Risk Management

Value at Risk

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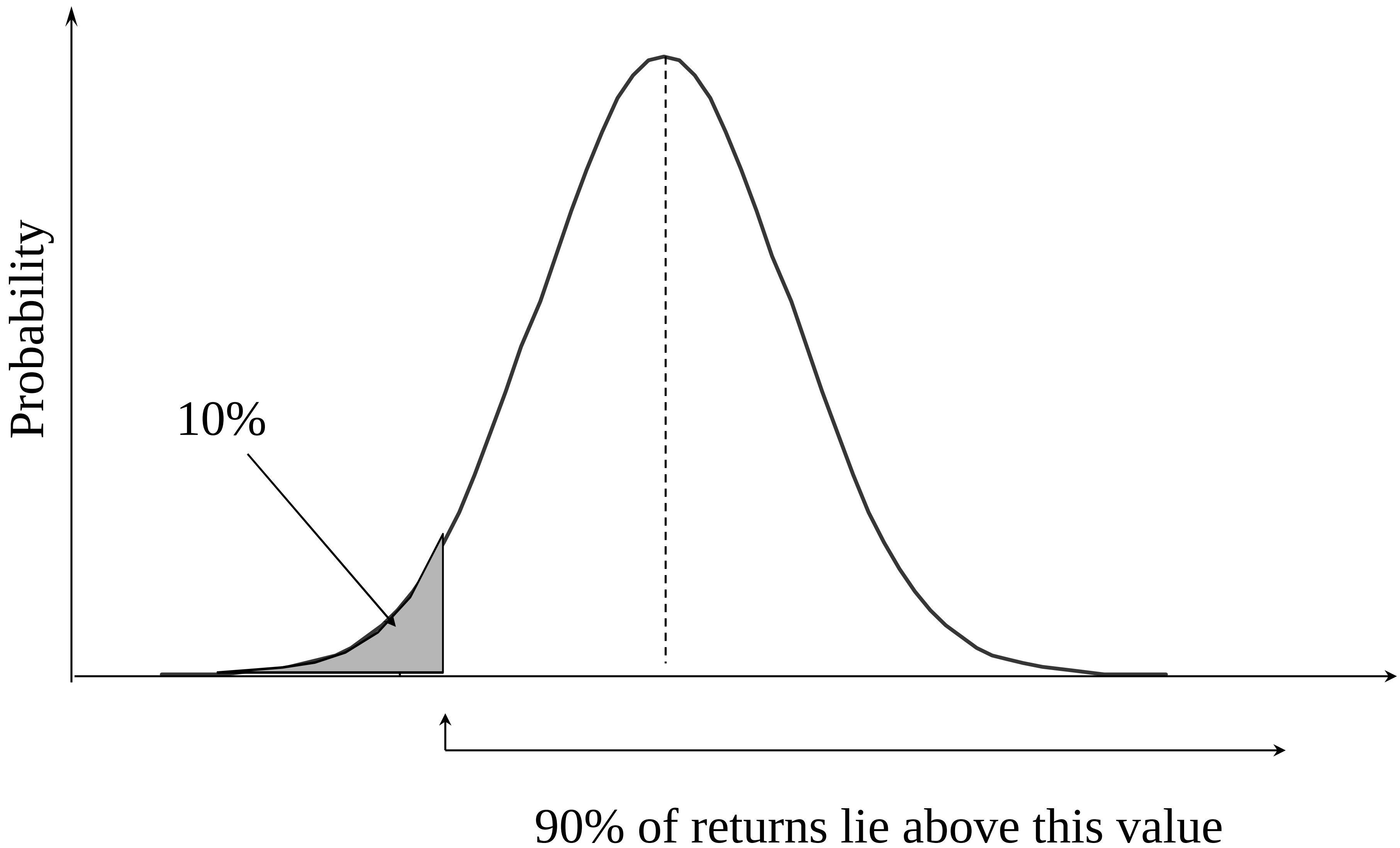
Agenda

- Statistical Distributions
- Value at Risk
- Factors affecting VAR
- VAR & Derivatives
- The Delta-Normal Model
- Model testing
- Other VAR models

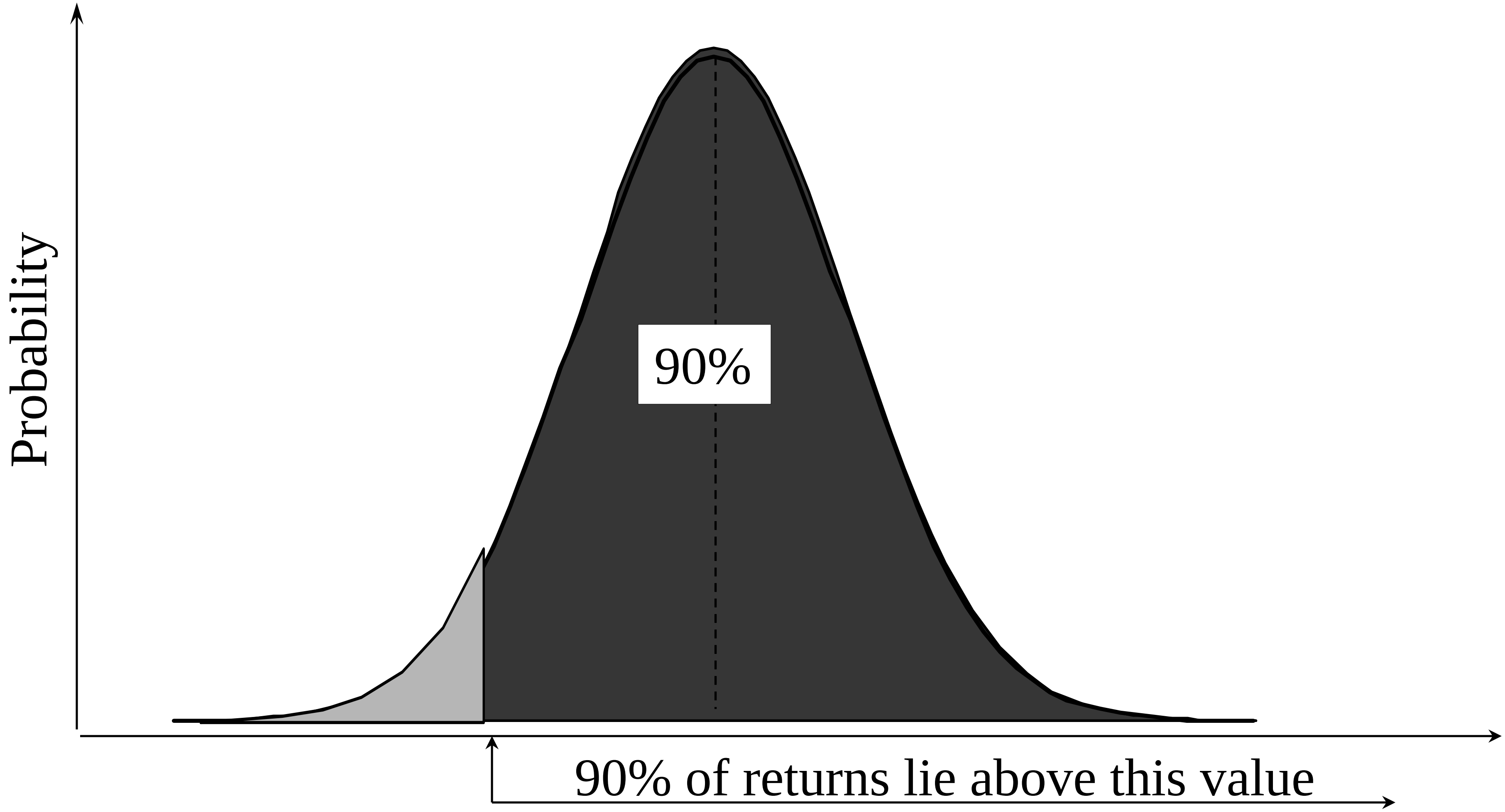
Confidence Intervals

- You can use normal probability tables to find:
  - ✓The probability of achieving a given minimum return
- Confidence Intervals
  - ✓Turn this idea round
  - ✓Given a specified probability, what's the minimum return?
- Example: 90% confidence interval
  - What minimum rate of return can I expect to achieve 9 times out of 10?

Confidence Intervals



Confidence Factor

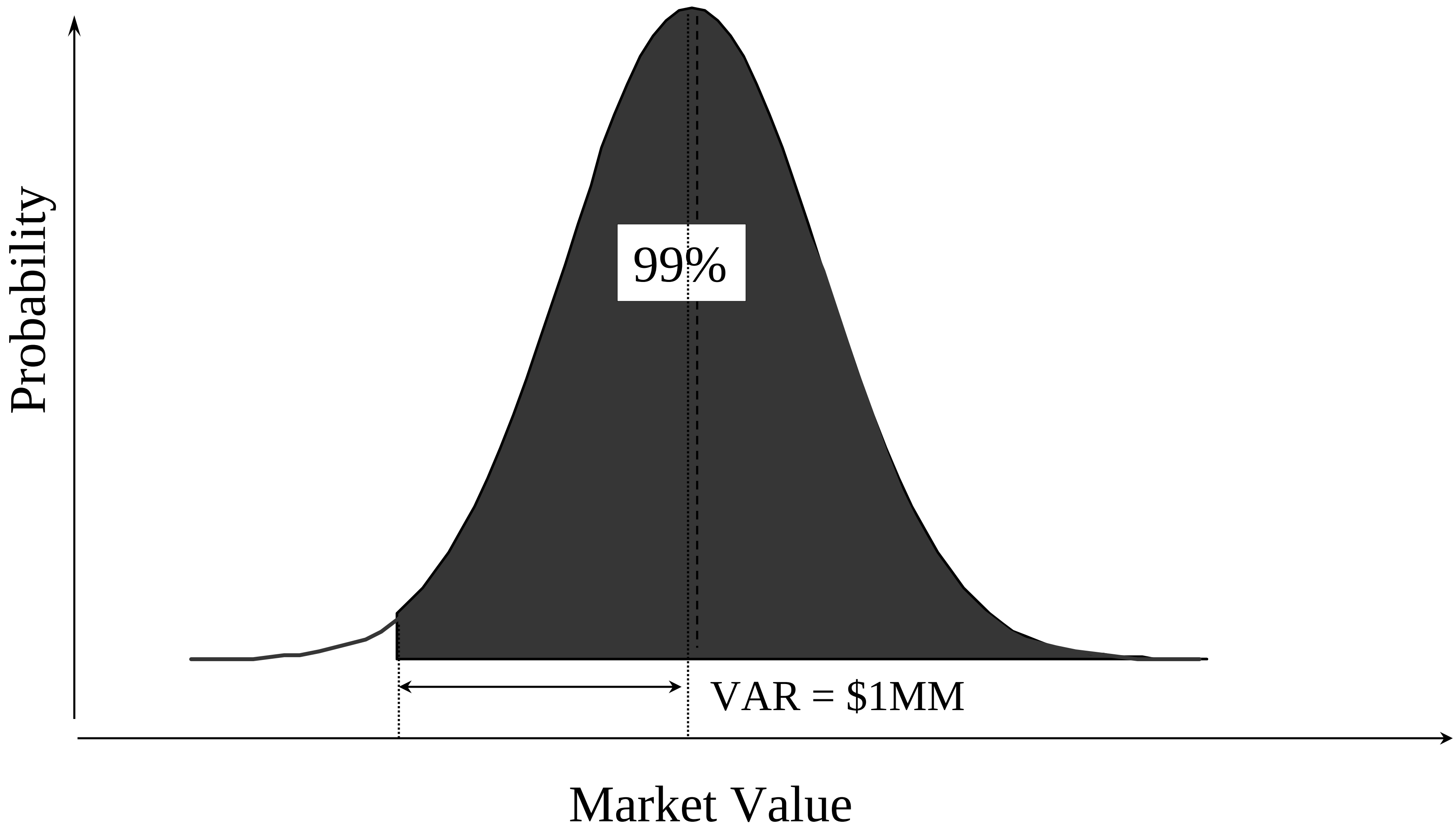


CF = 1.28

Value at Risk

- Measures the *maximum expected loss*
  - For a given *holding period*
  - For a given *confidence level*
    - ✓ chosen by the portfolio manager
- Example
  - Portfolio with daily VAR of $1MM with 99% confidence
    - ✓ There is a 1% chance that the portfolio will lose more than $1MM in the next 24 hours

Value at Risk



Using VAR for Simple Risk Comparisons

- VAR gives a simple yardstick:
  - Portfolio A: VAR $30MM over 30 days, 99% conf.
  - Portfolio B: VAR $10MM over 30 days, 99% conf.
  - If both A & B have same value, which is riskier?

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VAR Question

- A VAR of $1MM, with 87% conf means:
  - Portfolio is expected to return at least 13%?
  - Portfolio manager is 87% sure he will earn less than $1MM?
  - Portfolio manager is 87% sure he will not lose more than $1MM?
  - There is a 13% chance that the portfolio will earn more than $1MM?

Calculating VAR

- Portfolio VAR
  - VAR = *Market Value x Confidence Factor x Volatility*
- Example:
  - Portfolio value $10MM
  - Daily volatility 5%
  - Confidence level = 88%
    - ✓ CF = 1.17
  - DAILY VAR = $10MM x 1.17 x 0.05 = $585,000
    - ✓ Interpretation: there is a 12% chance the portfolio will lose more than $585,000 in a day

Portfolio Volatility & VAR

- Use historical correlations to estimate portfolio volatility

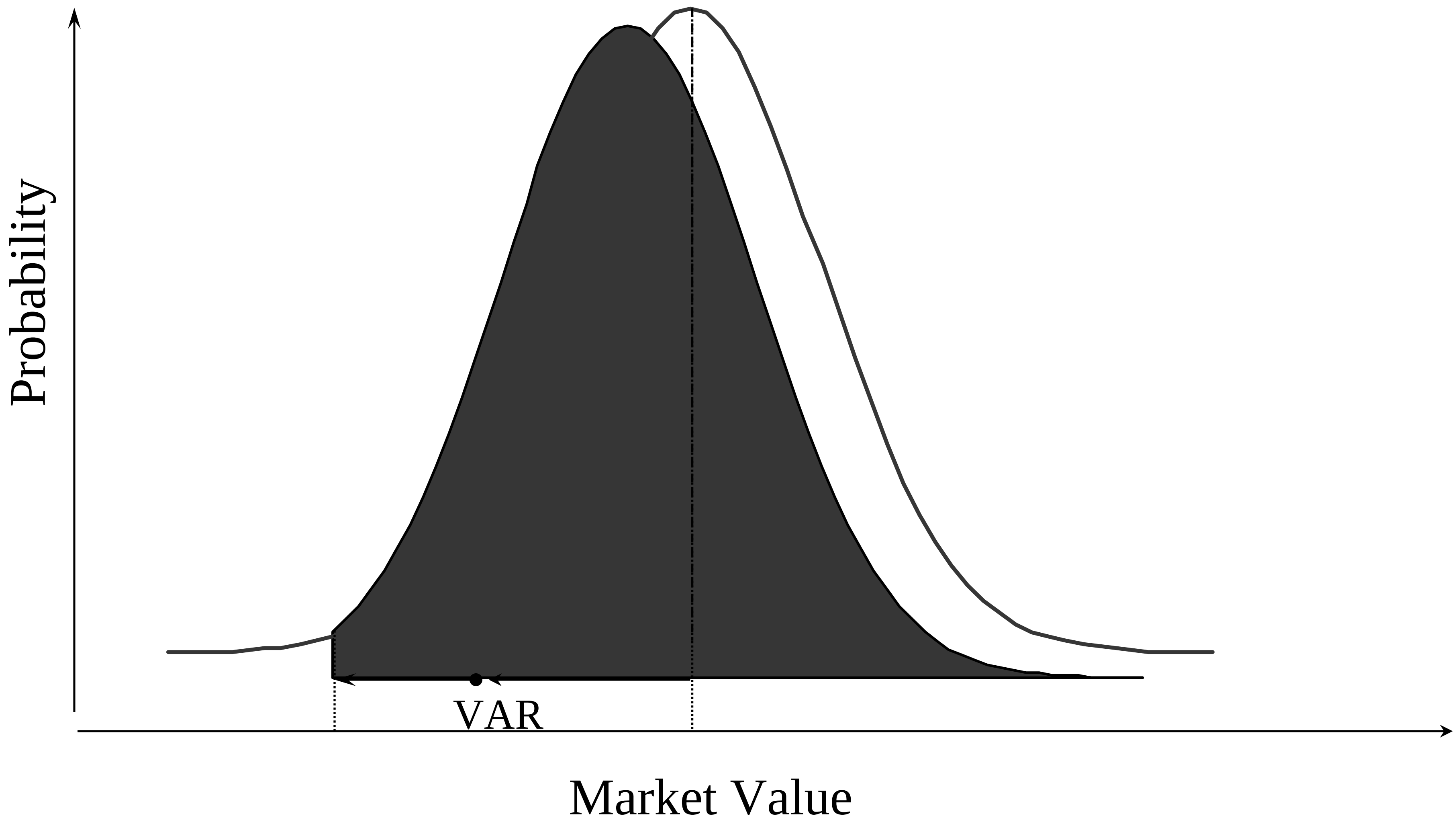
$$\sigma_p = \sqrt{[\sum \omega_i^2 \sigma_I^2 + \sum_I \sum_{I \neq J} \omega_I \omega_J \rho_{IJ} \sigma_I \sigma_J]}$$

  - RiskMetrics (JP Morgan)
    - ✓provides estimates of volatilities and correlations of returns (daily, monthly) using 75 day data

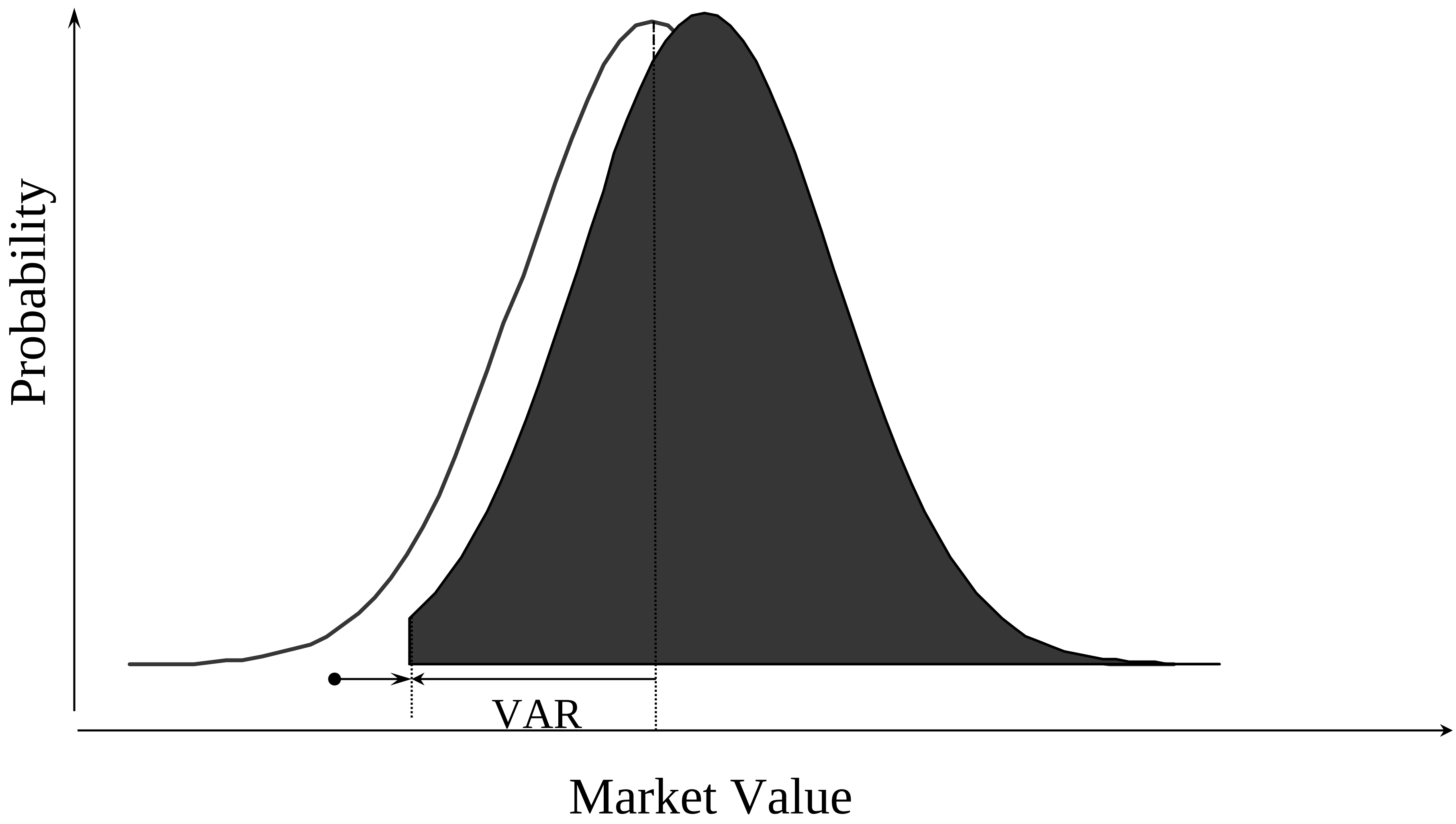
Factors Affecting VAR

- VAR reflects 3 key aspects of Risk Mgt.
  - The *probability distribution* of portfolio returns
    - ✓ Parameters: mean, volatility
  - The degree of *risk aversion* of the portfolio manager
  - The *holding period* of concern

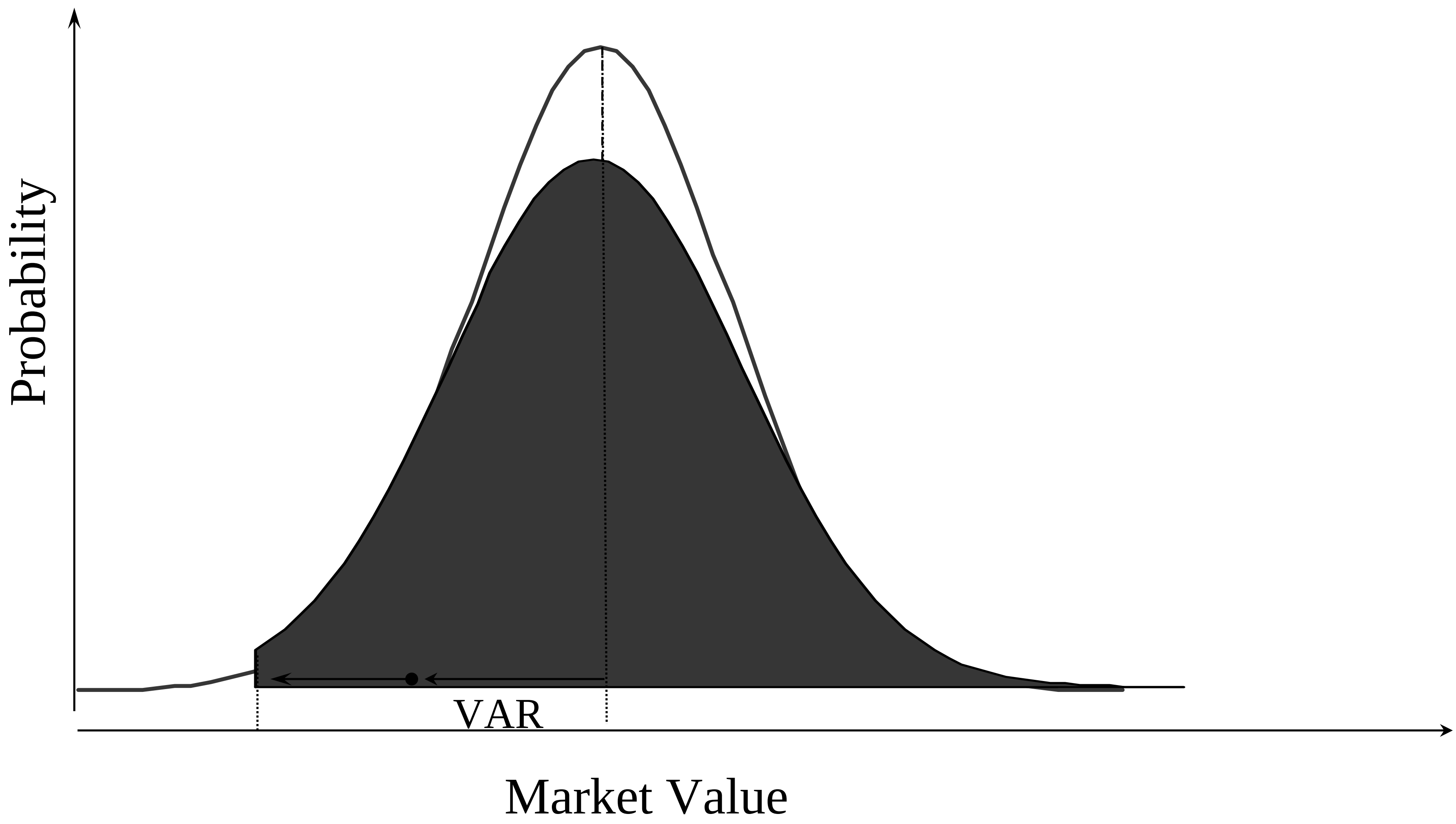
VAR & Lower Average Returns



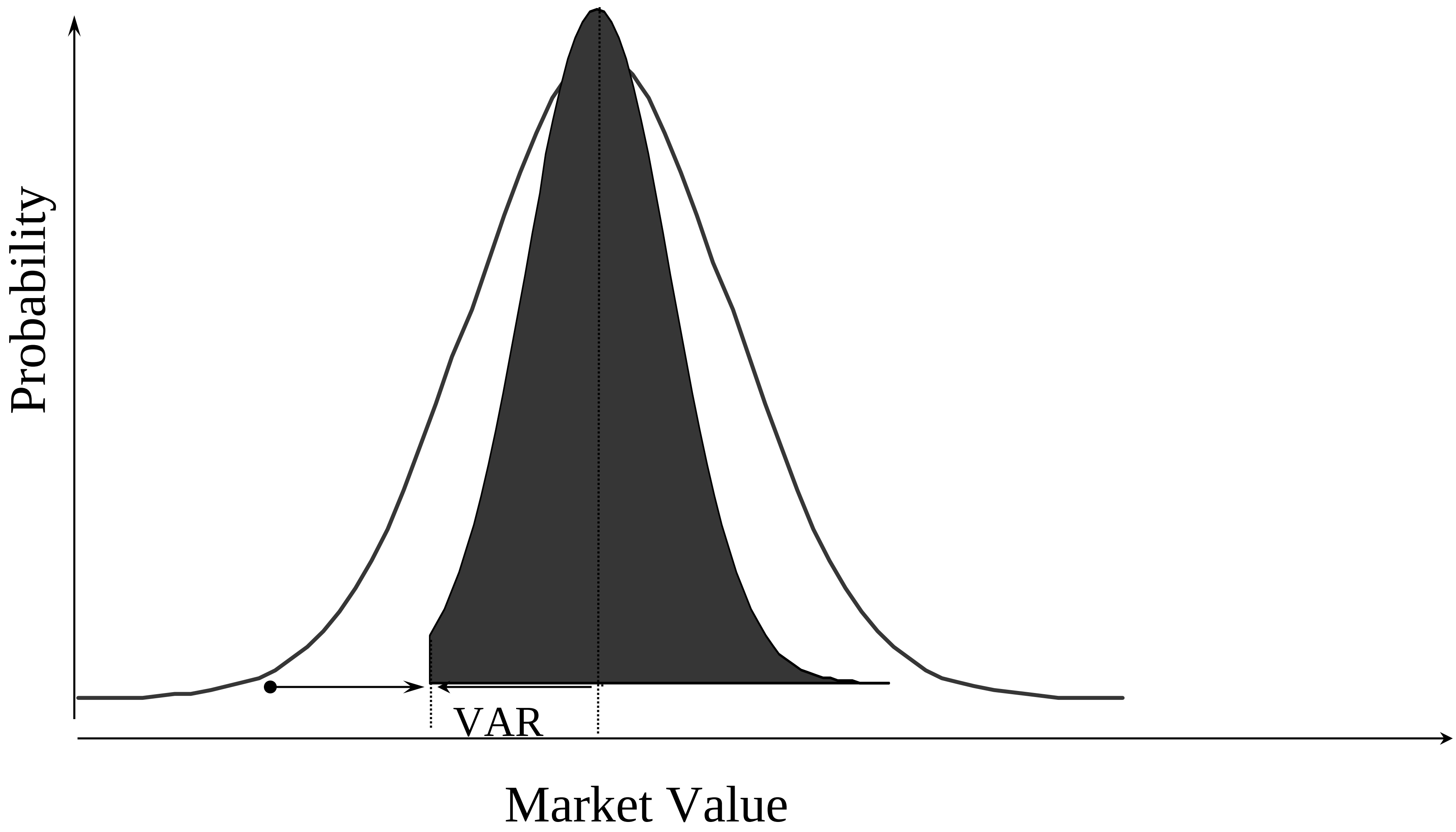
VAR & Higher Average Returns



VAR & Higher Volatility



VAR & Lower Volatility



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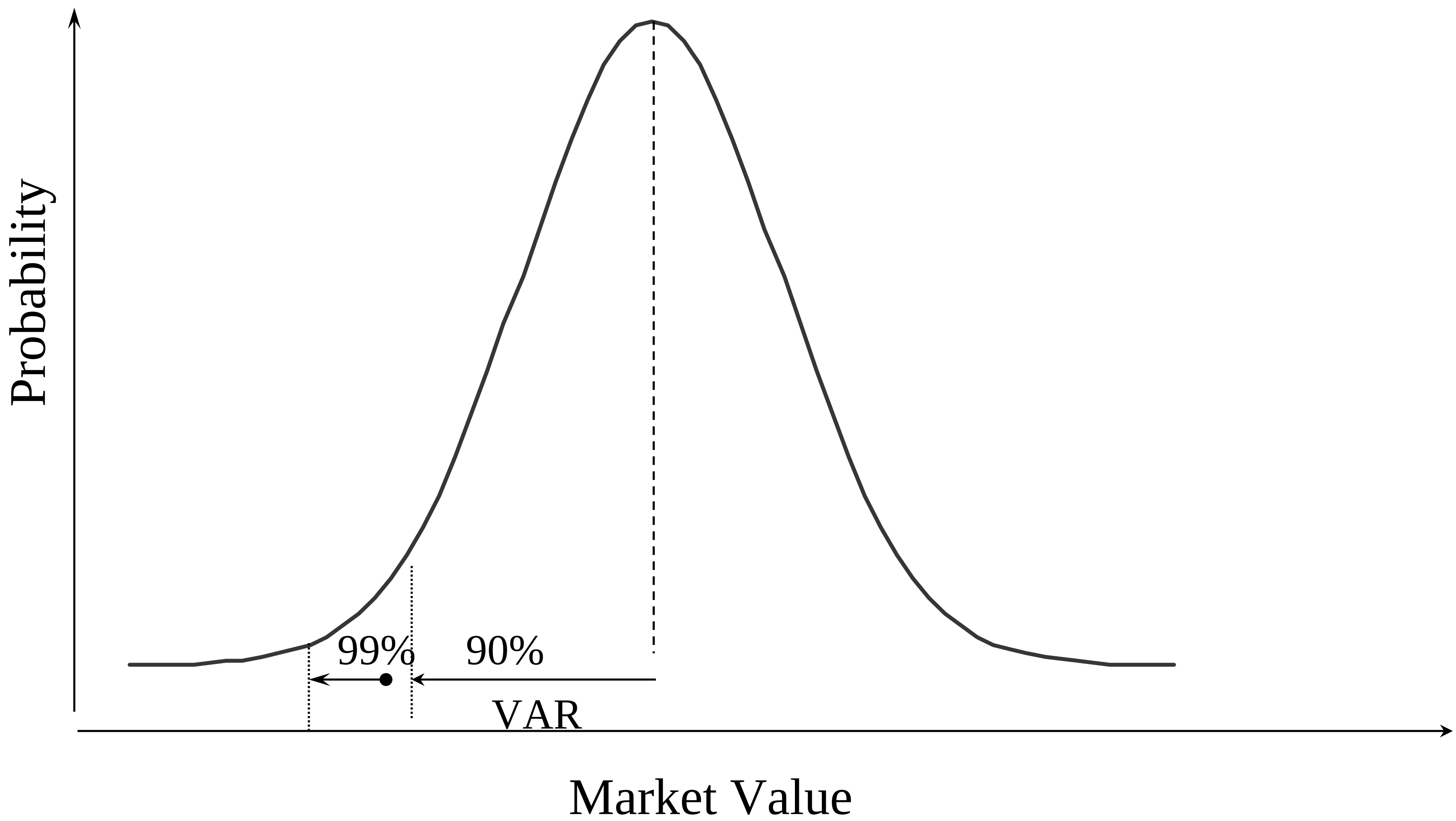
VAR & Distribution of Returns

- VAR will *increase*
  - With rising volatility
  - Lower average returns
- VAR will *decrease*
  - With falling volatility
  - Higher average returns
    - ✓NB effect of changing average returns is usually negligible for short holding periods

VAR and Confidence Levels

- A more risk-averse manager will want to determine VAR with greater confidence
  - Increasing the confidence level will increase VAR
  - Decreasing the confidence level will decrease VAR

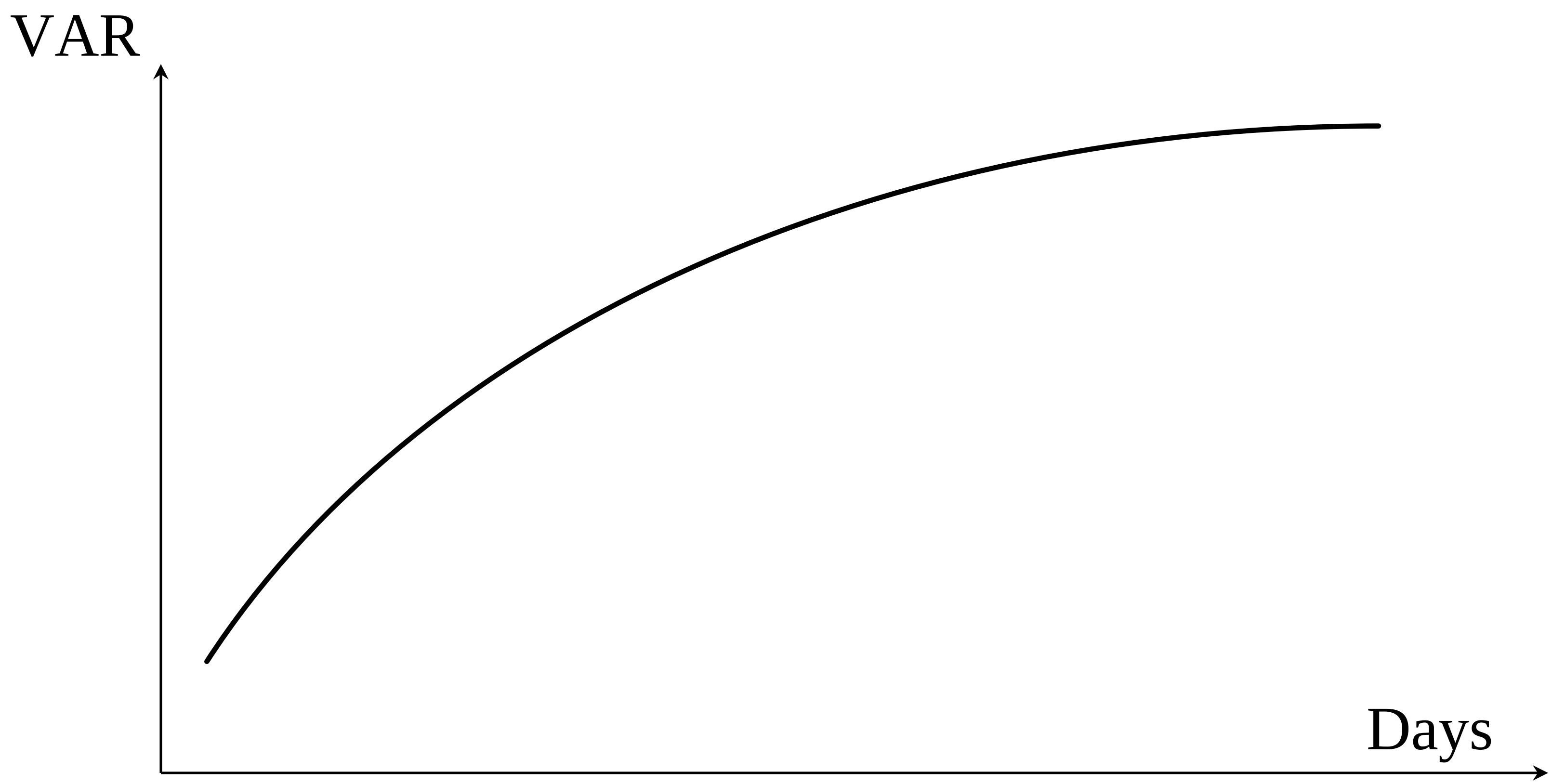
VAR & Confidence Levels



VAR and Holding Period Formula

- Volatility normally quoted on annual basis
  - ✓Hence previous formula gives VAR on annual basis
- Holding Period Adjustment (Sqrt time rule)
  - VAR = Market Value x Confidence Factor x Volatility x $T^{1/2}$
    - ✓Volatility is in % per annum
  - $T = \dfrac{\text{\# days in holding period}}{\text{\# trading days in year (252)}}$

VAR and Holding Period



✓As Holding Period lengthens, VAR increases
✓As Holding Period shortens, VAR decreases

Selection of Confidence Levels

- Confidence intervals
  - Basle Committee: 99%
  - Others:
    - ✓Chase-Chemical 97.5%
    - ✓BoA, JP Morgan 95%
- Holding period
  - Basle Committee: 10 days
  - Others:
    - ✓Investment management: 1 month typical
    - ✓Trading house: 1 day

VAR and Derivatives

- VAR for a derivative portfolio:
  - Portfolio value changes with underlying asset
  - Hence adjust formula for portfolio delta
  - May also need to adjust for non-linear, second-order effects (Gamma, convexity)

Normal Models

- Delta-Normal
  - ✓Simple, linear model uses derivatives delta
  - ✓Ignores high-order effects
- Non-Linear Model
  - ✓Makes adjustments for non-linear effects (Gamma risk)
  - ✓Important for derivatives portfolios

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The Delta-Normal Model

- VAR = *Market Value x Confidence Factor x Volatility x Delta*

  - Same as previous model, just incorporating delta
    - ✓ NB: stock portfolio: delta = 1
  - A simple linear function of delta
  - Assumes that returns are normally distributed
  - If necessary, adjust volatility by $T^{1/2}$ for appropriate holding period, as before

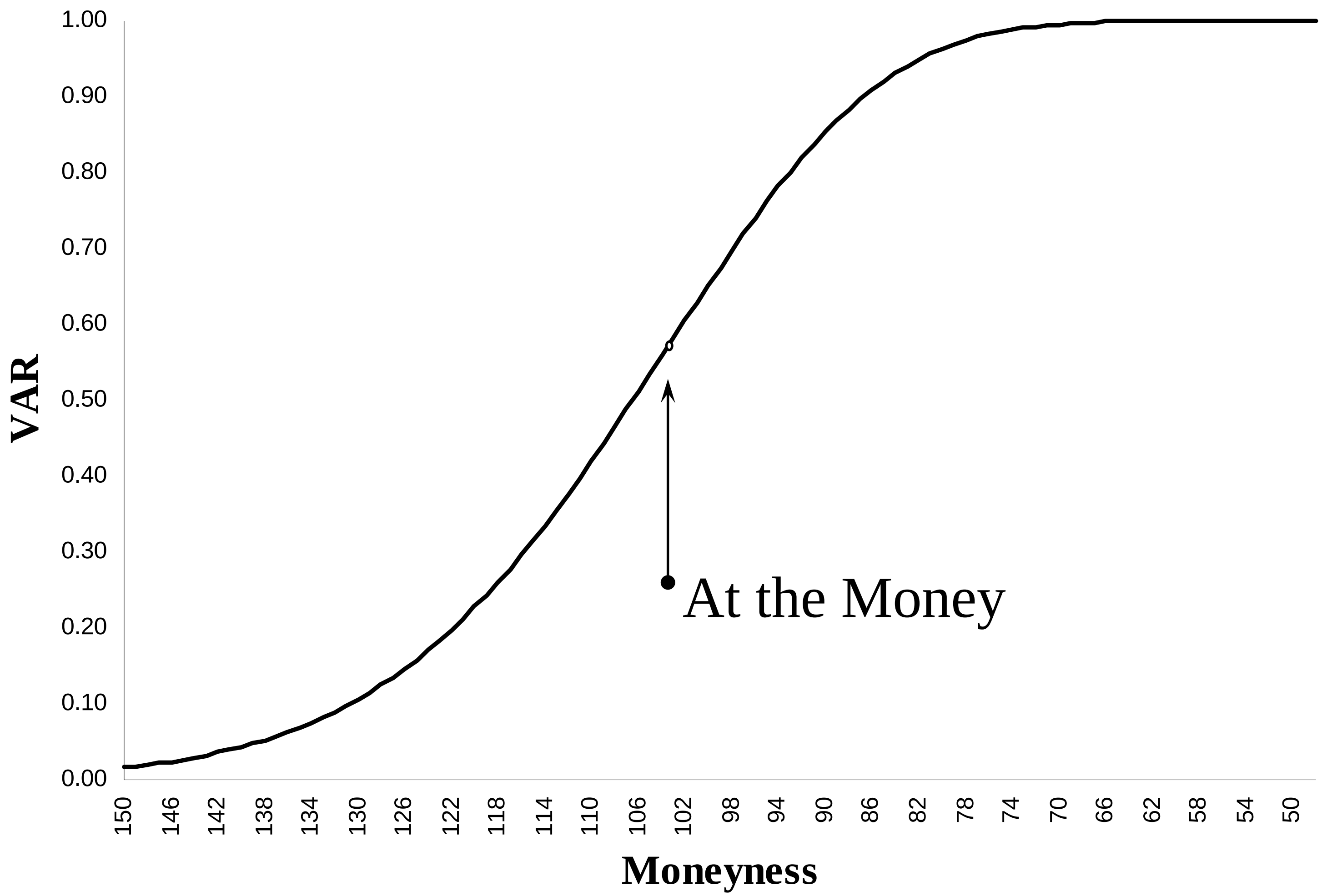
Derivative Portfolio VAR Example

- Long S&P100 OEX index calls
  - Market value $9.45MM
  - Daily volatility 1%
  - Option delta 0.5
  - Confidence level 99% (CF = 2.33)
- VAR = $9.45 x 2.33 x .01 x 0.5 = $110,000
  - There is a 1% chance that the call portfolio will lose more that $110K in a day

Options & VAR

- A deep In-the-Money option
  - Has approximately same VAR as underlying stock
    - ✓ Assuming equal $ amounts invested in each
  - Only true for short-term holding periods - Why?
  - Answer:
    - ✓ Delta of ITM option ~ 1
    - ✓ For long holding period, greater chance that option will expire OTM

VAR & Delta


1.00
0.90
0.80
0.70
0.60
0.50
0.40
0.30
0.20
0.10
0.00
VAR
At the Money
150
146
142
138
134
130
126
122
118
114
110
106
102
98
94
90
86
82
78
74
70
66
62
58
54
50
Moneyness

VAR and Delta

- VAR increases with Delta
  - Minimum for OTM options, delta ~ 0
  - Maximum for ITM options, delta ~ 1
  - Changes most rapidly for
    - ✓ATM options
    - ✓Short maturity options
      - because of Gamma

Delta-Normal VAR: Equivalent Formulation

☞ VAR = $S \; x \; CF \; x \; \sigma \; x \; \Delta_p$

- S = underlying (stock) price
- CF = confidence factor
- σ = volatility
- Δ_p = Delta of portfolio

VAR and Gamma

- Gamma adjustment required for
  - ATM options
  - Short-dated options
- Gamma risk is minor for
  - Deep ATM/OTM long dated options
  - Short holding periods

VAR Formula - Gamma Adjusted

- VAR = $S \times CF \times \sigma \times [\Delta_p^2 + {}^1/_2 (S \times \sigma \times \Gamma)^2]^{1/2}$
  - S = underlying stock price
  - CF = confidence factor
  - σ = volatility
  - Γ = Gamma
  - Δ_p = Delta of portfolio

✓Note: same as Delta-Normal model when Γ = 0

VAR & Gamma Risk Example

- Long 1000 ATM calls
  - Stock price is $50
  - Daily volatility 1.57% (25% annual)
  - Portfolio delta 700
  - Gamma is 27.8
  - Confidence level 95% (CF = 1.65)
- VAR = $50 x 1.65 x 0.0157 x $[700^2 + {}^1/_2(50 \times 0.0157 \times 27.8)^2]^{1/2}$ = $907
  - There is a 5% chance that the call portfolio will lose more that $907 in a day

Limitations of Delta Normal Model

- Formula allows us to compute VAR for different holding periods
- Can be misleading, because depends on delta
- Two main sources of error:
  - Delta changes as underlying changes
    - ✓Use Gamma adjustment where significant
  - Gamma changes over time
    - ✓Gamma increases as maturity approaches
    - ✓High-order effect produces very rapid changes in VAR

Model Testing

- *Back testing* recommended by Basle Committee
- Check the *failure rate*
  - Proportion of times VAR is exceeded in given sample
  - Compare proportion p with confidence level
- Problem:
  - Hard to verify VAR for small confidence intervals
    - ✓ Need very many sample periods to obtain adequate test

Failure Rates

- Test Statistic: see Kupiec 1995
  - $1 + 2Ln[(1-p)^{T-N}p^{N}] - 2Ln[(1-(N/T))^{T-N} (N/T)^{N}]$
    - ✓ChiSq distribution, 1d.f.
    - ✓N = # failures; T = Total sample size
- Example: Confidence level = 5%
  - Expected # failures = 0.05 x 255 = 13
  - Rejection region is 6 < N < 21
    - ✓If # failures lies in this range, model is adequate
    - ✓If N > 21, model underestimates large loss risk
    - ✓If N < 6, model overestimates large loss risk

Additional Tests

- Christofferson (1996)
  - Interval test for VAR
  - Very general approach
- Zangari Excessive Loss Test (1995)
  - Calculates expected losses in “tail” event

$$E\,[R_t \mid R_t < \alpha\sigma_t] = -\sigma_t f\,(\alpha) / F\,(\alpha)$$

  - F and f are standard Normal density / Distribution fns
  - Use t-test to compare sample mean losses against expected

Lopez Probability Forecasting Approach

- Most tests have low power
  - Likely to misclassify a bad model as good
  - Especially when data set is small
- Lopez Approach
  - Uses forecasting loss function
    - ✓ e.g Brier Quadratic Probability Score

$$QPS = 2\sum_{t=1}^{T}(p_t^f - I_t)^2/T$$

    - ✓ p_t is forecast probability of event taking place in interval t
    - ✓ I_t takes value 1 if event takes place, zero otherwise
  - Correctly identifies true model in large majority of simulated cases

Adjusting for Fat Tails

- Standardized residual process

$$R_r/\sigma_t = \varepsilon_t$$

  - ε_t ~ Normal(0, 1)

- Zangari's Normal Mixture Approach

$$R_r/\sigma_t = \varepsilon_{1,t} + \delta_t \varepsilon_{2,t}$$

  - δ_t is binary variable, usually 0, sometimes 1
  - $\varepsilon_{2,t}$ ~ Normal (μ_t, σ_{2t})

Zenari's GED

- Generalized Error Distribution

$$f(\varepsilon_t) = \frac{\upsilon e^{-(1/2)|\varepsilon_t/\lambda|^{\upsilon}}}{2^{(1+1/\upsilon)}\Gamma(1/\upsilon)}$$

$$\lambda = [2^{-(2/\upsilon)}\upsilon\Gamma(1/\upsilon)/3]^{1/2}$$

  - Normal distribution when v = 2
  - Probability of extreme event rises as v gets smaller

Empirical Tests

- Zengari (1996)
  - Tested Standard Normal, Normal Mixture and GED VAR
    - ✓ 12 FX and equity time series
  - All performed well at the 95% confidence level
  - Normal Mixture ad GED performed considerably better at 99% confidence level
- Conclusion
  - Both NM and GED improve on Normal VAR

Position Mapping

- Huge amount of data required for VAR computations
  - N separate volatilities
  - N(N-1)/2 cross-correlations
  - Large variance-covariance matrices tend not to be positive definite
- Need to reduce dimensionality of problem

Mapping Procedures

- RiskMetrics Approach
  - Select core set of instruments as representative
    - ✓Core currencies
    - ✓Equity indices
    - ✓Zero coupon bonds
    - ✓Commodity futures contracts
  - Map portfolio to representative securities
  - Calculate VAR based on core instruments

Multivariate Analytical Mapping

- Factor or Principal Component Analysis
  - Portfolio returns are analyzed into a (small) number of different, independent factors
  - Drastically reduces dimensionality of problem
    - ✓Factors are independent, hence all covar terms are 0
  - Each instrument is mapped to some linear combination of the factors
    - ✓Linear parameters estimated by regression analysis
  - Technique is most useful for large, highly correlated portfolios

VAR Building Blocks

- VAR for FX Positions
  - VAR = $-\alpha\sigma_E xE$
    - ✓ σ_E is the exchange rate volatility
    - ✓ xE is the size of the position in domestic currency units
    - ✓ α is the confidence factor

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VAR for Equity Positions

- Suppose we do not know the volatility and correlation data for a specific equity
- Use CAPM: $R_A = R_f + \beta(R_M – R_f)$
  - Volatility of firm's returns is:

$$\sigma_A = \sqrt{\beta_A^2 \sigma_M^2 + \sigma_{\varepsilon A}}$$

  - VAR = $-\alpha\sigma_A x$
- For large diversified portfolio, firm specific risk is eliminated
  - VAR = $-\alpha\beta_A\sigma_M x$

VAR for Bonds

- Map to ZCB's
  - VAR = $-\alpha\sigma_R B \approx -\alpha D y \sigma_y B$
    - ✓ D is modified duration
    - ✓ Y is yield to maturity
    - ✓ σ_R is return volatility
    - ✓ σ_y is yield volatility
    - ✓ B is PV of bond cashflow
- Timing of cash flows
  - e.g. 6 year cash flow
  - Map to linear combination of 5 and 7 year cashflows

VAR for Forwards / Futures

- VAR = $-\alpha\sigma_F xF$
  - σ_F is volatility of forward / futures price
  - xF is value of our forward position

VAR for More Complex Positions

- Coupon Bonds
  - Decompose into ZCB's
- FRN's
  - Equivalent to par coupon bonds
- Interest Rate Swaps
  - Equivalent to long coupon bond, short FRN
- Options
  - Compute VAR for replicating stock/bond portfolio
- FRA's
  - Long long-term ZCB, short short-term ZCB

Other Approaches to VAR

- Historical Simulation
- Stress Testing
- Monte-Carlo Simulation

Historical Simulation

- Calculation
  - Calculate return on portfolio over past period
  - Calculate historical return distribution
  - Look at -1.65σ point, as before
- Pros & Cons
  - Does not rely on normal distribution
  - Only one path (could be unrepresentative)

Stress Testing

- Scenario approach:
  - Simulates effect of large movements in key financial variables
- E.g. Derivatives Policy Group Guidelines
  - ✓Parallel yield curve shifts +/ 100 bp
  - ✓Yield curve twist +/ 25 bp
  - ✓Equity index values change +/ 10%
  - ✓Currency movements +/ 10%
  - ✓Volatilities change +/ 20% of current values
- Pros & Cons
  - ✓More than one scenario
  - ✓Validity of scenarios is crucial
  - ✓Handles correlations poorly

Monte Carlo Simulation

- Sometimes called “full valuation” method
- Widely applicable
  - Does not assume Normal distribution
  - Handles all types of securities

Monte-Carlo Methodology

- Simulate movement in asset value
  - Repeat 10,000 times, get 10,000 future values
  - Create histogram
    - ✓Find cutoff value such that 95% of calculated values exceed cutoff
- Cutoff value is the portfolio VAR
    - ✓For given confidence level (95%)
    - ✓For given holding period

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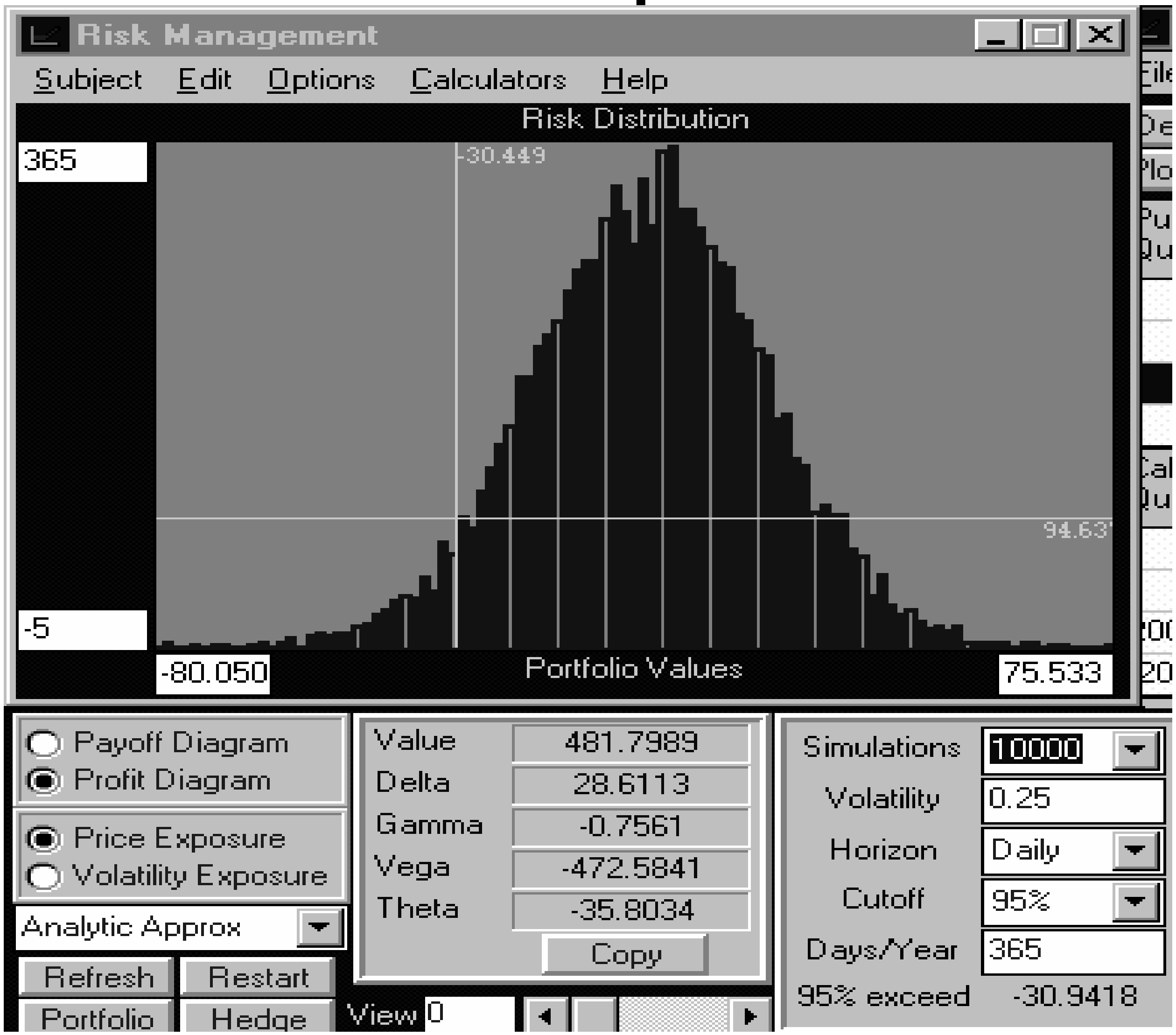
Generating Simulated Values

- $\Delta S = S_0 \text{ x } (\mu + \sigma\varepsilon)$
  - ✓ ΔS is change in value
  - ✓ S_0 is initial value
  - ✓ μ is average daily return
  - ✓ σ is daily volatility
  - ✓ ε is random variable
- Procedure:
  - Generate ε (random)
  - Compute change in portfolio value
  - Repeat many times (10,000+)
  - Create a histogram of portfolio values

Example


Risk Management
Subject
Edit
Options
Calculators
Help
Risk Distribution
365
-30.449
94.63
-5
-80.050
Portfolio Values
75.533
Payoff Diagram
Profit Diagram
Price Exposure
Volatility Exposure
Analytic Approx
Refresh
Restart
Portfolio
Hedge
Value 481.7989
Delta 28.6113
Gamma -0.7561
Vega -472.5841
Theta -35.8034
Copy
View 0
Simulations 10000
Volatility 0.25
Horizon Daily
Cutoff 95%
Days/Year 365
95% exceed -30.9418

Lab: Implementing a VAR Model

- Implementing a VAR model:
  - Delta normal
  - Delta-Gamma
  - Monte-Carlo simulation
- Hedging
  - Delta neutral
  - Delta-Gamma
- Worksheet: Risk Management

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Summary

- Value at Risk
- Factors affecting VAR
- VAR & Derivatives
- The Delta-Normal Model
- Other VAR models