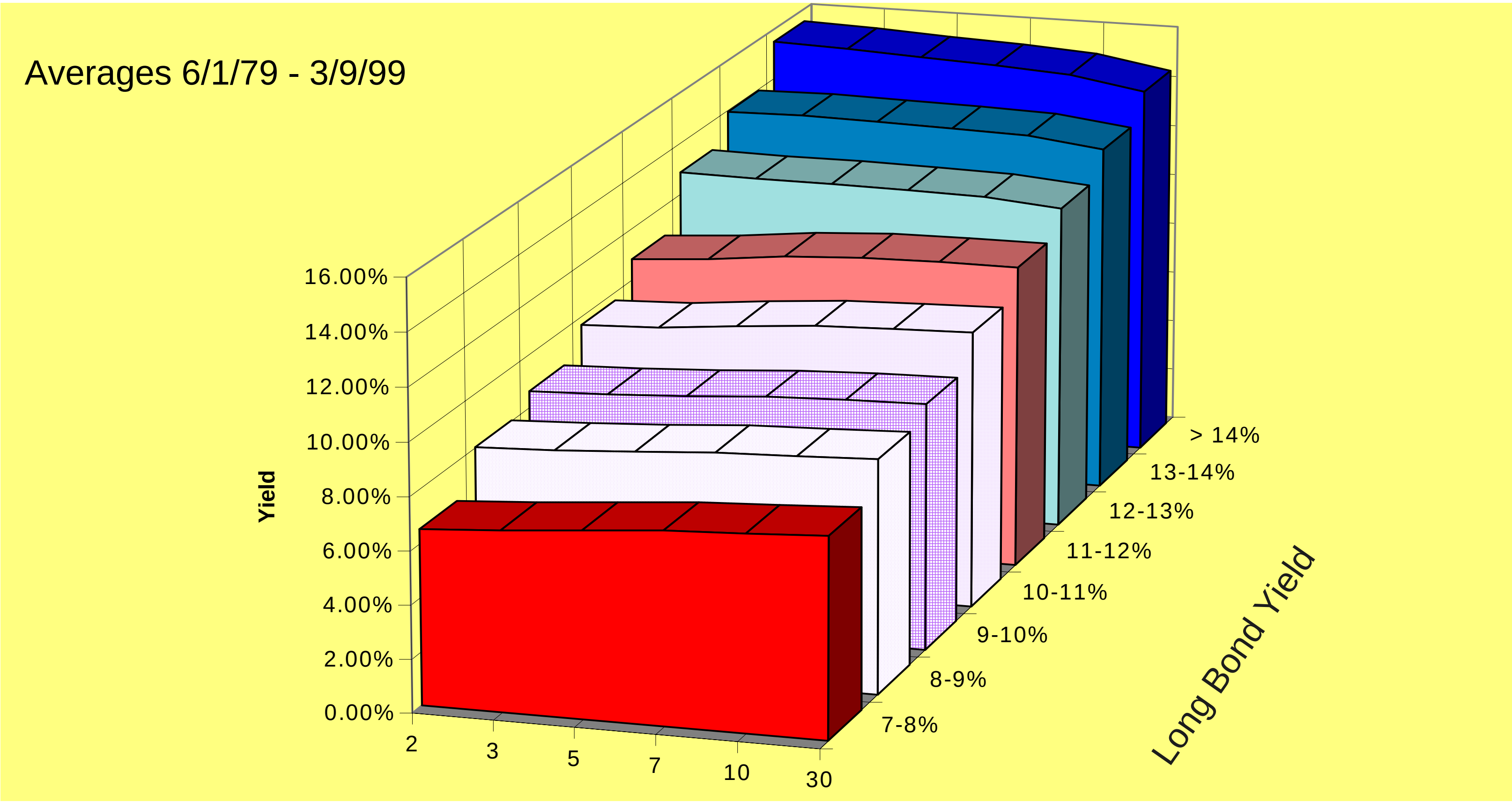
Implications for Yield Curve Shape

- 2y Note much safer than 30y Bond
    - (holding period days rather than years)
- As investor extends along yield curve, probability of losing money rises
- Hence must receive risk premium in higher yields
- CONCLUSION: Yield curve +ve slope

Yield Curve Shape & Yield Level

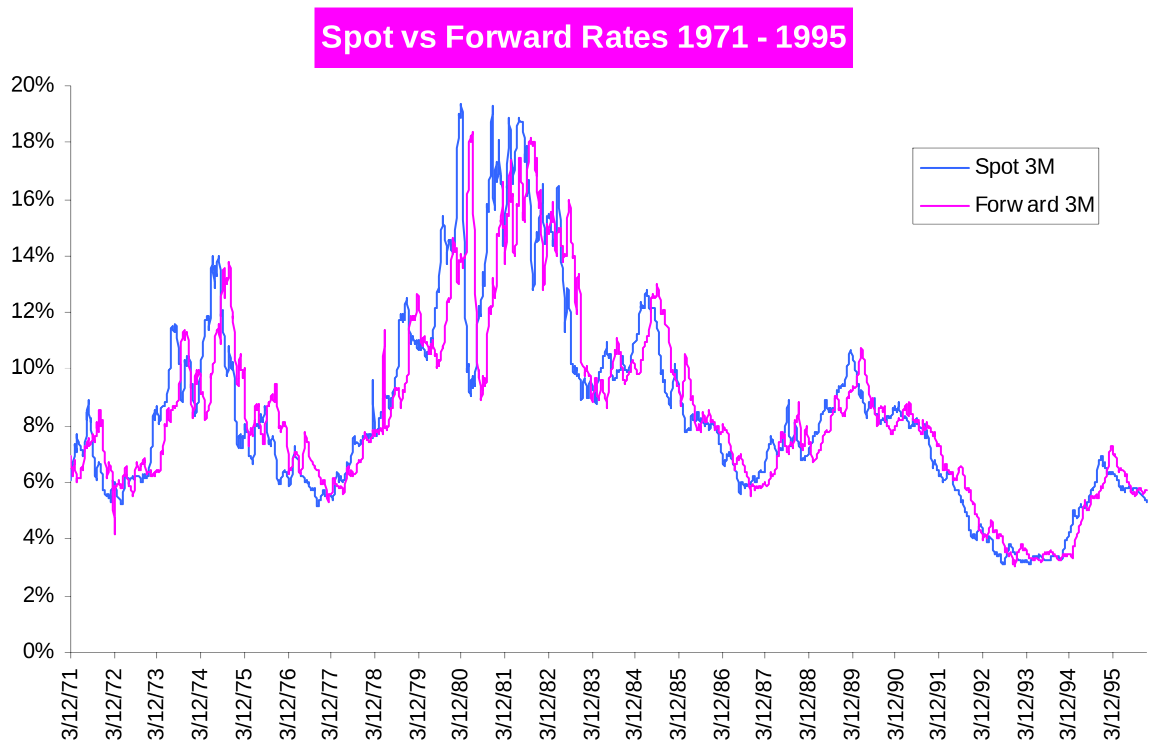
- Curve has +ve slope at low yields
- Curve has -ve slope at high yields
- Why?
- As yields *increase*:
  - Probability of Zero Loss *rises*
  - Risk of long-maturity issue relative to short-maturity issue *falls*
  - Investors buy the long end, yield curve flattens, then inverts

Shape of Yield Curve Changes with Yield Level



Empirical Tests

- Forward rates: biased or unbiased forecast of future spot rates?



Yield Curve Regression Model

- Regression Model
  - $S_t = a_0 + bF_{t-3} + \varepsilon_t$
    - S_t is spot rate at time t
    - F_t is 3m forward rate
    - ε_t is a white noise process: IID ~ No(0, σ^2)
- Expectations theory: b = 1
- Liquidity/risk theory: b < 1
  - Forward typically exceeds future spots rates
  - By an amount, which is the liquidity/risk premium
- See lab exercise

Regression Analysis

Regression Statistics	
Multiple R	89%
R Square	79%
Adjusted R Square	79%
Standard Error	1.56%
Observations	1294

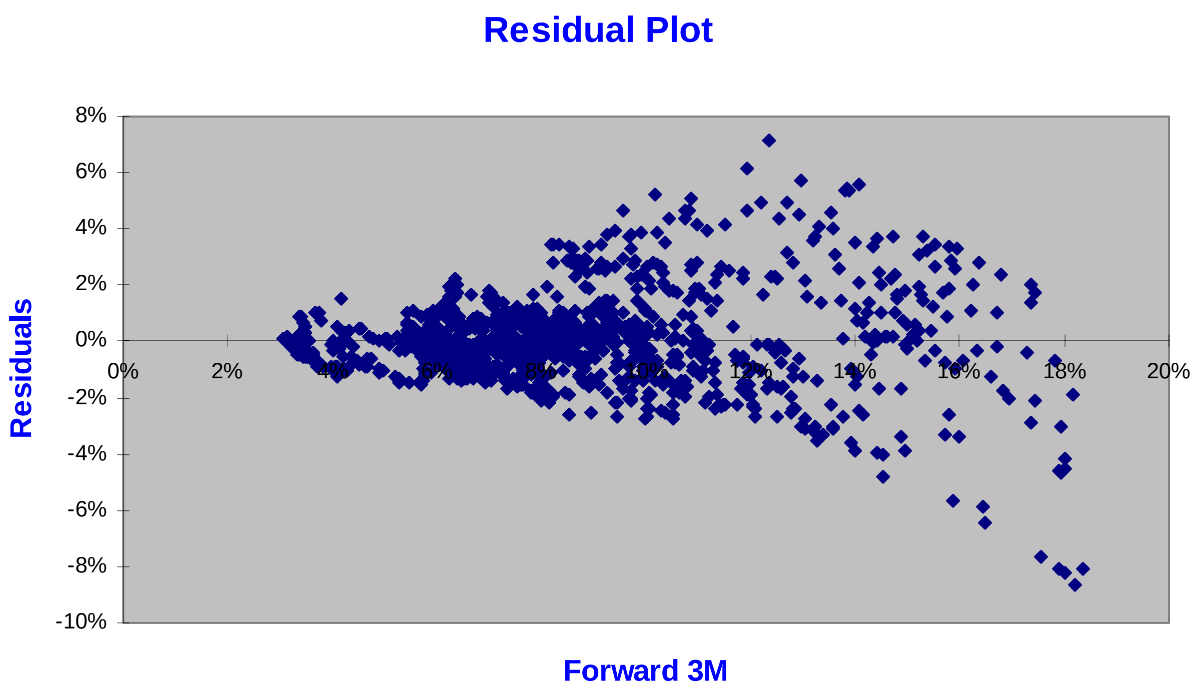
	Coefficients	*Std. Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*
Intercept	0.0046	0.0012	3.6847	0.0002	0.0021	0.0070
Forward 3M	0.9476	0.0138	68.7944	0.0000	0.9206	0.9746

b < 1: indicates expectations theory does not hold (reject at the 5% confidence level)

Residual Plot

- Evidence of violation of model assumptions
  - Residual variance is not constant

es



Other Empirical Evidence

- Fama (1976, 1984), Shiller (1979), Mankiw & Miron (1986)
  - Predictive power of forward rate is weak
  - Varies dramatically over sub-periods
  - Due to term premium (he conjectured)
- Buser, Kayroli, Sanders (1996)
  - Variation is due to the term premium
  - Model term premium using GARCH model
  - Adjusted forward rate is good predictor of spot

Summary: Yield Curve Theories

- Expectations Hypothesis
  - $F_T = E(S_T)$
  - Empirical evidence suggests otherwise
- Liquidity Preference
  - Investors require a liquidity premium to hold long term securities
  - Liquidity Premium: $L_T = F_T - E[S_T]$
  - Idea: why not try to capture L_T ?
- Risk Theory
  - Probability of zero loss
- Empirical evidence: favors liquidity/risk model