



Interest Rate Derivatives

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Investment Analytics



Interest Rate Derivatives

- Swap futures & forwards
- Caps/Floors/Collars
- Swaptions
- Callable & putable bonds
- Floating rate notes
- Inverse floaters
 - Range floaters
 - Inverse floaters
 - Step-up MTN's



Futures on Swaps

- Obligation to deliver a swap with known swap rate
- Traded on CBOT
 - Competes with IMM Euro\$ futures
- Similar uses: lock in future borrowing cost
 - Not useful for hedging swaps - standardization does not work in this context!



Forward Swaps

- Swap will start at a later date
- Swap rate fixed now
- Application: Anticipating financing
 - E.g. need swap in 6 months.

Pricing of Forward Start Swaps



- Pricing
 - Similar vanilla swap
 - Except that swap coupon set so that swap is expected to be fairly priced at the start date





Lab: Pricing a Forward Swap

- Currently 1-Mar-91
- Price a forward interest rate swap
 - Start 8-Feb-94
 - Tenor 2 years
 - Quarterly resets
 - \$100MM notional principal
- Excel Workbook: Yield Curve Modeling.xls
 - Labs: Forward swap

Solution: Pricing a Forward Swap

Two Year Q-Q Swap Starting on 8 Feb. 94

Principal: 100,000,000

Swap Coupon:

8.009%

8.043%

Dates	Days	Lin D.F	Fixed CF	PV	Fixed CF	CS Disc	PV
8-Feb-94	1075	0.8309	0.00	0.00	0.00	0.8310	0.00
8-May-94	1164	0.8154	1,979,939.84	1,614,359.95	1,988,342.34	0.8152	1,620,948.27
8-Aug-94	1256	0.7992	2,046,679.39	1,635,734.58	2,055,365.12	0.7989	1,642,061.88
8-Nov-94	1348	0.7831	2,046,679.39	1,602,692.58	2,055,365.12	0.7827	1,608,810.14
8-Feb-95	1440	0.7669	2,046,679.39	1,569,650.58	2,055,365.12	0.7668	1,576,100.19
8-May-95	1529	0.7521	1,979,939.84	1,489,096.49	1,988,342.34	0.7518	1,494,805.61
8-Aug-95	1621	0.7370	2,046,679.39	1,508,419.83	2,055,365.12	0.7366	1,513,937.29
8-Nov-95	1713	0.7219	102,046,679.39	73,670,043.59	102,055,365.12	0.7216	73,644,210.01
			Fixed CF NPV:	83,089,997.59			83,100,873.39
			FRN NPV	83,089,997.59			83,100,873.39
			Swap NPV	0.00			0.00



Caps, Floors & Collars

- Very popular instruments
 - Great demand for caps due to increased interest rate volatility
 - Market very liquid
 - Used to calculate market's view of interest rate volatility



Caps, Floors & Collars

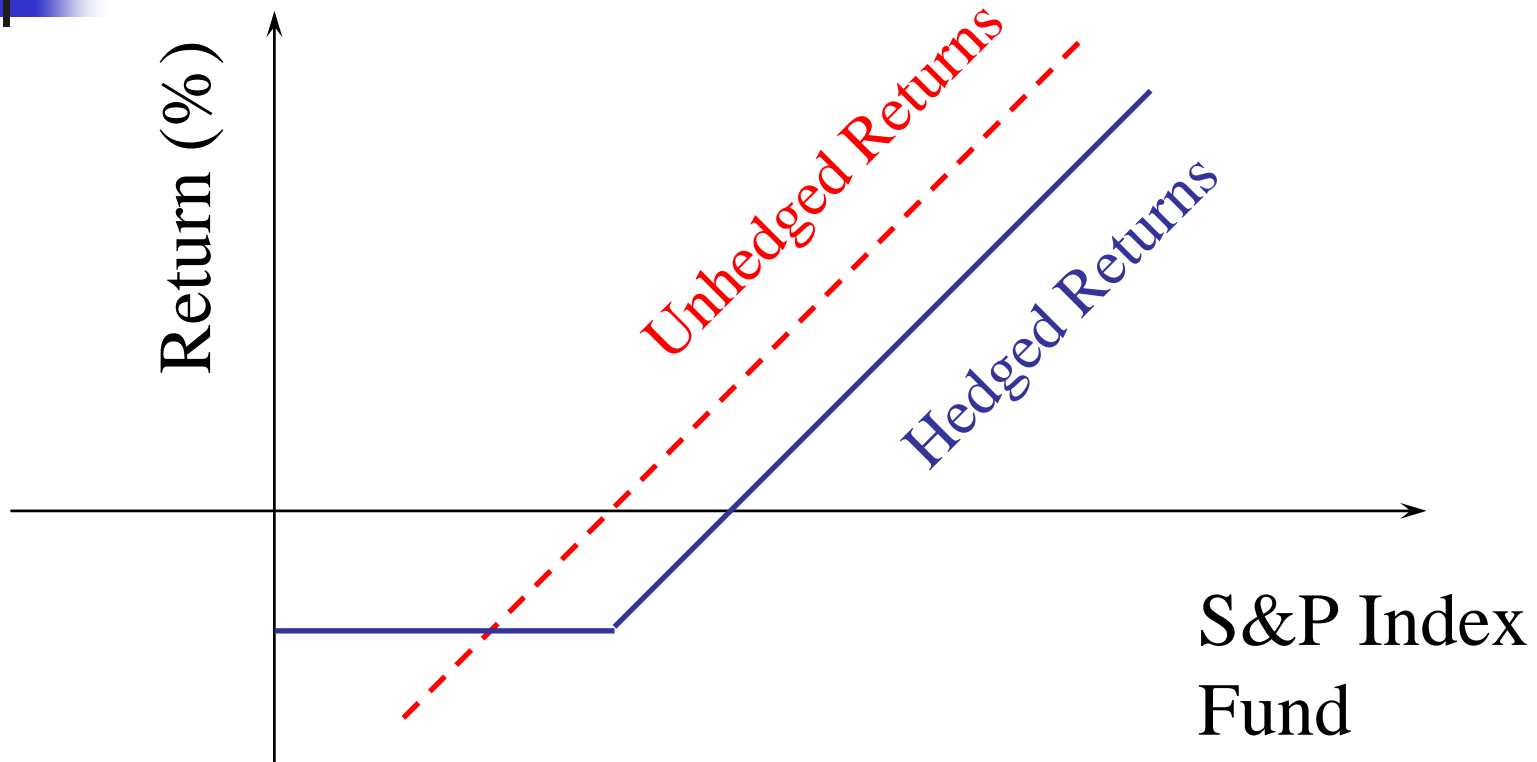
- Caps:
 - Limits Upside Risk / Gain
 - Series of interest rate call options
 - Caps interest rate, or equity index return
- Floor:
 - Limits Downside Risk / Gain
 - Series of interest rate put options
- Collar
 - Combines Cap & Floor
 - Fixes interest rate or equity index within a band



Equity Index Floor

- Limits the downside exposure
- Typical Portfolio Application:
 - User: stock portfolio manager
 - Hedge against market retracement after rally
 - Mechanism: Purchase S&P put option
- Swap Application
 - User: S&P receiver in equity swap
 - Set minimum return from the swap
 - Mechanism: Purchase series of European put options maturing on reset dates

Floor Example: S&P Index Fund



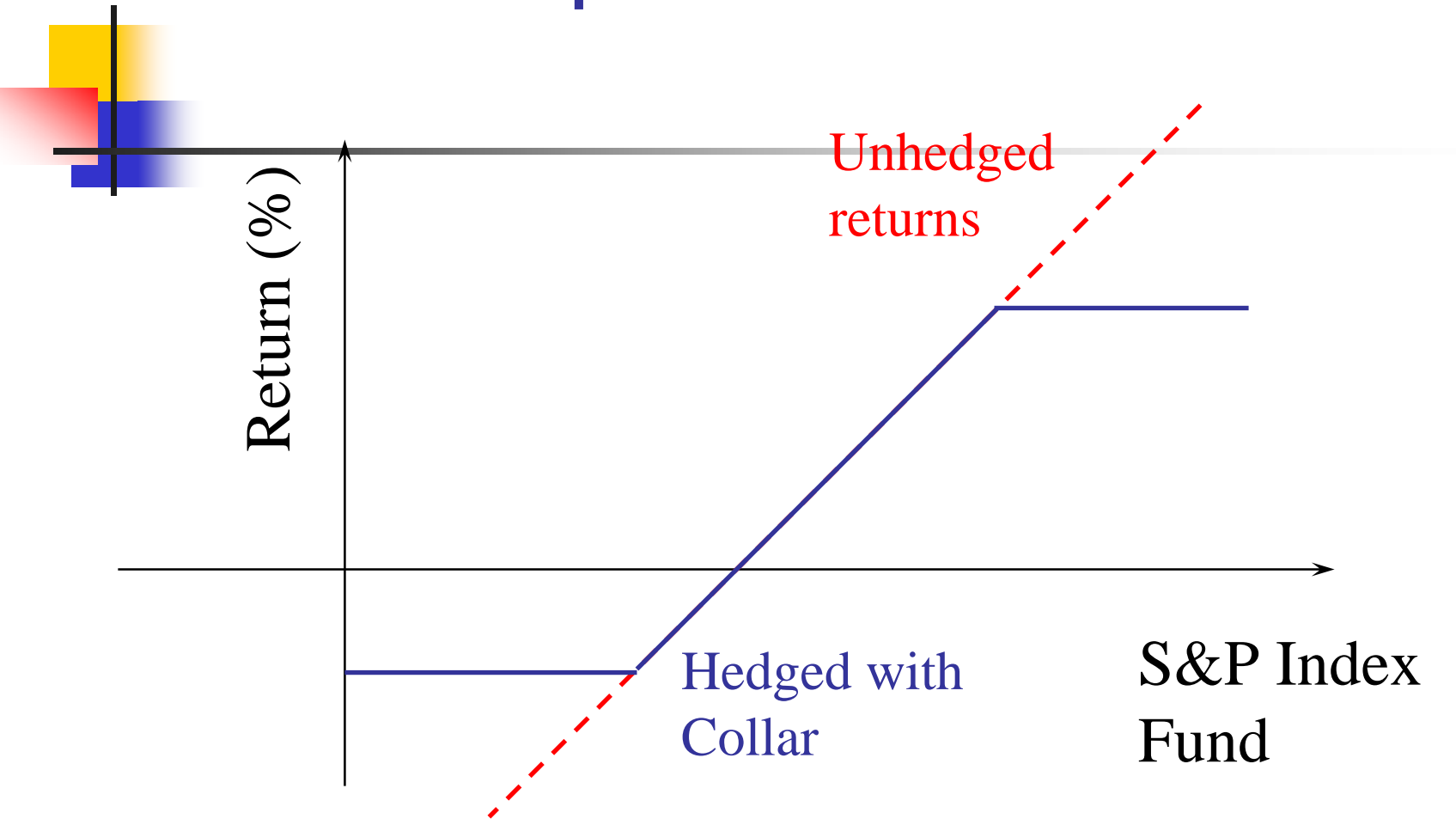
- $\text{Stock} + \text{Floor} = \text{Stock} + \text{Put} = \text{Call}$



Collar

- Combines Floor(s) and Cap(s)
 - Limits upside potential and downside risk
 - Sale of call(s) & purchase of put(s)
 - Premium from calls offsets cost of puts
- Zero Cost Collar:
 - Special case where Put Premium = Call Premium
 - Net cost is zero
- Typically used to lock in gains after market rally

Collar Example: S&P Index Fund





Interest Rate Caps

- Contract terms
 - Cap strike rate, R_x (7%)
 - Term (3 years)
 - Reset frequency (quarterly)
 - Reference rate (LIBOR)
 - Principal (\$1MM)
- Payment from seller to buyer:
 - $0.25 \times \$1\text{MM} \times \text{Max}(\text{LIBOR} - R_x, 0)$
 - In arrears, usually starts after 3 months
 - Each piece is called a “caplet”



Capped FRNs

- Floating Rate Note, but rate is capped
 - Can also set floor or collar
 - Growing market given events of 1994



Capped Swaps

- Swaps, but floating rate is capped
 - Typically, floor on rate as well
- Short Swap (pay floating)
 - hedge rising rates with a cap
- Long Swap (pay fixed)
 - Hedge falling rates with a floor
- Can embed caps, floors collars within swap
- Applies to any kind of swap (interest, equity)

Black's Model

Simple extension of Black-Scholes

- Originally developed for commodity futures
- Used to value caps and floors
- Let F = forward price, X = strike price
- Value of call option:

$$C = e^{-rt} [FN(d_1) - XN(d_2)]$$

$$d_1 = \frac{\ln(F / X) + (\sigma^2 / 2)t}{\sigma \sqrt{t}}$$

$$d_2 = d_1 - \sigma \sqrt{t}$$



Application to Caps

- Example: 1-year cap
 - NP = notional principal
 - R_j = reference rate at reset period j
 - R_x = strike rate
 - Then, get NP x $\text{Max}\{R_j - R_x, 0\}$ in arrears
 - But this is an option on R_j , not F_j
 - Use F_j as an estimator of R_j and apply Black's model to F_j
 - Previously was a forward price, now a forward rate



Black's Model for Caps

- Payments: $NP \times \text{Max}\{R_j - R_x, 0\}$ in arrears
- These are a series of options:
 - One for each R_j , the future spot interest rate
 - Called *caplets*
- Let $F_j =$ forward rate from j to $j+1$
- Value of caplet j :
 - Discount by $(1 + F_j)$ as paid in arrears

$$C = NP \times e^{-rt} [F_j N(d_1) - R_x N(d_2)] / (1 + F_j)$$



Black's Model - Example

- 8% cap on 3-m LIBOR ($R_x = \text{Strike} = 8\%$)
 - Capped for period of 3m, in 1-year's time
 - $f = 1\text{-year forward rate for 3m LIBOR is } 7\%$
 - $R_f = 1\text{-year spot rate is } 6.5\%$
 - Yield volatility is 20% pa
- See Excel workbook Swaps.xls
 - Black's Model - Example Spreadsheet

Black's Model - Example

Strike Term Fwd Vol R_f C/P E/A HCost
Rate

■ $C = \text{BSOpt} (8\%, 1, 0, 7\%, 20\%, 6.5\%, 0, 0, 0)$

■ Holding Cost

■ $\text{Hcost} = (R_f - d)$ for stocks, 0 for Futures

■ Cap Premium = 0.00211

■ Convert to %: $C\% = C \times t / (1 + F * t)$

■ $0.00211 \times 0.25 \times 1 / (1 + 7\% \times 0.25)$

■ Cap Premium % = 0.0518% (5.18bp)

■ So cost of capping \$1000,000 loan would be \$518

Black's Model - Equivalent Formulation in Terms of Price

- Cap = *Put option on price*
 - Equivalent of *call option on rate*
 - Useful if know price volatility rather than yield vol.
- $F = 1 / (1 + f \times t)$ is *forward price*
 - $F = 1 / (1 + 7\% \times 0.25) = 0.982801$
- $X = 1 / (1 + R_x \times t)$ is *strike price*
 - $X = 1 / (1 + 8\% \times 0.25) = 0.980392$
- Require *price volatility*
 - Other parameters as before



Lab: Cap, Floor & Collar pricing - Black's Model

- Excel Workbook, Swaps.xls
 - Black's Model - Worksheet
 - See lab writeup, written solution & solution spreadsheet
- Pricing a 1 year cap on 3-m LIBOR
 - Quarterly resets, so 4 caplets
 - Given price volatility, so use price formulation
 - Back out forward rates from spot rates

Solution: Cap, Floor & Collar pricing - Black's Model

CAP	03-Aug-96	15-Oct-96	15-Jan-97	15-Apr-97	15-Jul-97
Strike Rate		7.500%	7.500%	7.500%	7.500%
Strike Price		0.9850	0.9812	0.9816	0.9814
Caplet Cost		0.0299%	0.0686%	0.1531%	0.2190%
Cap Cost		0.470497%			
FLOOR	03-Aug-96	15-Oct-96	15-Jan-97	15-Apr-97	15-Jul-97
Strike Rate		4.736%	4.736%	4.736%	4.736%
Strike Price		0.9905	0.9880	0.9883	0.9882
Floorlet Cost		0.083%	0.123%	0.126%	0.138%
Floor Cost		0.470497%			



Limitations of Black's Model

- Problems:
 - Unbiasedness: empirically false
 - Option on R_j not same as option on F_j
 - Discount rate: fixed - but F_j variable
 - Rates both stochastic and fixed!
 - If applied to prices the additional problem
 - Assumes prices can be any positive number
 - But can't exceed value of future cash flows



Swaptions

- Option on a swap
 - Right to enter a swap at known fixed rate
 - *Receiver Swaption*: right to receive fixed
 - *Payer Swaption*: right to pay fixed
 - Essentially a bond option with strike = notional
 - When exercised, will exchange floating for fixed
 - Fixed payments correspond to a bond



Swaptions: Applications

- Anticipatory financing
 - manage future borrowing cost
 - e.g., bidding for a contract; if get it, will want to swap, lock in today's rates
 - e.g., option to build a plant, so buy swaption
- Change terms of existing swap
 - Cancelable, putable swap



Example Swaption Strategies

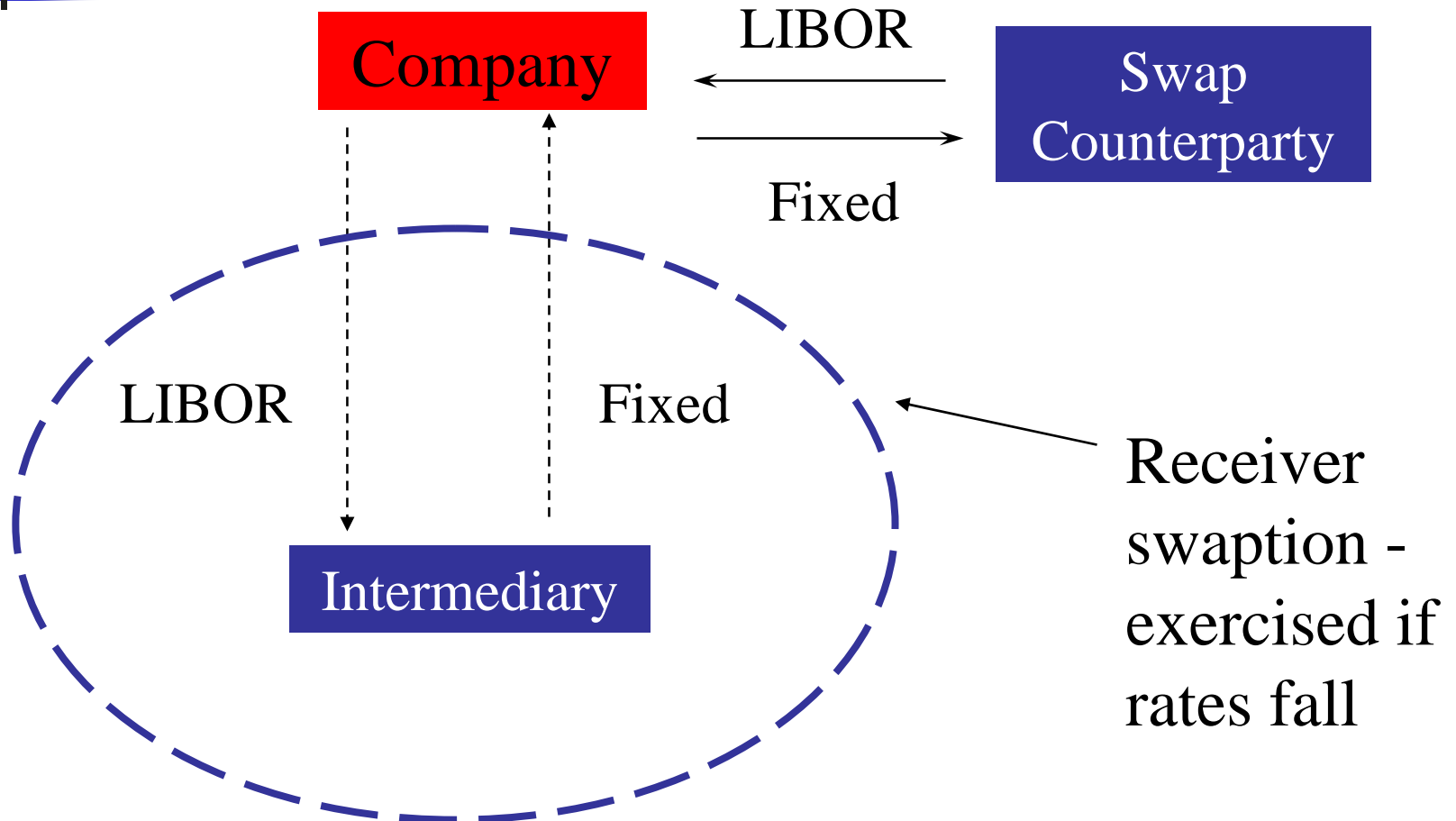
- Floating rate borrower doesn't believe rates will fall. How can he reduce funding cost?
 - Sell floor = sell receiver swaption
- Borrower wants to delay decision to lock in rates at 9% for 1 year
 - Buys payer swaption
- Speculator believes rates will rise next year
 - Buy payer swaptions
 - Sell receiver swaptions



Swaptions: Creating Swap Variants

- **Extendible Swap:**
 - Fixed payer can extend life of swap
 - = payer swap + payer swaption
 - e.g. uncertain about term of financing
- **Putable Swap: can cancel swap**
 - = payer swap + receiver swaption
 - e.g. financing need disappears
- **Cancelable Swap: counterparty can cancel**
 - = payer swap - receiver swaption
 - e.g. credit rating worsens

Putable Swap





Swaptions & Asset Swaps

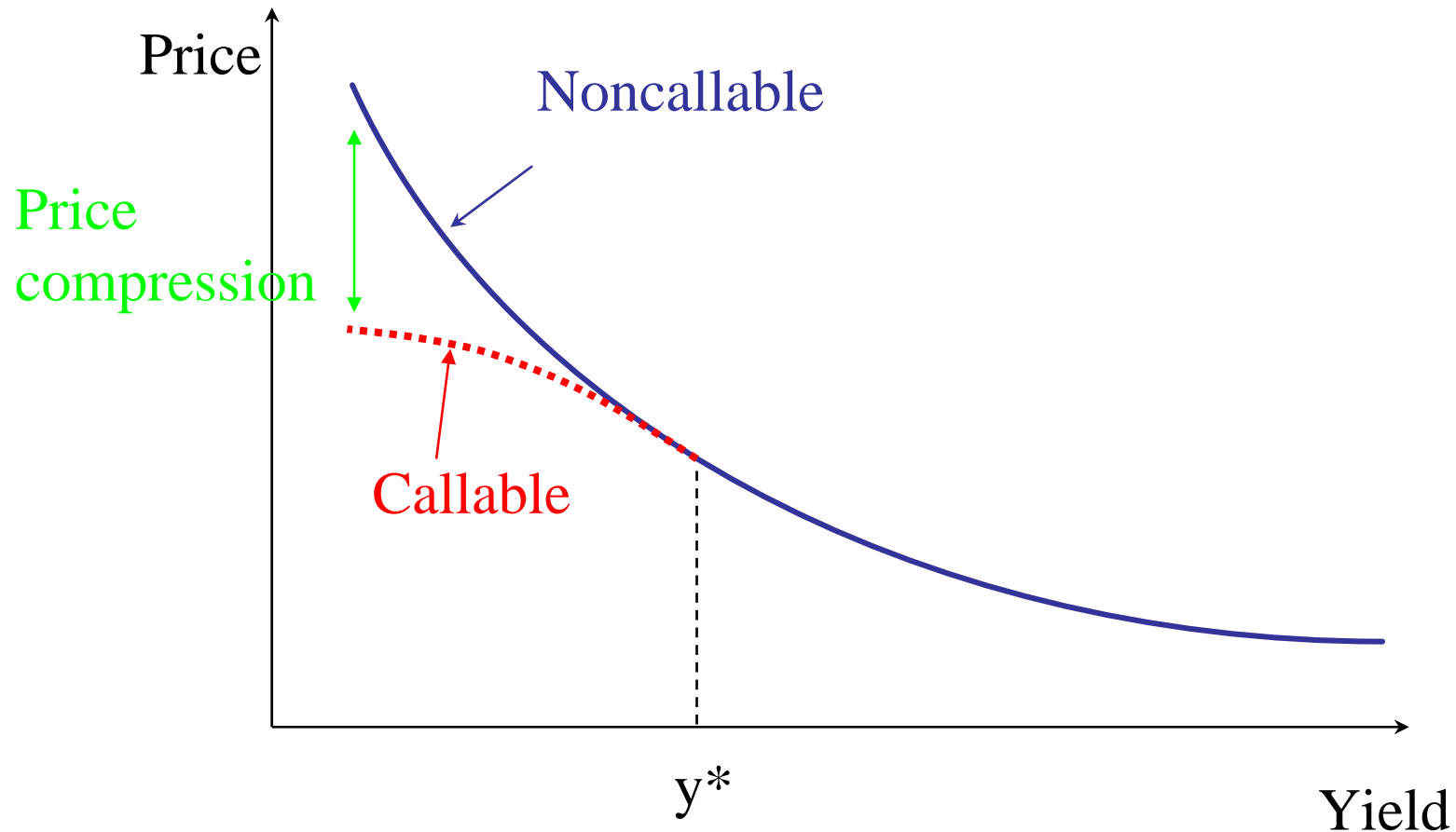
- Another application: callable bonds
- Asset swap: issue fixed-rate bond, swap to floating
- If issuer calls bond (most corporates are callable), he is left with swap
- Swaption: allows swap to be cancelled
 - Payer swaption



Bonds with Embedded Options

- Callable bonds
 - Give issuer right to redeem the issue
 - Usually at par, after *non-call period*
- Call risk
 - As yields fall likelihood increases that issuer will call
 - Investor faces reinvestment risk
 - Compensated by higher potential yield
- Puttable Bonds
 - Investor has right to sell the bond back to issuer
 - Investor has purchased a put option, hence lower yield

Price-Yield Relationship for Callable Bonds





Features of Callable Bonds

- Price compression
 - Limited price appreciate as yields decline
- Negative convexity
 - As yields fall:
 - Duration *increases* (as for non-callable)
 - Then duration *decreases*



Components of a Callable Bond

- Callable Bond = Straight Bond - Call Option
 - Higher yield due to call option premium received
 - As yields decline, value of call option increases, hence price compression
- Pricing of callable bonds
 - Price straight (i.e. non-callable) bond
 - Price call option
 - Interest rate option model
 - $\text{Price}_{\text{CB}} = \text{Price}_{\text{NCB}} - \text{Price}_{\text{CO}}$



Market Conditions & Callable Bonds

- Flat yield curve
 - Very little yield pickup extending along the curve
- High implied volatility
 - Options are expensive
- Tight credit spreads
 - Minimal yield pickup from riskier paper
- Implications
 - Investors sell calls to issuers to enhance yield
 - Callable bonds
 - Issuers attempt to arbitrage



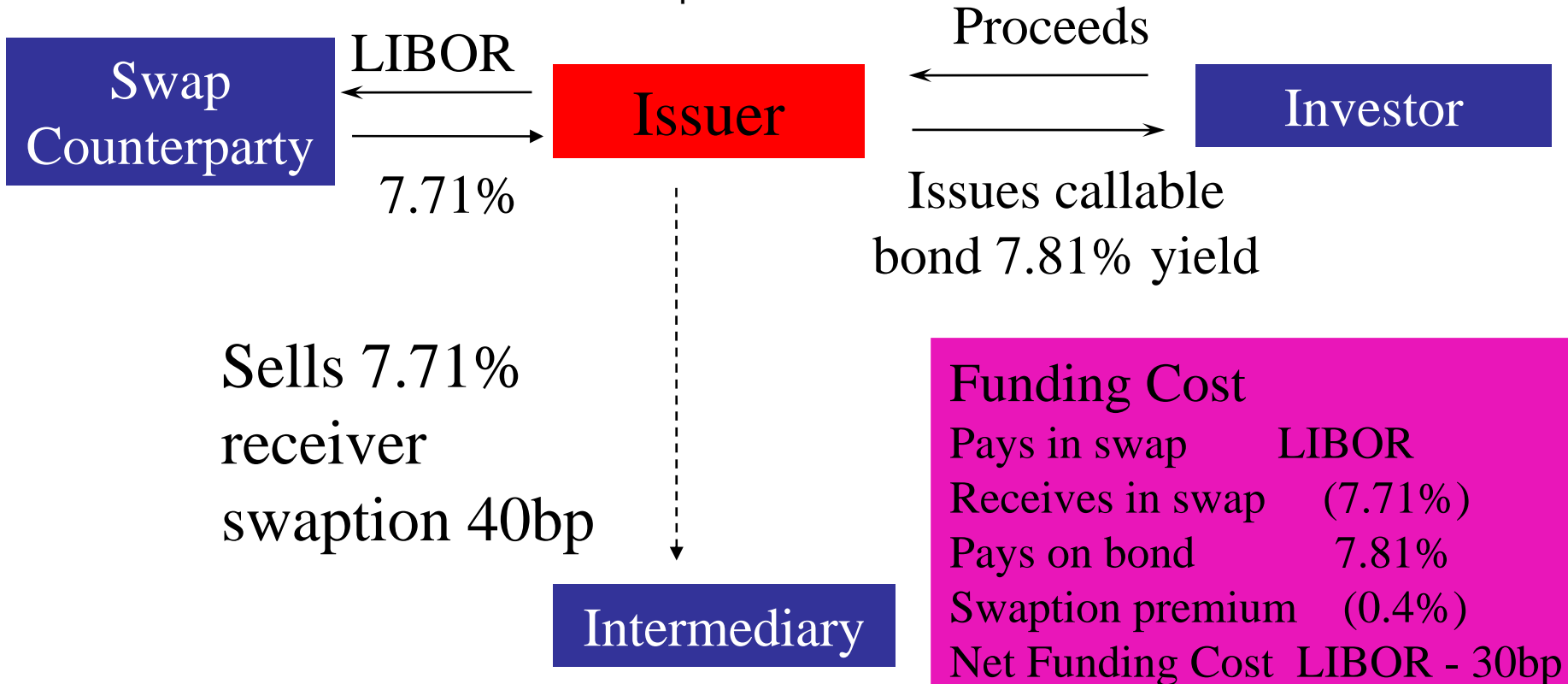
Swaption Arbitrage with Callable Bonds

- Strong demand for receiver swaptions
 - From swap buyers (paying fixed), concerned about rates falling
- Supply: from issuers of callable bonds
- Arbitrage
 - Issuer sells receiver swaption
 - Swaption premium > extra yield on callable bond
 - Nb match term of call provision to term of swaption
 - e.g. callable after 2 years, sell 2-year European swaption

Swaption Arbitrage Example

- Action

- Rates rise: no action; swaption not exercised
- Rates fall; swaption exercised; bond called



Market Conditions & Puttable Bonds



- Flat yield curve
 - Very little yield pickup extending along the curve
- High implied volatility
 - Options are expensive
- Wide credit spreads
 - Investors heavily invested in riskier paper
- Implications
 - Investors willing to give up some yield in return for valuable put options
 - Issuers - seeking way to reducing financing cost



Putable Bond Arbitrage

- Issue
 - 10 year bonds putable after 3 years
 - 8% yield
- Investor
 - Will put bonds if rates $>$ 8% in 3 years
- Issuer
 - Hedges refinancing risk by purchasing 3 year 8% payer swaption
- Arbitrage
 - If cost of put sold $>$ cost of swaption purchased



Floating Rate Notes

- Like the floating side of a swap
 - A way to issue floating rate debt
- Value:
 - On reset dates, equal to par
 - Suppose 1 period
 - Current rate is r
 - In one period get $F(1+r)$
 - Present value is F
 - For longer periods, do by induction
 - In between reset dates: accrued interest calculation



FRN Types

Type	Description	Embedded Option	Investor View
Straight	pays index + spread	none	rising rates
Collared	index within range	cap + floor	rates rising less than fwds
Range	payment only if index in range	binary options	volatility less than priced



Inverse Floaters

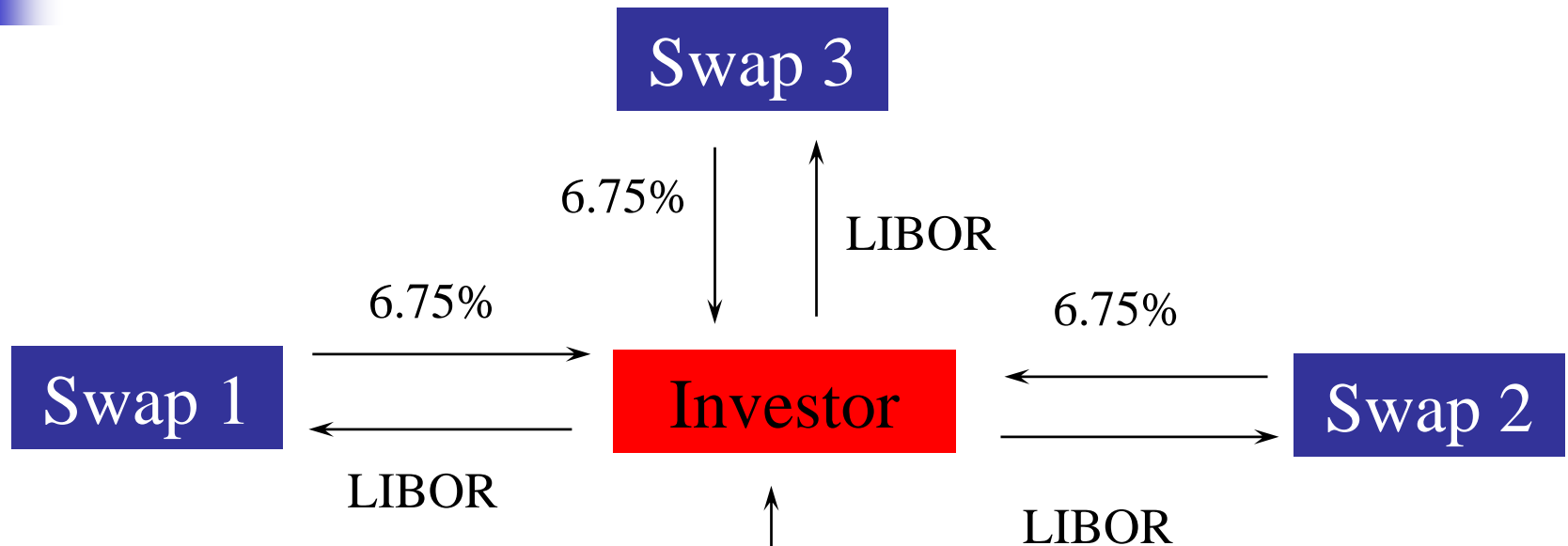
- Floating rate note, but coupon changes inversely with interest rates
 - Coupon (floating rate) rises as rates fall
 - Betting on falling rates
 - Used by banks to hedge against falling deposits if rates fall
 - In steep yield curve of 1992-93, betting on mean reversion
 - Used by Orange County
- Sample Contract
 - Floating rate = $12\% - \text{LIBOR}$, not < 0
 - Floating rate = $20\% - 2 \times \text{LIBOR}$, not < 0
 - Here, leverage is 2



Constructing an Inverse Floater

- Construction: from fixed income security
 - Split into two tranches: floater plus inverse floater
- Example: \$100MM, 7.5% fixed rate bond
 - Floater: LIBOR + 1%
 - Inverse Floater: 14% - LIBOR
 - Then \$100MM split into two tranches of \$50MM:
 - $(1/2)(\text{LIBOR} + 1\%) + (1/2)(14\% - \text{LIBOR}) = 7.5\%$
 - Floater capped at 15%

Double LIBOR Inverse Floater



Cash Flow Summary

Receive on swaps:	+ 3 x (6.75%)
Pay on swaps:	- 3 x LIBOR
Receive on FRN	+ LIBOR
Net:	20.25% - (2 x LIBOR)



Rationale for Structured Notes

- Low interest rates
 - Investors seeking yield enhancement
- High implied volatility
 - Arbitrage opportunity
- Steep yield curve
 - Implies higher rates in future
 - Opportunity to market securities to investors with contrarian opinion



Range Floaters

- Typical Structure
 - 4 Year FRN
 - Coupon LIBOR + 50bp
 - Only paid if LIBOR in range
 - Year 1-2 range 5% - 6%
 - Year 3-4 range 6% - 7%
 - Ranges increase due to upward sloping forward curve
- Investor has written series of binary calls and puts
 - Compensated by higher spread
 - Taking advantage of high implieds
 - Betting that volatility will be lower than anticipated



Callable Step-Up MTN

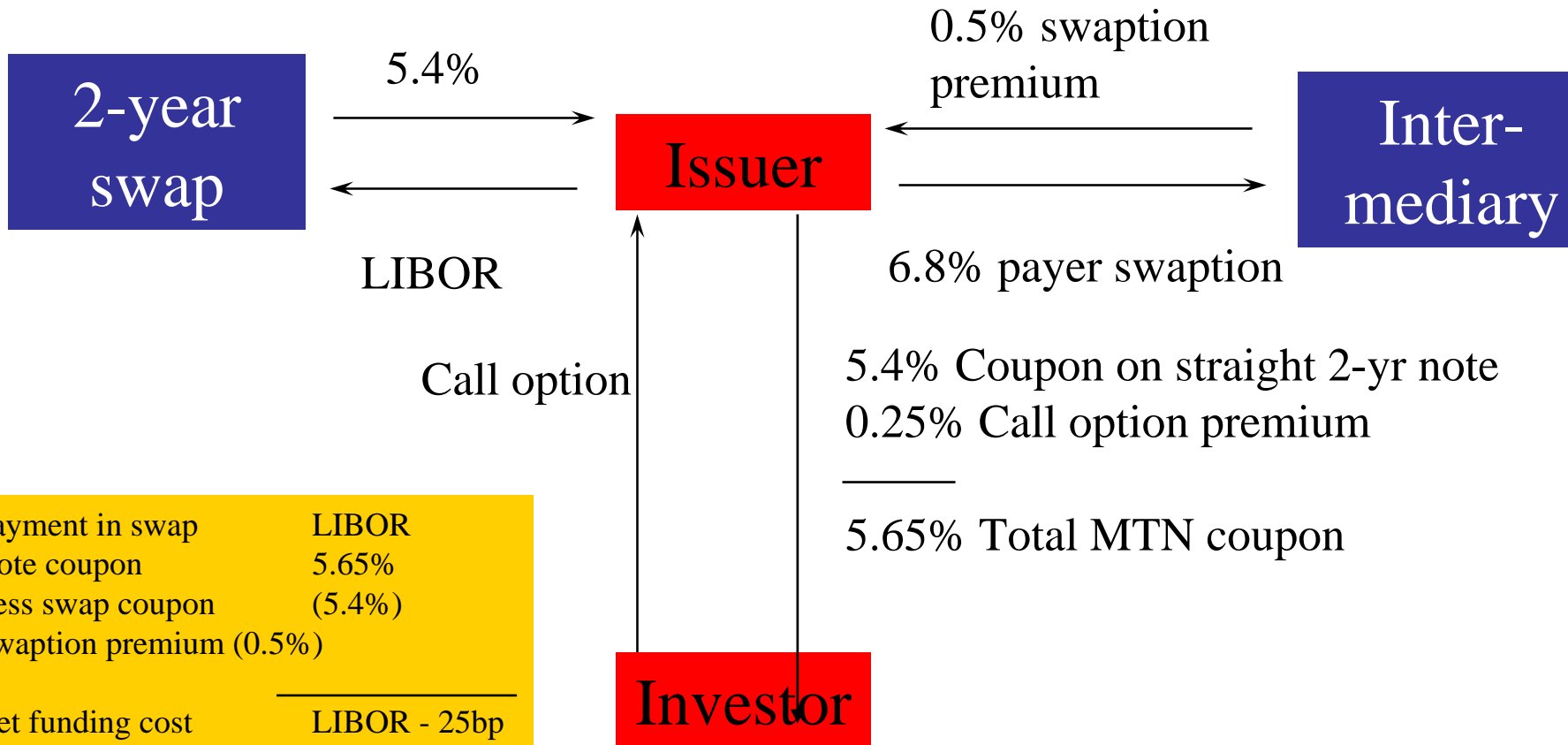
- Maturity 3 years
- Coupon (semiannual)
 - 5.65% years 1 and 2
 - NB coupon on comparable vanilla 2-year debt is 5.4%
 - 6.8% year 3
- Callable at par after 2 years
- What is the target investor group?
 - Investors looking for yield enhancement in the three year sector;
or
 - Investors who do not believe rates will rise significantly



Target Investor Group

- Investors in the three year sector
 - Given steep yield curve, better off in straight 3 yr notes
- Investors who think rates are going to rise:
 - Better off with vanilla FRN
- Attractive to 2-year sector investors
 - Looking for yield enhancement
 - Don't believe rates will rise significantly

Step-Up MTN Structure





Step-Up MTN Action Matrix

Issuer

Intermediary

Rates remain
below step-up rate

Calls note;
Refinances at
lower rate

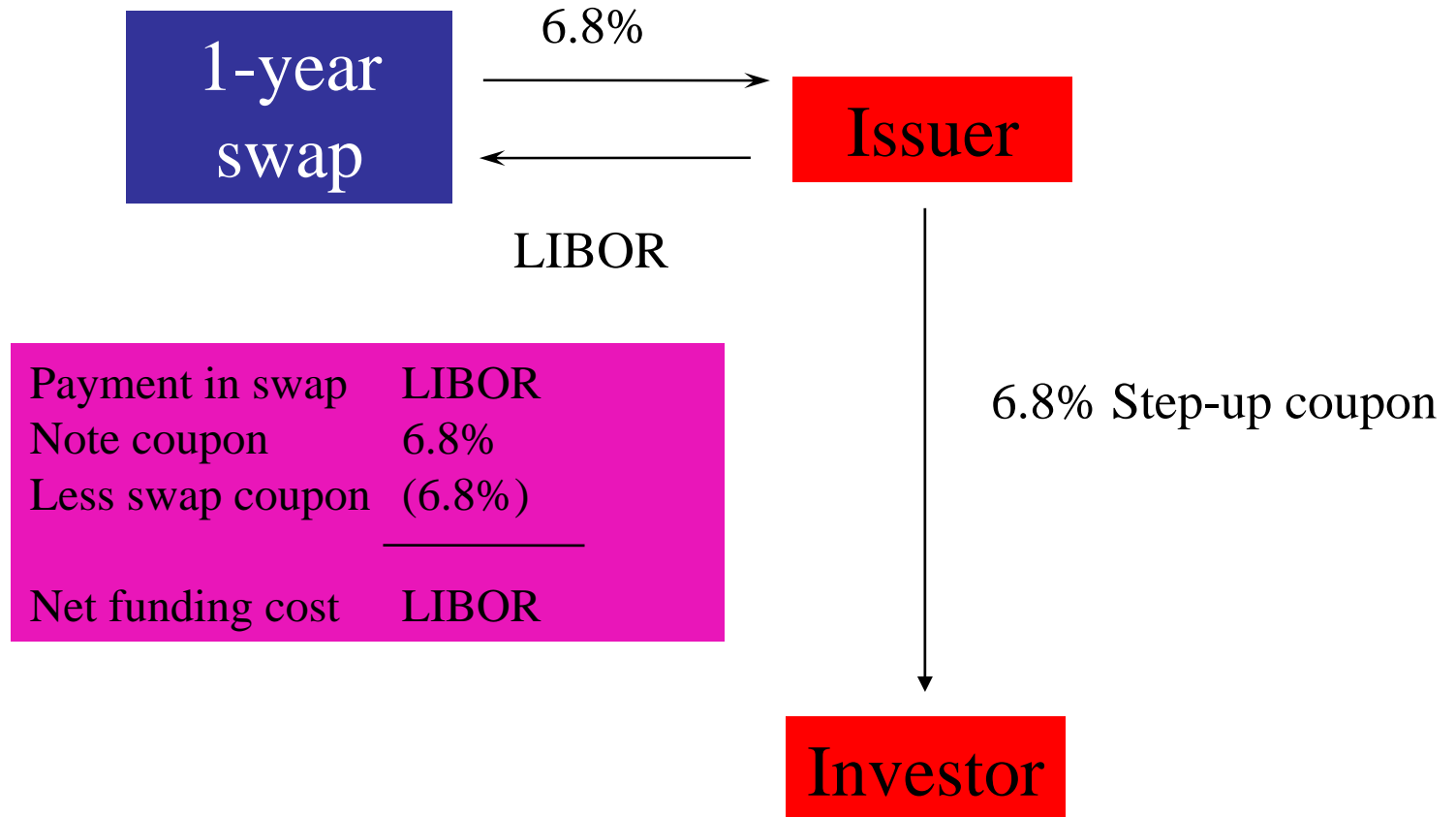
Lets swaption expire

Rates rise
above step-up rate

Does not call;
Pays step-up
coupon

Exercises swaption;
Pays fixed

Year 3: Rates rise above step-up coupon





Step-Up MTN: Investor's Perspective

- Investor has sold call option for 25 bp
- Issuer has resold option for 50bp
 - As payer swaption
- What is investor's motivation?
 - Depends on view of rates



Lab: Step-Up MTN

- Question 1
 - What is YTM of the issue?
- Question 2
 - What is breakeven rate (in year 3)?
 - Above this rate investor would be better off purchasing a 2yr note & reinvesting in yr 3



Solution: Step-Up MTN

- YTM on note
 - 6.011%
- Breakeven rate
 - 7.35%
- If rates rise beyond this level at end year 2, investor would be better off purchasing 2yr note and reinvesting



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- Inverse floaters
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 - Step-up MTN's