Portfolio Management Theory

Modern Portfolio Theory

- Capital Allocation Line
- Naive diversification
- Mean-variance criterion
- Markowitz diversification
- The Efficient Frontier

Portfolio Returns

- A portfolio is a diverse collection of assets $A_1, A_2, \ldots, A_N$
- We invest a proportion w_i in asset A_i
- The w_i are called *weights* and sum to 1.
- The Expected Return on the portfolio is

$$W_1 E(r_1) + W_2 E(r_2)$$

  - $E(r_1)$ is the expected return on asset A_1

Portfolio Risk

- For a two asset portfolio:

$$\sigma_p = \sqrt{[w_1^2\sigma_1^2 + w_2^2\sigma_2^2 + 2w_1w_2\sigma_{12}]}$$

  - where σ_{12} is the covariance between assets 1 and 2

- Estimate portfolio standard deviation using:

$$Sd_p = \sqrt{w_1^2 Sd_1^2 + w_2^2 Sd_2^2 + 2w_1w_2 \operatorname{cov}(1,2)}$$

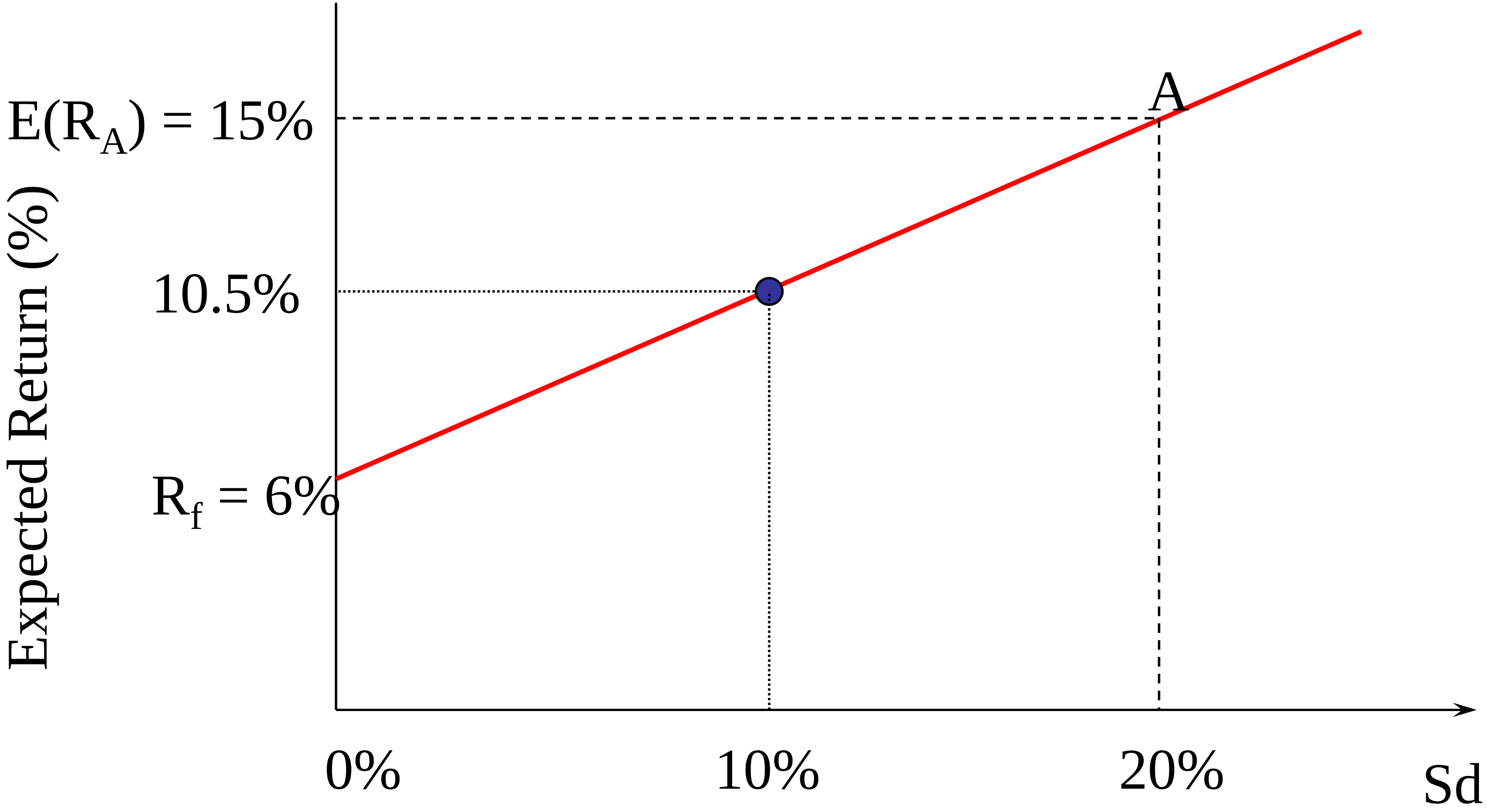
Combining Risky and Risk-Free Assets

- The standard deviation of the risk-free asset is zero
- The correlation (covariance) between the risk free asset and any other asset is zero
- Construct a two-asset portfolio P:
  - Invest an amount w in the risky asset A and (1-w) in the risk-free asset
- The expected return is:
  - $E(R_P) = wE(R_A) + (1-w)R_f$
- The standard deviation is:
  - $\sigma_P = w\sigma_A$

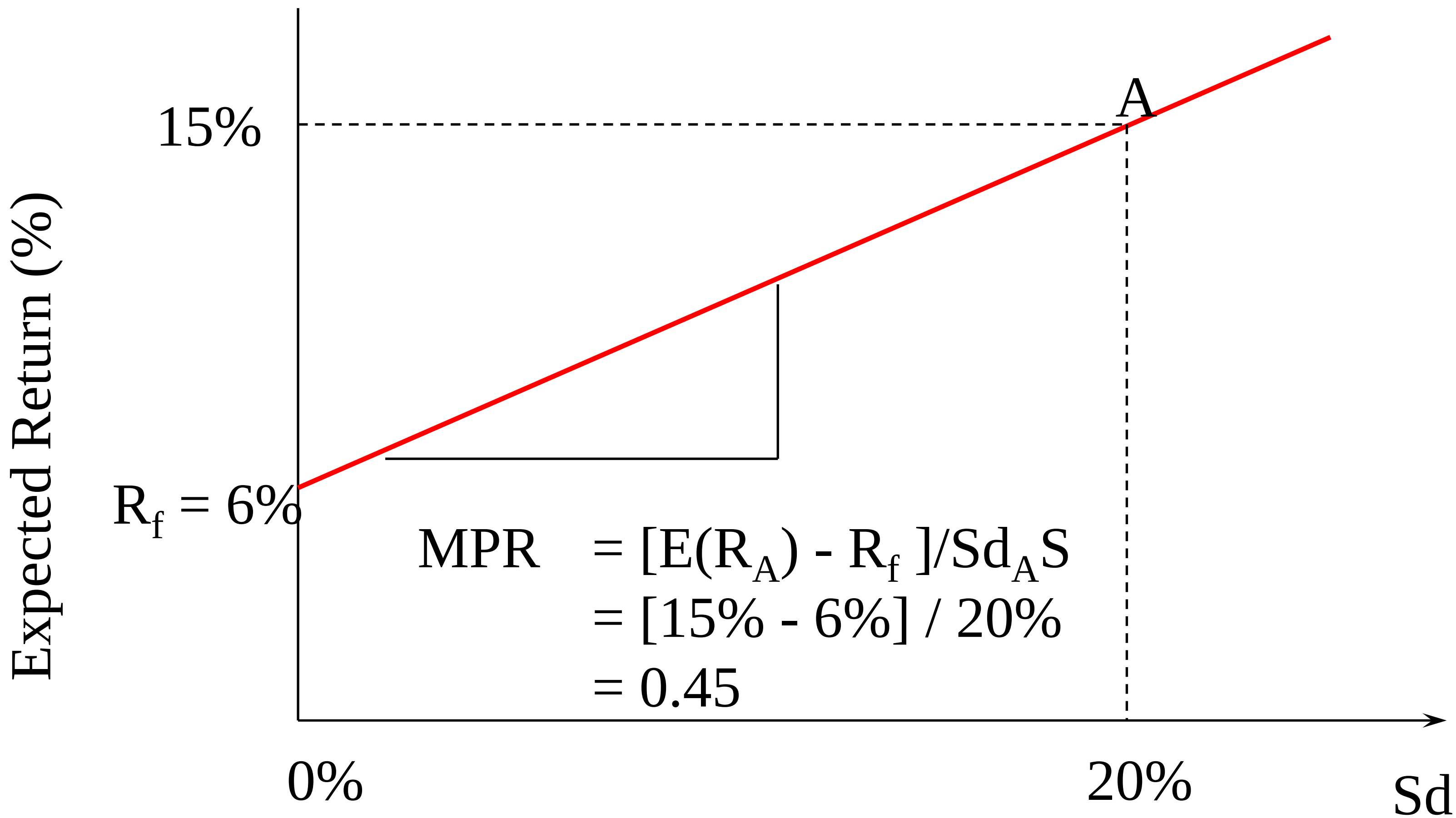
Risky & Risk-Free Assets Example

- Risky asset with expected return 15% and standard deviation 20%
- Riskless asset with expected return 6% (Sd of zero)
- Portfolio consisting of 50% invested in each asset
  - the expected return is 0.5 x 15% + 0.5 x 6% = 10.5%
  - the portfolio Sd is 0.5 x 20% = 10%

Capital Allocation Line



Market Price of Risk
Reward-to-Variability Ratio



$$\text{MPR} = [E(R_A) - R_f]/Sd_A S = [15\% - 6\%] / 20\% = 0.45$$

Risk-Reward Choice

- The CAL tells us:
  - How much risk we must bear to achieve a certain target return
  - What rate of return we can expect to achieve given a certain level of risk
- The Market Price of Risk tells us:
  - How much extra risk we must accept to increase our return by a given amount
  - How much return we must expect to give up in order to reduce our risk a by a given amount

Naive Diversification

- A collection of assets $A_1, A_2, \ldots, A_N$
- We invest a proportion w_i in asset A_i
- The most obvious way to diversify:
  - allocate equal $ amounts to ever asset
  - if there are N assets, then $W_i = 1/N$

Lab: Using CAPM Tutor

- Step 1
  - Run CAPM Tutor
- Step 2
  - Run Excel, load in spreadsheet
- Step 3
  - Select Naive Diversification
- Step 4
  - Load data in CAPM Tutor using Excel Link

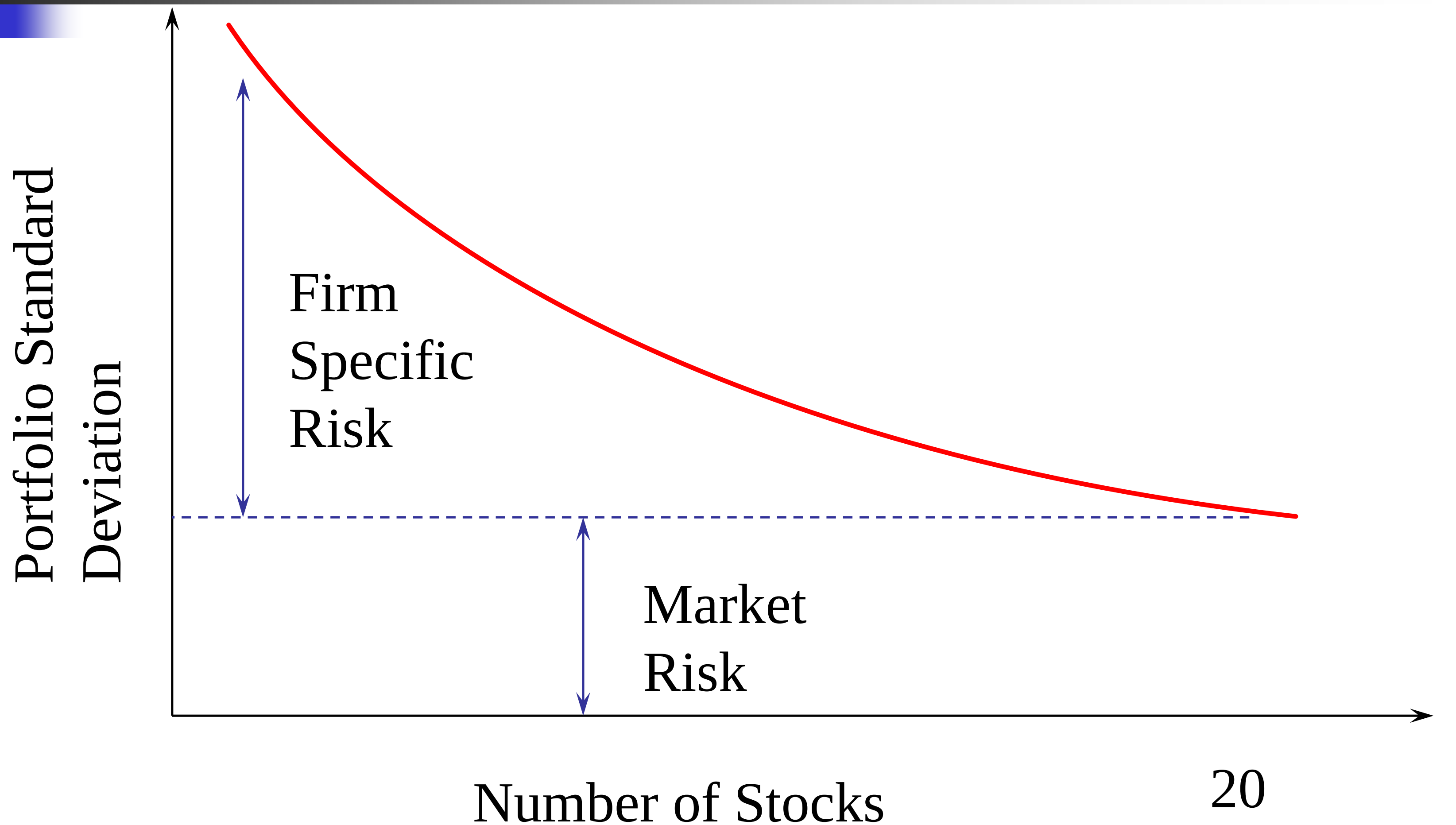
Exercise 1: Beating the S&P

- Exercise 1
  - Turn all stocks off except SPX
  - Plot SPX variance
  - Now turn SPX off
  - Select other stocks to include
  - Can you beat S&P (achieve lower variance)?

Exercise 2: Limits to Diversification

- Exercise 2
  - Click on the random button
  - Chooses portfolios at random
  - What happens to variance as # of stocks increases?
  - Is there a limit to risk reduction?
- Exercise 3
  - Now click the Plot Minima button
  - This plots the minimum variance for portfolios containing 1stock, 2 stocks, etc.
  - Check: can you beat the S&P?

The Limits to Diversification



Specific and Market Risk

- Specific Risk
  - Risk specific to individual firms
  - diversifiable
  - *A random portfolio of 20 stock will eliminate most specific risk*
- Market or Systemic Risk
  - Risk factors which affect all firms
  - therefore NOT diversifiable

Markowitz Diversification

- Naive diversification has equal weights across all assets
- Can we do better with unequal weights?

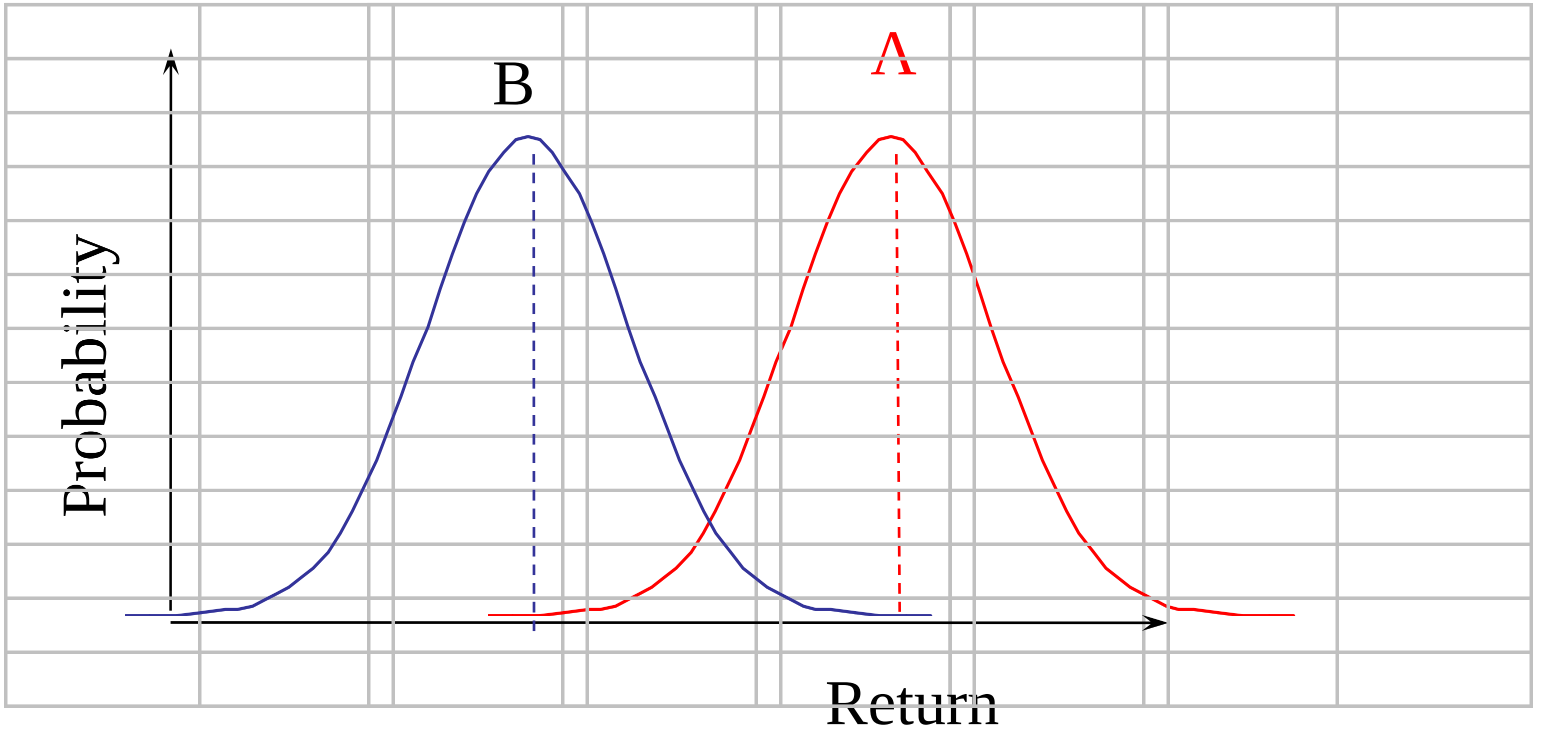
Lab: Worked exercise on Markowitz Diversification

- Use CAPM tutor
- Subject Markowitz diversification
- Use Finance data set
  - Options, Read data from file
  - Data format is *covariances*
- Follow notes on worked exercise

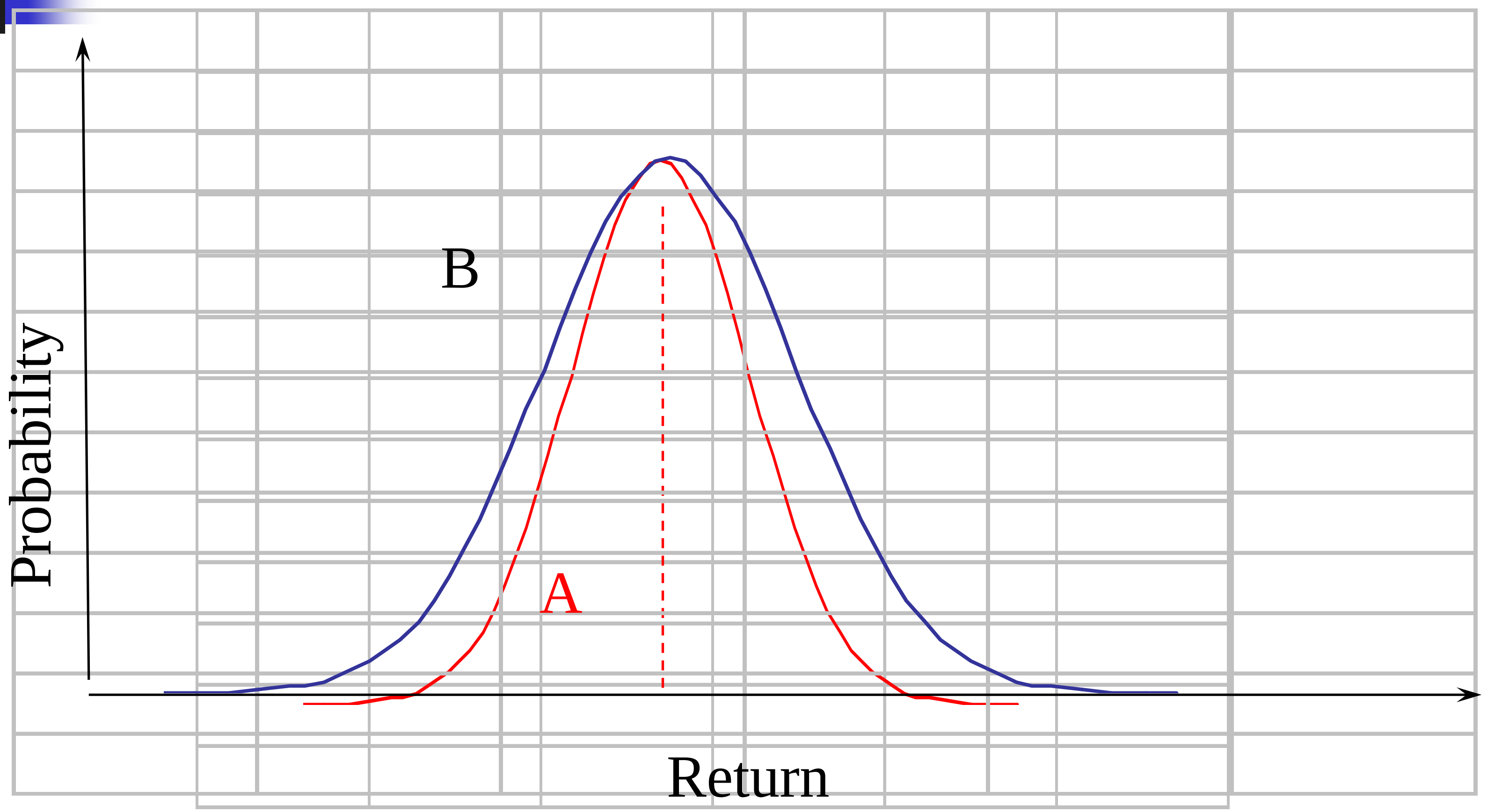
The Mean-Variance Criterion

- How do we judge if one investment is superior to another?
- Suppose we have two assets, A & B
- Expected returns are $E(r_A)$ and $E(r_B)$
- Then we say A dominates B if
  - $E(r_A) > E(r_B)$, and
  - $SD(A) <= SD(B)$

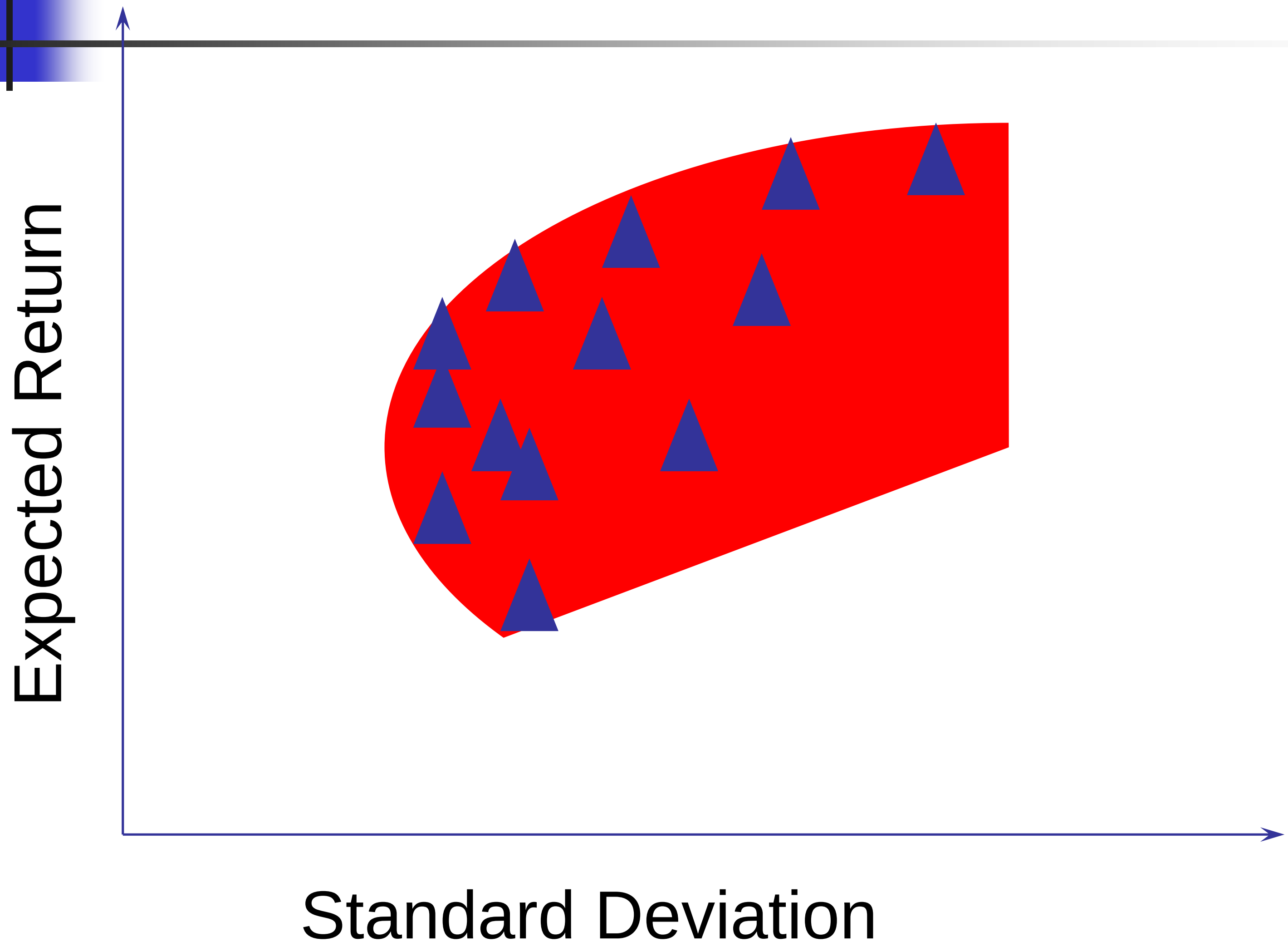
Mean-Variance Criterion Illustrated



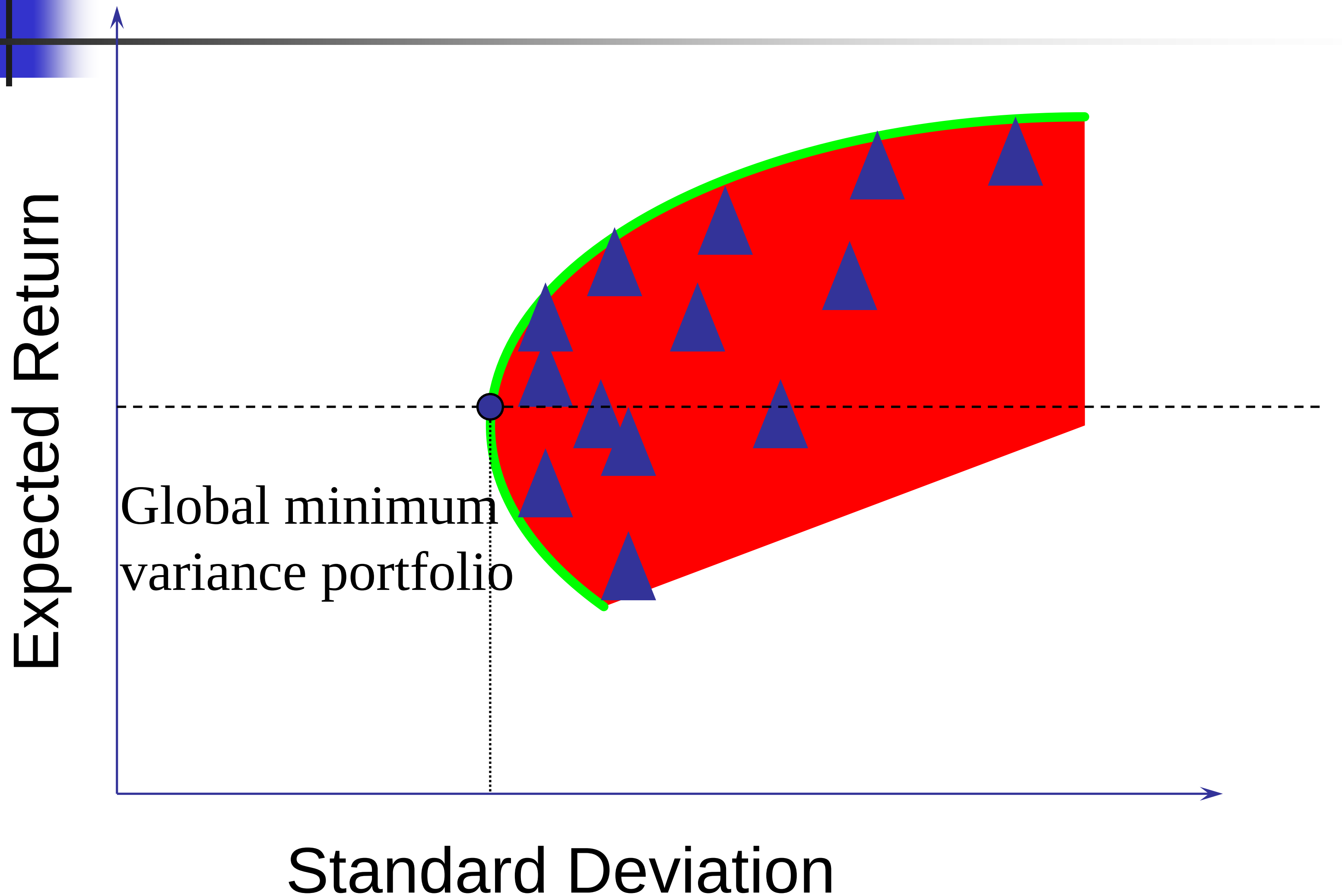
Mean-Variance Criterion Illustrated



Investment Opportunity Set



The Efficient Frontier



The Markowitz Solution

- Two-asset case:

$$w_1 = \frac{\sigma_2^2 - \sigma_{12}}{\sigma_1^2 + \sigma_2^2 - 2\sigma_{12}}$$

Lab: The Frontier of MMI Stocks

- What does the frontier of MMI stocks look like?
- Load MMI spreadsheet in Excel MMI-M2.XLS
- Load CAPM Tutor, Markowitz Diversification
- Options, Read data, Paste from Spreadsheet
  - Data format is prices

Lab: MMI Portfolio Exercise

- Construct the frontier with 20 stocks plus the S&P500
  - Does the S&P lie on the frontier?
  - What is the risk-return of the S&P500?
  - What is the minimum variance portfolio?
- Construct the frontier with only 20 stocks
  - Find the minimum variance portfolio giving you the same return as the S&P500
  - Find the global minimum variance portfolio

Lab: Portfolio Project

- CAPM Tutor
- Select Subject Project, load MMI data again
- Plot frontier from period 22 onwards
- See how it evolves
  - Rescale: top = 0.1
- How the frontier changes over time:
  - Top part moves around
  - Min variance point stable

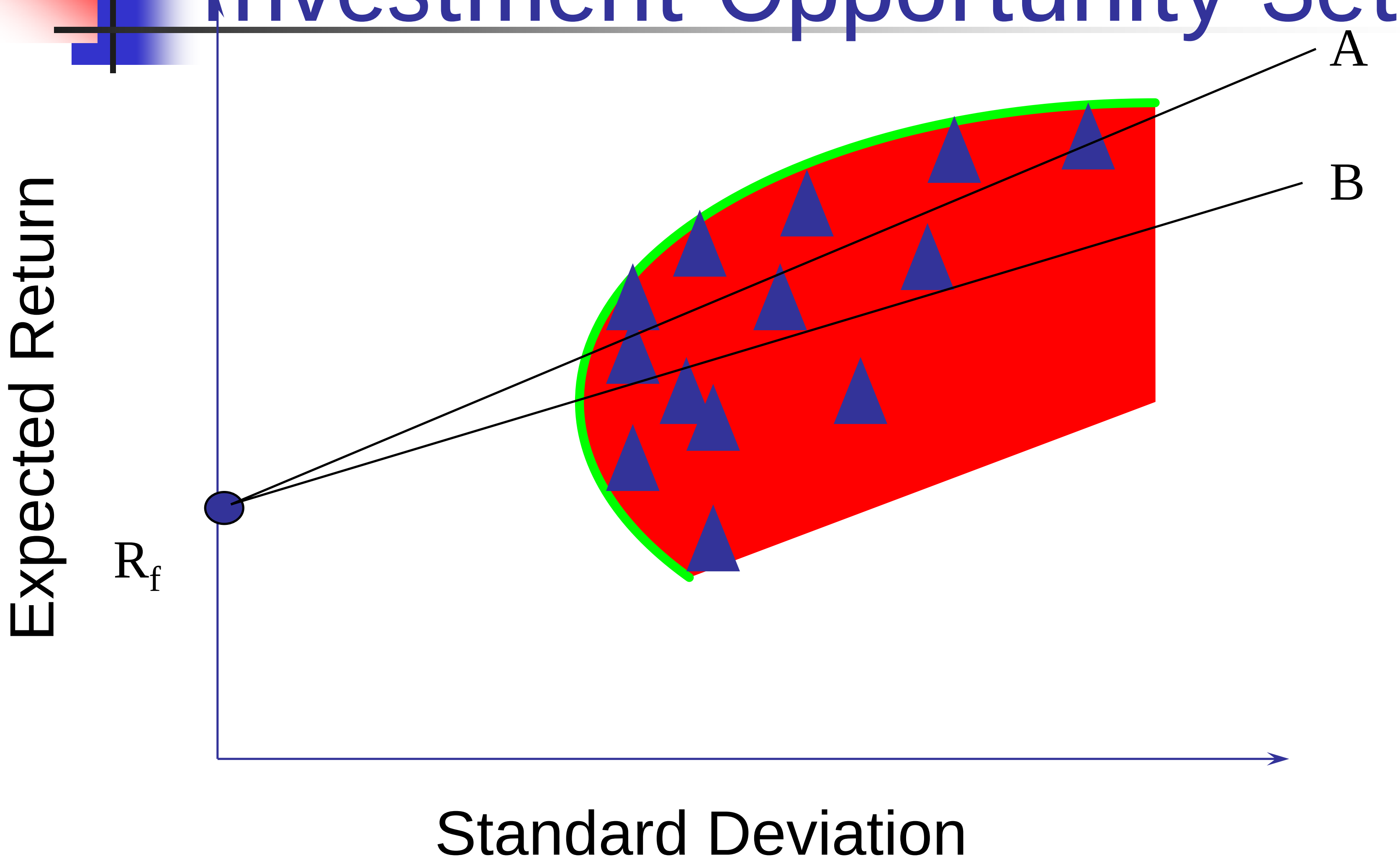
Lab: Portfolio Selection

- Objective:
  - Set target return
  - Find min. variance portfolio on efficient frontier which you expect to yield this return
- Buy & Hold Strategy
  - How much past data to use?
- Continuous Re-optimization Strategy
  - Use all data or “rolling block”?

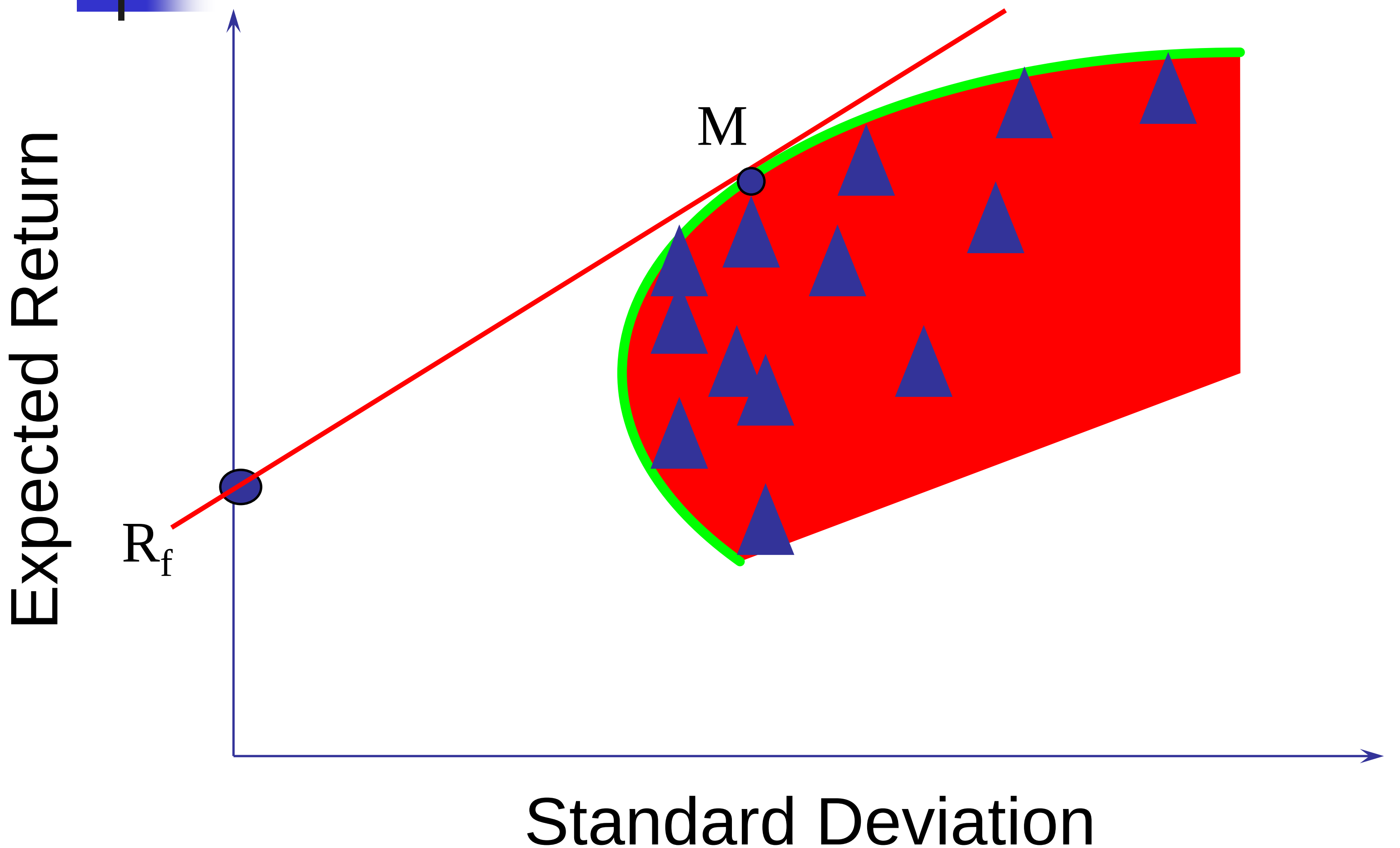
The Capital Asset Pricing Model

- Markowitz
  - Optimal portfolio weights
  - Implied investment strategy
- CAPM: goes much further
  - Strong implications for investment strategy

Capital Allocation Lines & Investment Opportunity Set



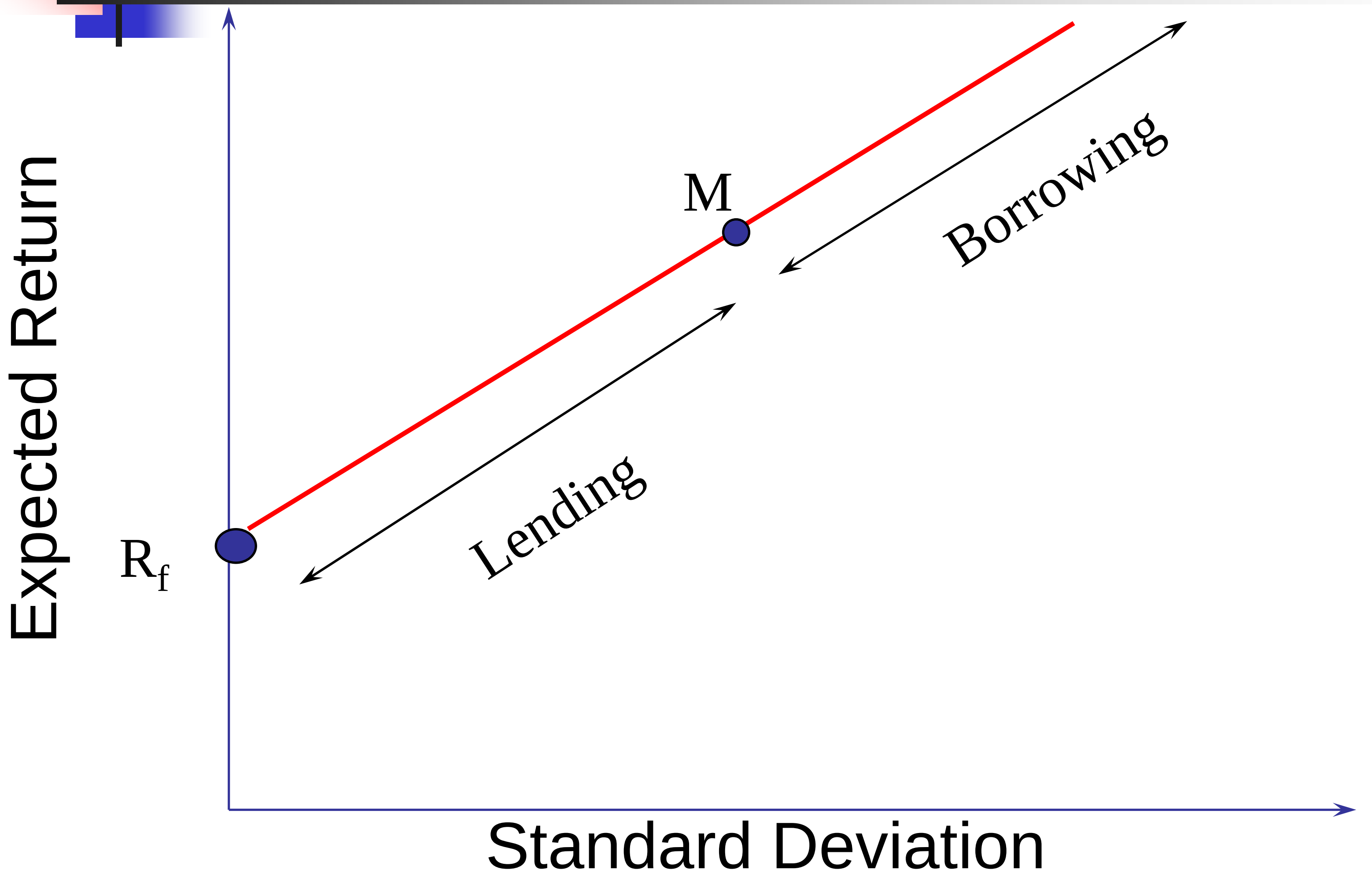
The Capital Market Line



CAPM

- Every investor will hold some combination of the risk-free asset and the portfolio M
  - If you are not holding M, you are carrying unnecessary risk
- M is the market portfolio
  - M is value-weighted portfolio of *all* risky assets
  - In practice, M is replaced by proxy, e.g. S&P500
- Mutual fund theorem
  - M lies on the Efficient Frontier
  - Passive strategy of investing in market index portfolio is efficient

Risk, Return & Leverage



Using CAPM to Predict Returns

- The CAPM Equation

$$E(r_A) = r_f + \beta_A [E(r_M) - r_f]$$

- Asset beta:
  - measures the proportion of the variance of the market portfolio contributed by asset A

$$\beta = \frac{Cov(r_A, r_M)}{\sigma_M^2} = \rho_{(A,,M)} \frac{\sigma_A}{\sigma_M}$$

Lab: Worked Exercise on CAPM

- Load CAPM tutor
- Choose Subject, Capital Asset Pricing Model
- Read in Finance data set

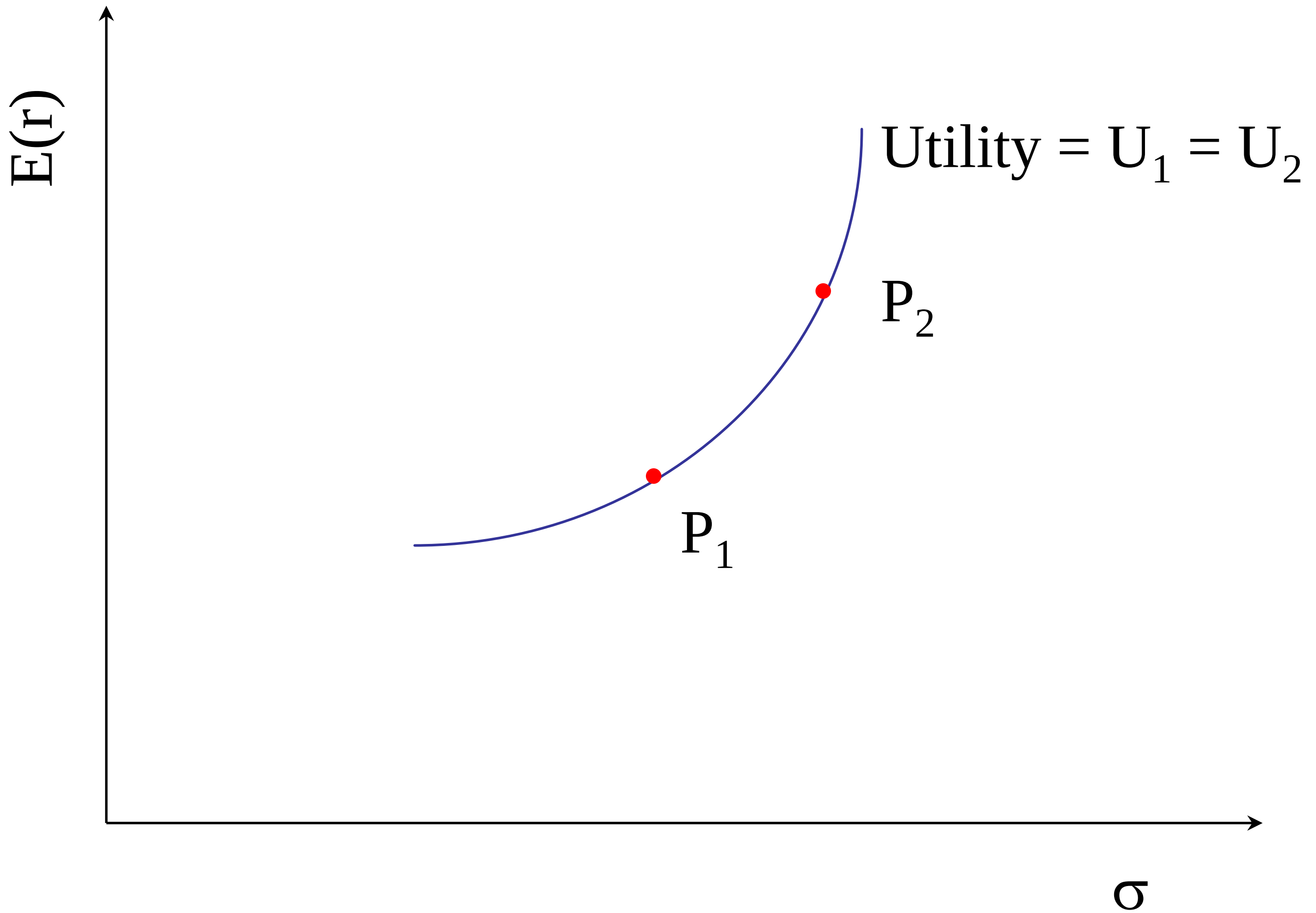
Questions on CAPM

- Assume you have no risk-free asset
  - Set a target return at e.g. 14%, 15%, 16%
  - Will the weights change?
- Repeat, assume you have a risk-free asset
- What happens to the market price of risk if the risk free rate falls?
- Try entering a risk-free rate of 25%
  - What happens? Why?
  - What would have to happen to stock returns?
  - What would this mean in terms of stock prices?

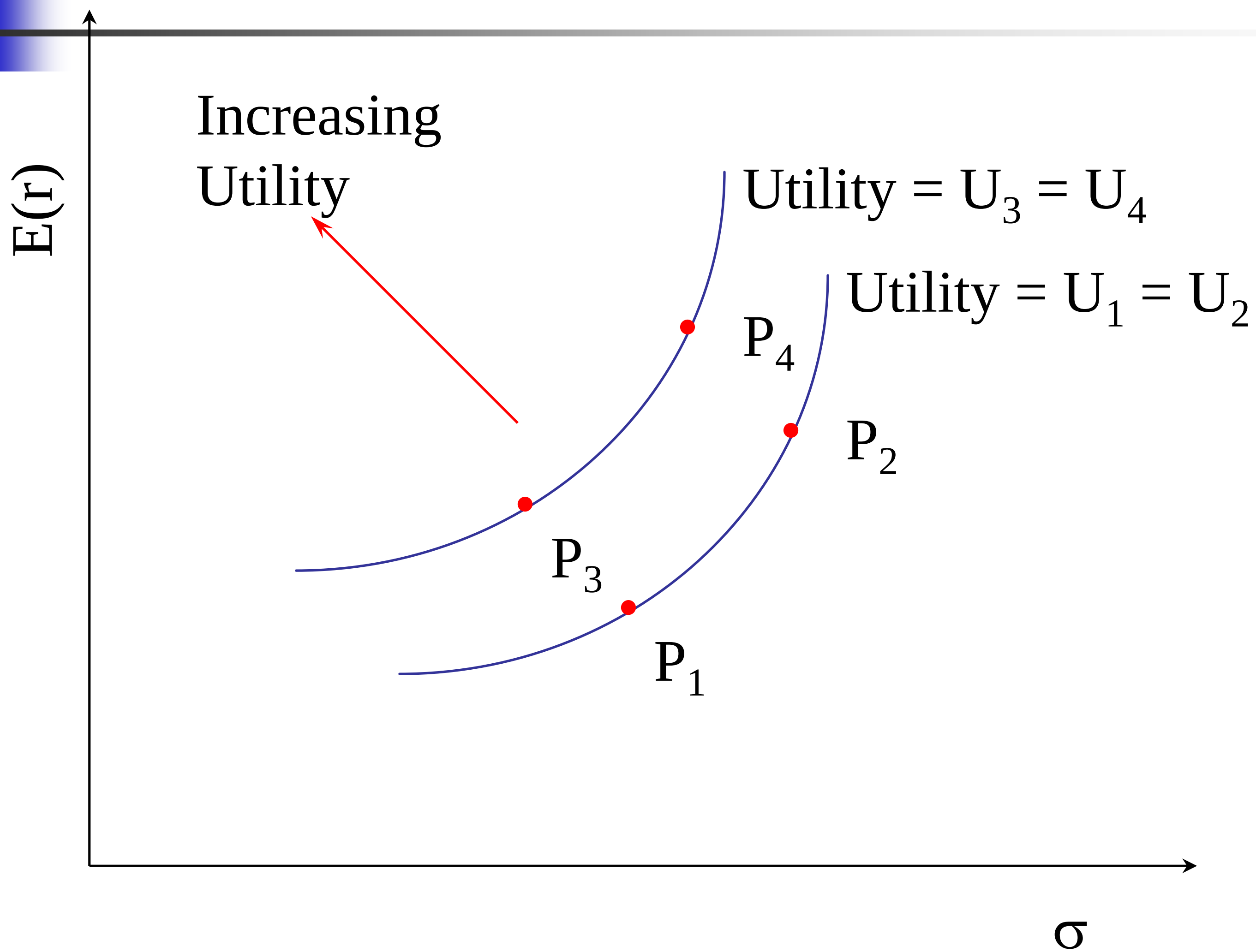
Risk & Utility

- Utility Function: $U = E(r) - 0.05 A\sigma^2$
- Suppose we have investment portfolios P_i
  - Each offering returns r_i , Sd σ_i , Utility U_i
- Suppose for P_1 and P_2, $U_1 = U_2$
  - the investor would be *indifferent* as to which portfolio s/he invested in
- Plot all the portfolios which have the same utility on the mean-variance chart
  - Forms a curve, known as the *indifference curve*

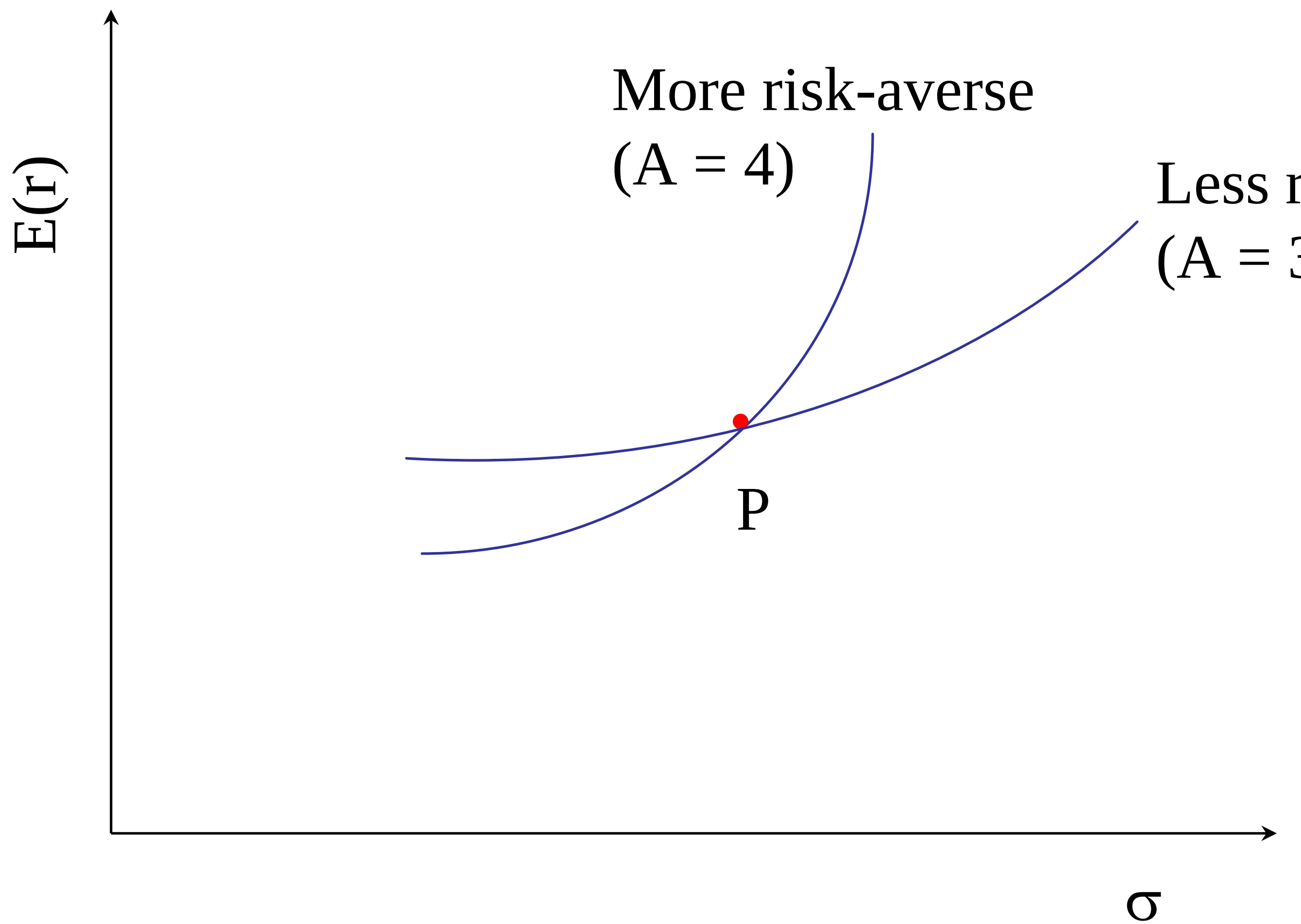
Indifference Curves



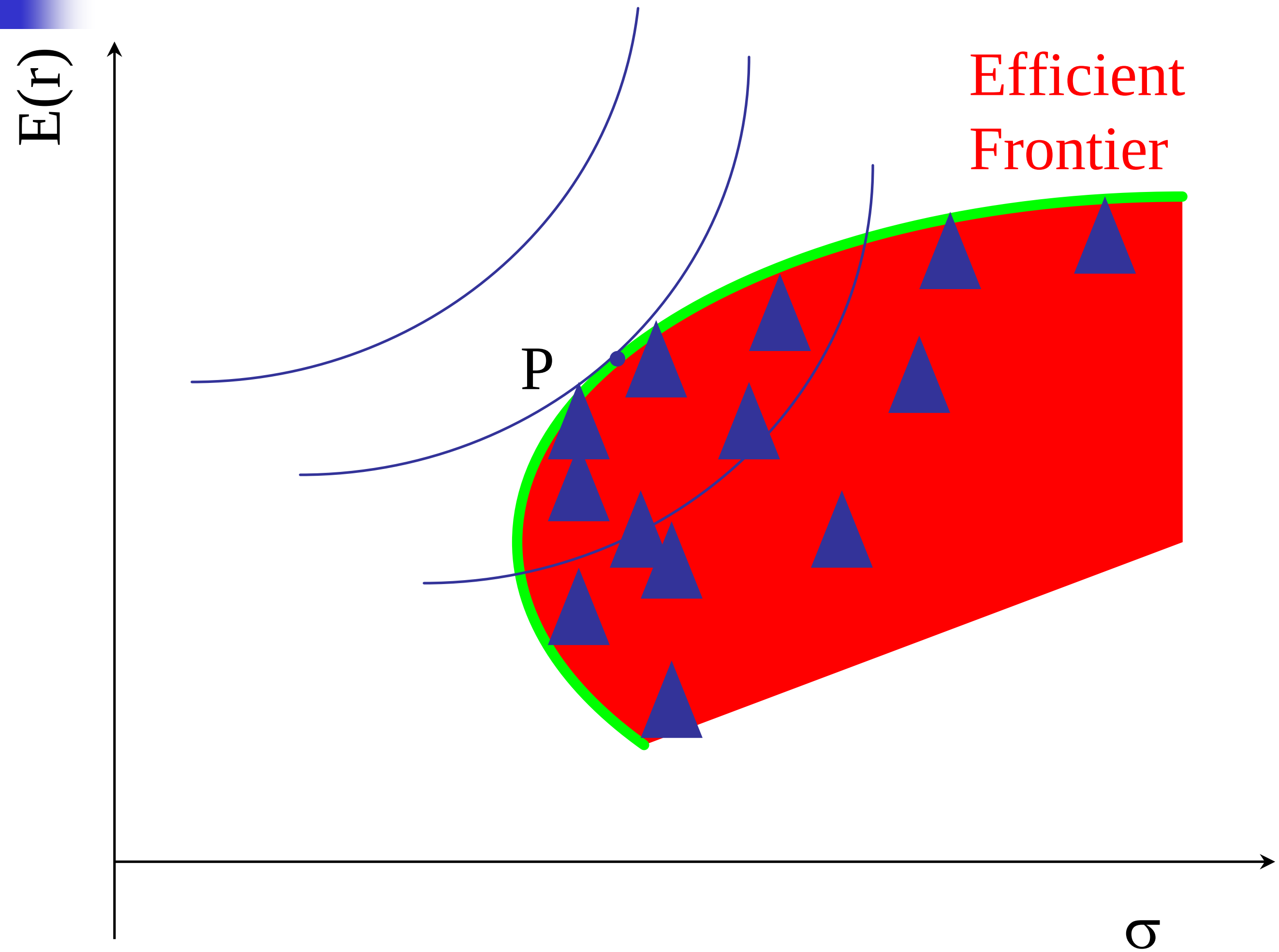
Indifference Curves



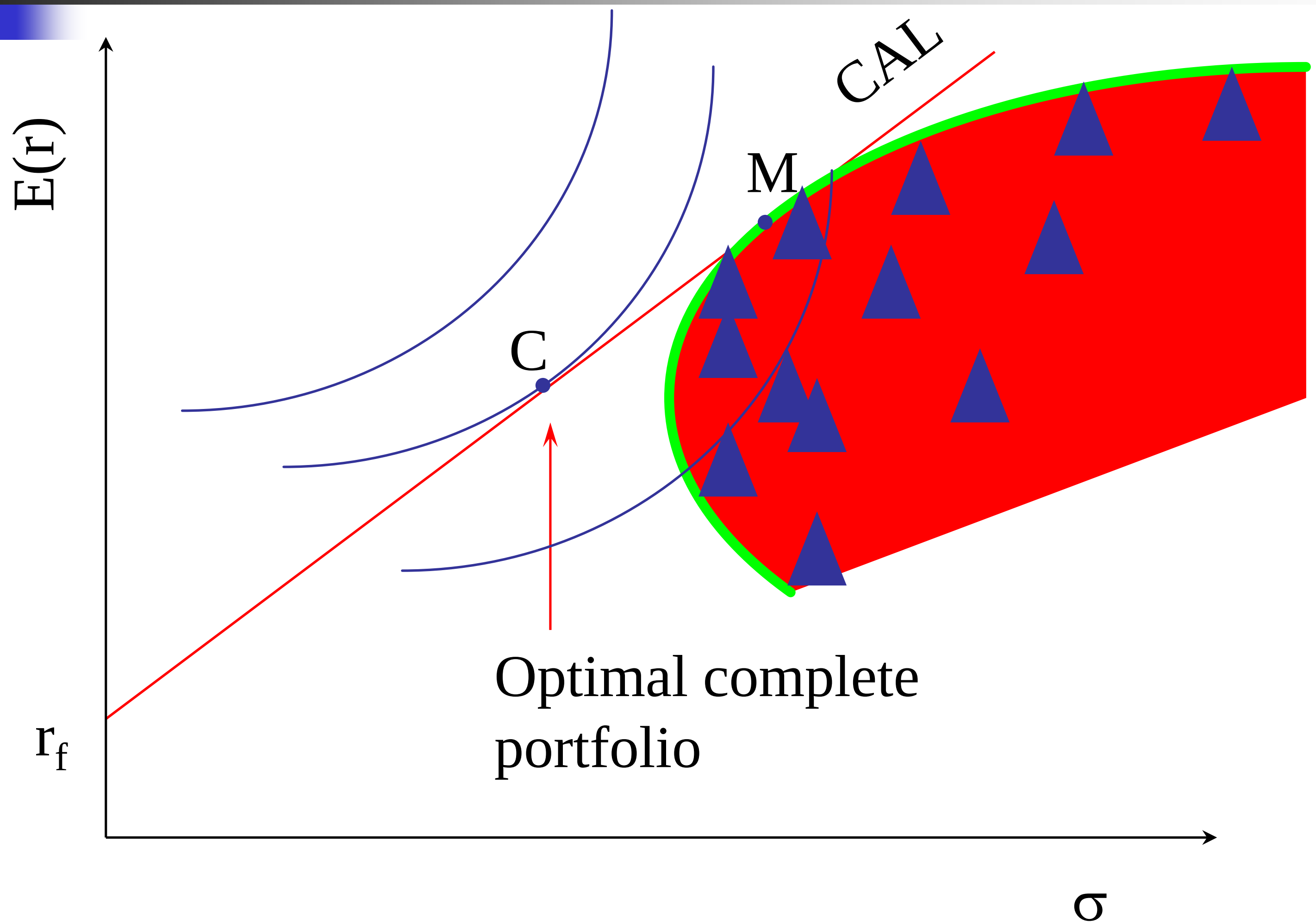
Indifference Curves & Risk Aversion



Indifference Curves & the Efficient Frontier



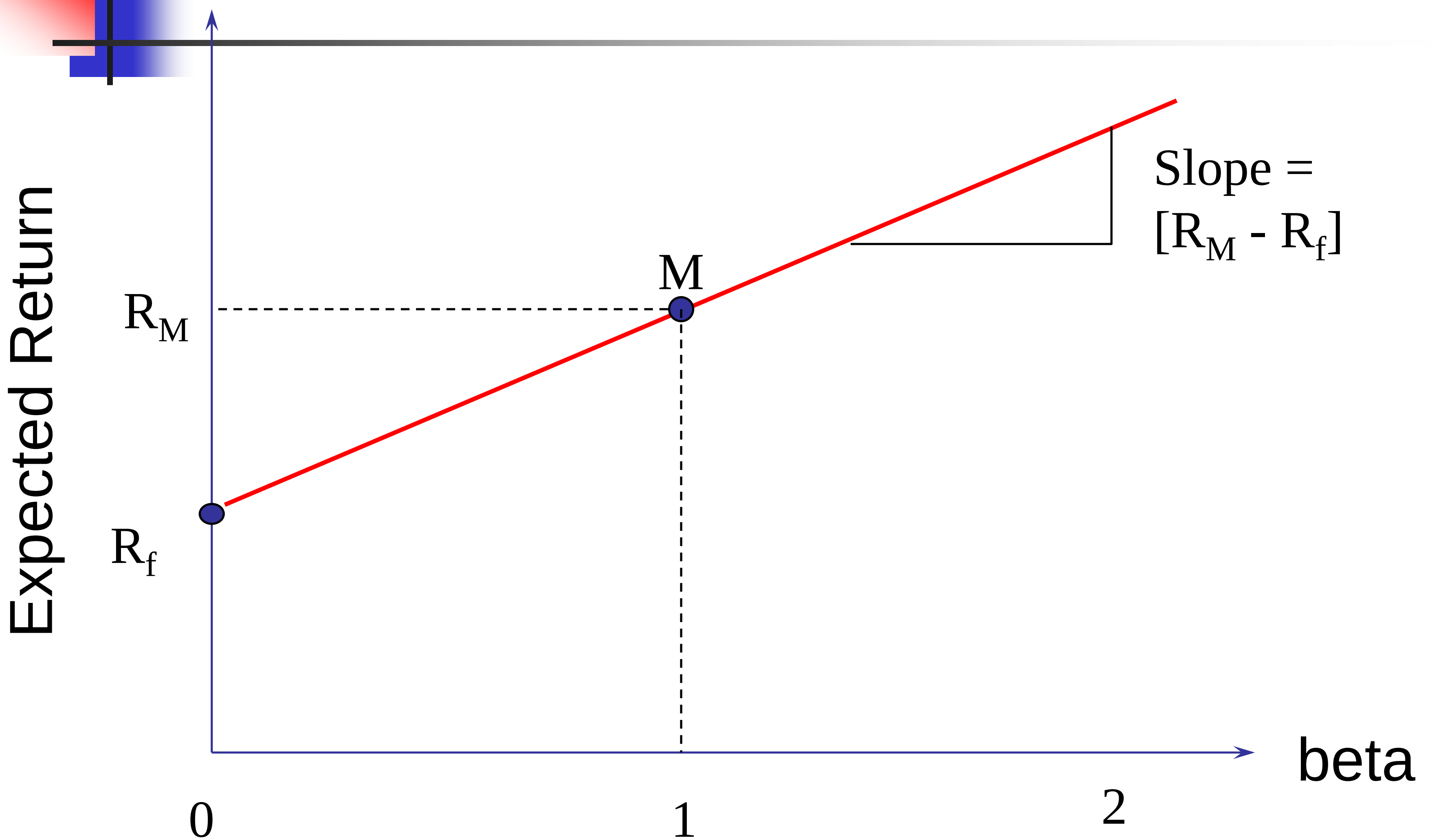
Indifference Curves & the CAPM



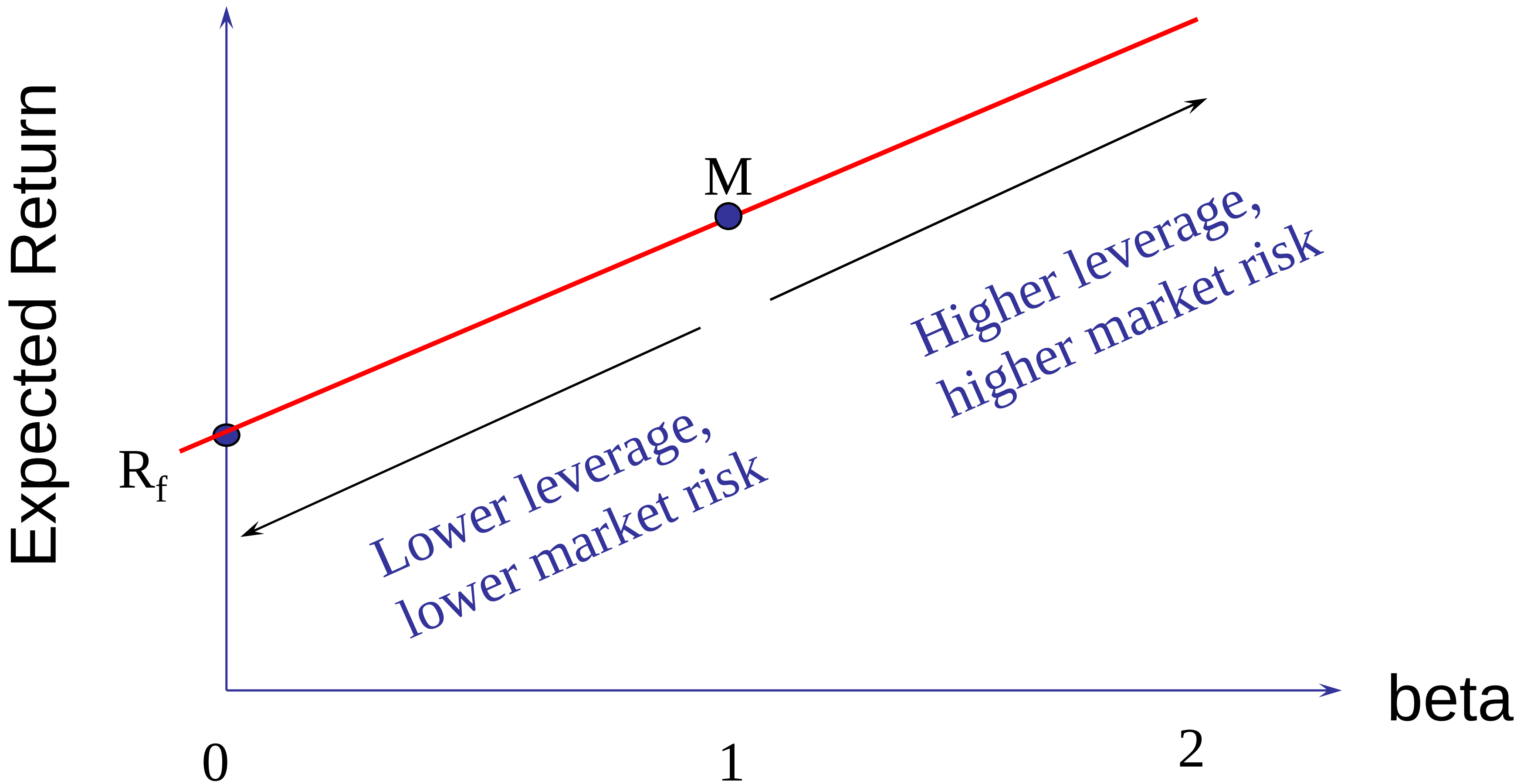
Lab: CAPM in Equilibrium

- How do risk preferences affect the CML?

The Security Market Line



Leverage & Return



Questions about Beta

- Rank the following stock in terms of their beta:
  - McDonalds
  - Netscape
  - Exxon
- Suppose the s.d of the market return is 20%
  - What is the standard deviation of returns on a *well-diversified* portfolio:
    - with beta 1.5?
    - with beta 0.5?
    - with beta of 0?
- A poorly diversified portfolio has an s.d. of 20%. What can you say about its beta?

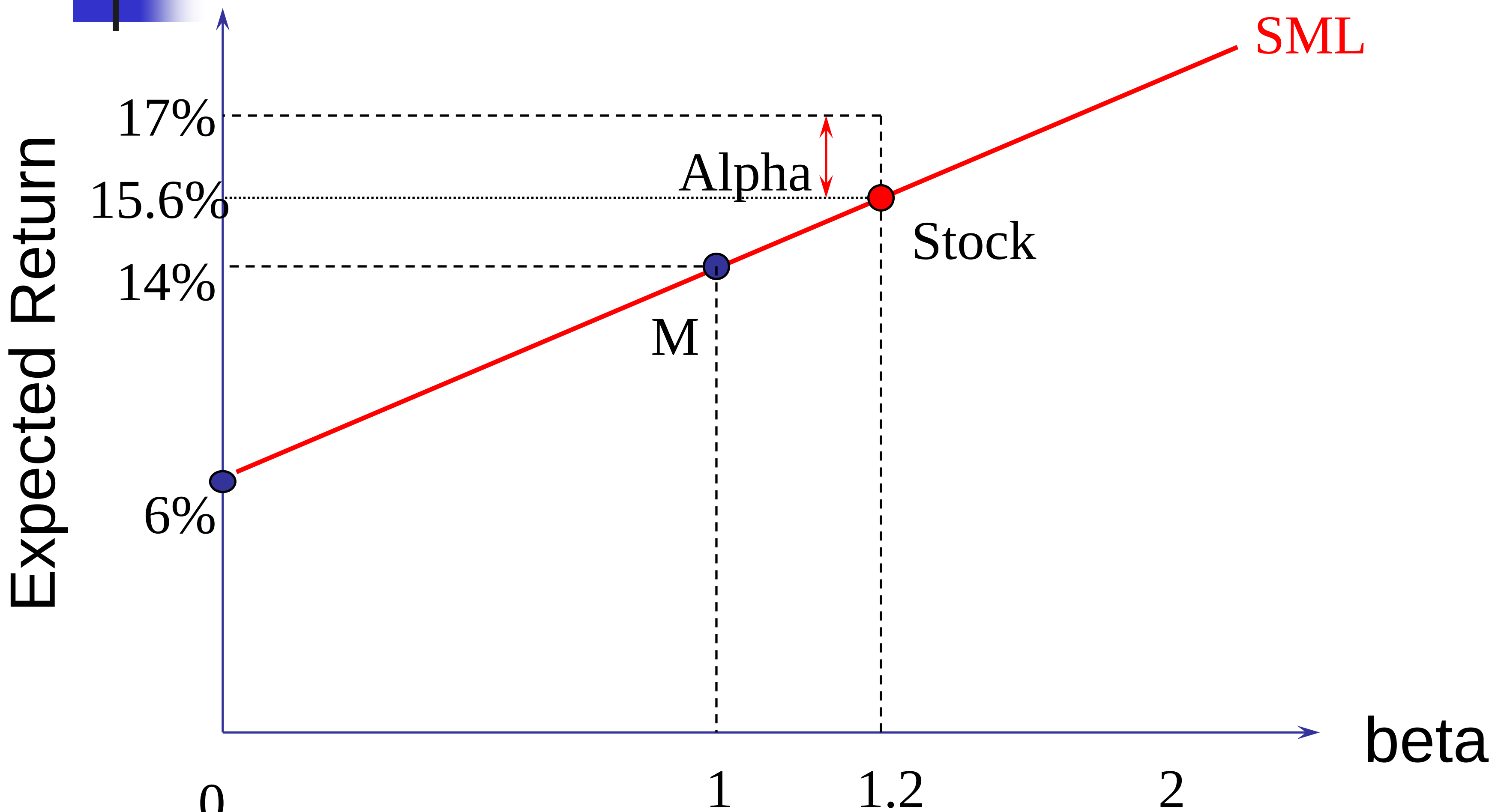
Another Beta Question

- A portfolio contains equal investments in 10 stocks. 5 have a beta of 1.2, 5 have a beta of 1.4
- What is the portfolio beta?
  - 1.3
  - More than 1.3, because the portfolio is not completely diversified
  - Less than 1.3, because diversification reduces beta

Stock Alpha & Beta

- CAPM predicts the *fair* rate of return for a stock
- EXAMPLE:
  - T-Bill rate is 6%
  - Expected market return is 14%
  - Stock has beta of 1.2
  - Then fair return is 6% + 1.2(14%-6%) = 15.6%
- Alpha: difference between *fair* and *expected* return
  - If we expect stock to earn 17%
  - Alpha is 17% - 15.6% = 1.4%

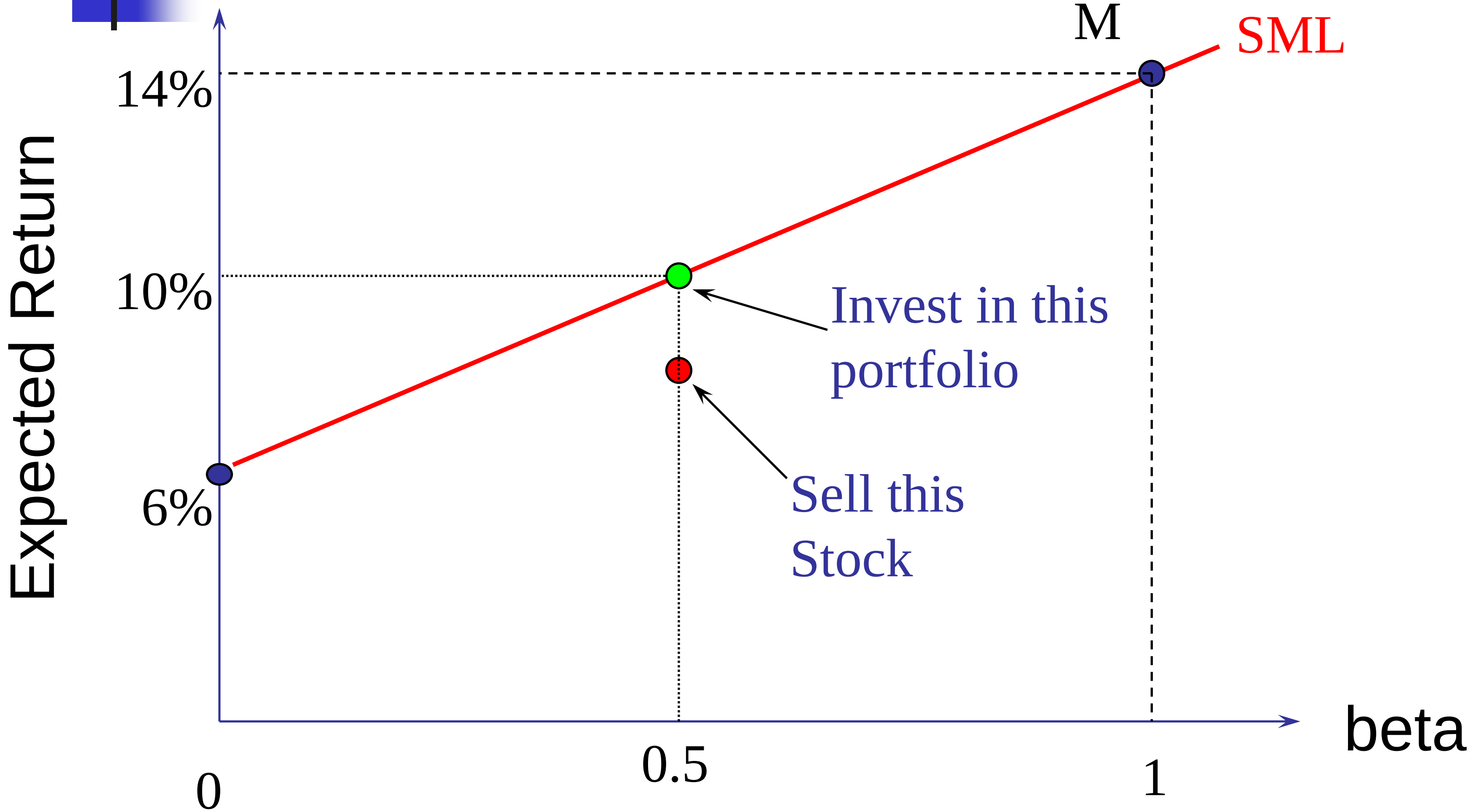
Alphas & SML



CAPM and Security Valuation

- CAPM: stock alphas should be zero
- All securities lie somewhere on the SML - why?
- Example:
    - R_f = 6%, R_M = 14%, stock beta = 0.5
    - CAPM: expected return is 6% + 0.5(14%-6%) = 10%
    - Suppose expected stock return is only 8% (alpha is -2%)
    - Then you would do better to invest 50% in the market portfolio and 50% in the riskless asset
- Arbitrage Argument
    - People would sell securities expected to underperform

CAPM & Arbitrage



Applications of CAPM

- Investment strategy
  - Hold R_f and M in some combination
- Forecasting returns
  - Using asset beta and CAPM equation
  - Check out Merrill's beta book
- Capital budgeting
  - Firm considering project
  - CAPM equation gives required rate of return (hurdle rate) given firm's beta

Issues with CAPM

- The market portfolio is not observable
- Many assumptions
  - Not taxes, costs
  - All investors analyze securities in same way
- Empirical evidence is mixed
- Roll's critique; CAPM not testable!

Summary: CAPM

- The market (index) portfolio M is efficient
- All investors should invest in combinations of M and R_f
- The CAPM equation predicts security returns
- A stock's beta measures its variability relative the the market portfolio

Equity Portfolio Management

- Active vs. Passive Management
- Objectives of Active Management
- Sharpe Ratio
- Market Timing
- Security Selection
- Appraisal Ratio

Passive Management

- Avoids any security selection decision
  - Example: Index Tracking Fund
- Advantages:
  - A good choice for many investors - from CAPM
  - Low cost
  - Free-rider benefit:
    - knowledgeable investors will ensure securities are fairly priced

Rationale for Active Management

- Economic Argument
  - If everyone chooses passive funds, funds under active management will dry up
  - Profits will fall
  - Expensive analysis will be cut
  - Prices will fail to reflect fair value
  - Active Management will be worthwhile again

Rationale for Active Management

- Empirical Argument
  - Some active fund managers outperform over long periods
  - Well known, persistent anomalies
  - Noisy data: some managers may have produced small, but significant abnormal returns
- Motivation: *profitability*

Active Management

- Asset Allocation
  - Choosing between broad asset classes
    - e.g. stocks vs. bonds
- Security Selection
  - Choosing particular securities to include in a portfolio
- Market Timing
    - Asset allocation in which investment in the market is increased when market is forecast to outperform T-bills
  - Market Timer: speculates on broad market moves rather than specific securities

Objectives of Active Management

- Risk-neutral Investor:
  - Maximize expected return
- Risk-averse Investor:
  - Objectives depend on degree of risk aversion
  - Consult every client? No!
  - Form the single optimal risky portfolio M
  - Each client decides how to apportion between the risky portfolio M and riskless T-bills

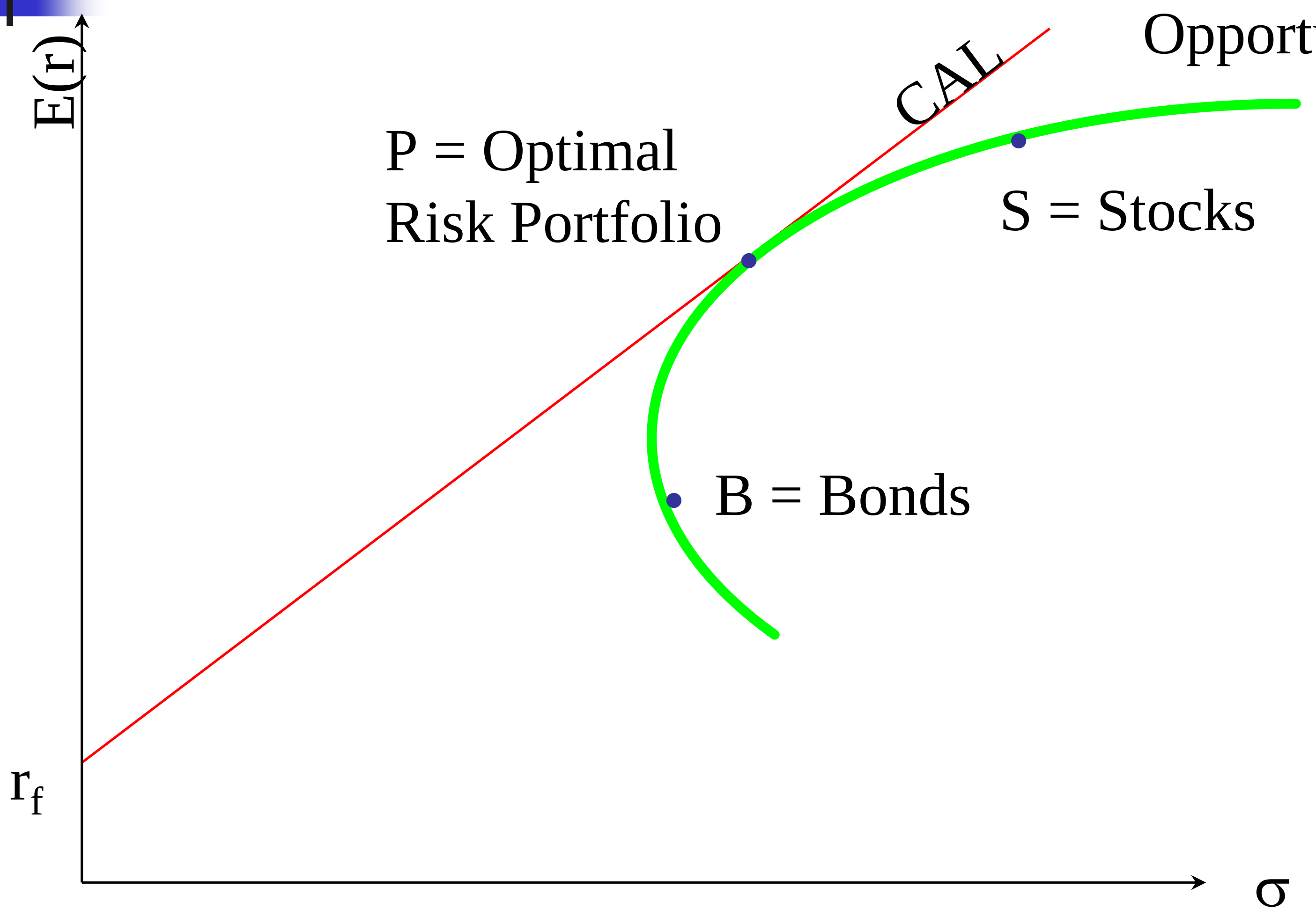
Sharpe Ratio

- How do we find the optimal portfolio M?
- M maximises the reward-to-variability ratio.
- Sharpe Ratio: $S = [E(r_p) - r_f] / \sigma_p$
- A good manager:
  - Maximizes the Sharpe Ratio
  - Maximizes the slope of the CAL
  - Has a steeper CAL than a passive strategy

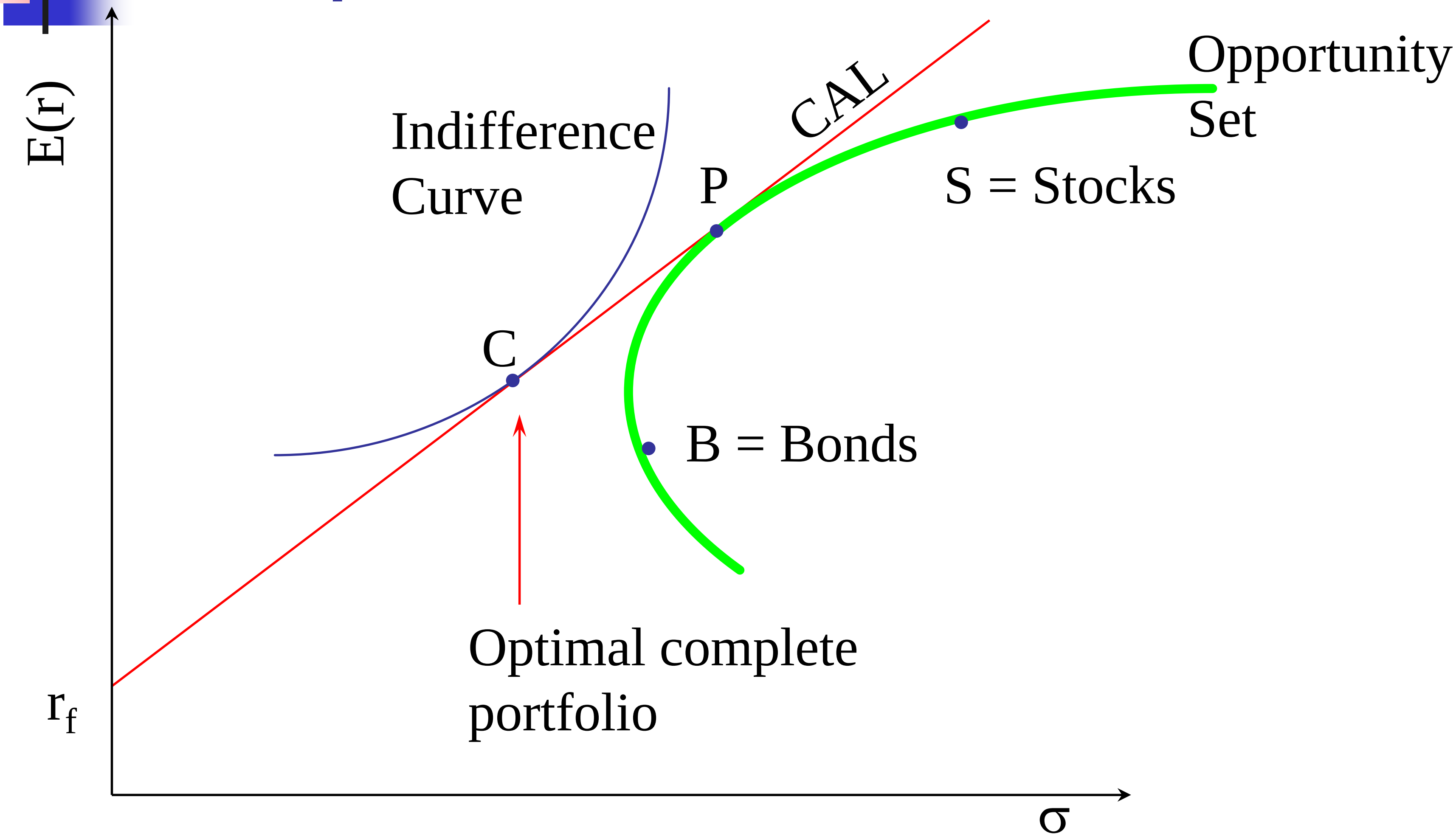
Asset Allocation

- What proportion to hold in stocks, bonds and bills
- Probably the most important investment decision
- How to proceed: Markowitz!

Asset Allocation - Optimal Risky Portfolio

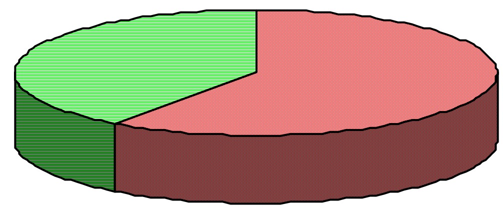


Asset Allocation - Optimal Complete Portfolio



The Optimal Risky Portfolio

- Composition



- Risk-Return Characteristics
  - Expected Return: $E(r_p) \sim 11\%$
  - Risk: $\sigma_p \sim 14\%$

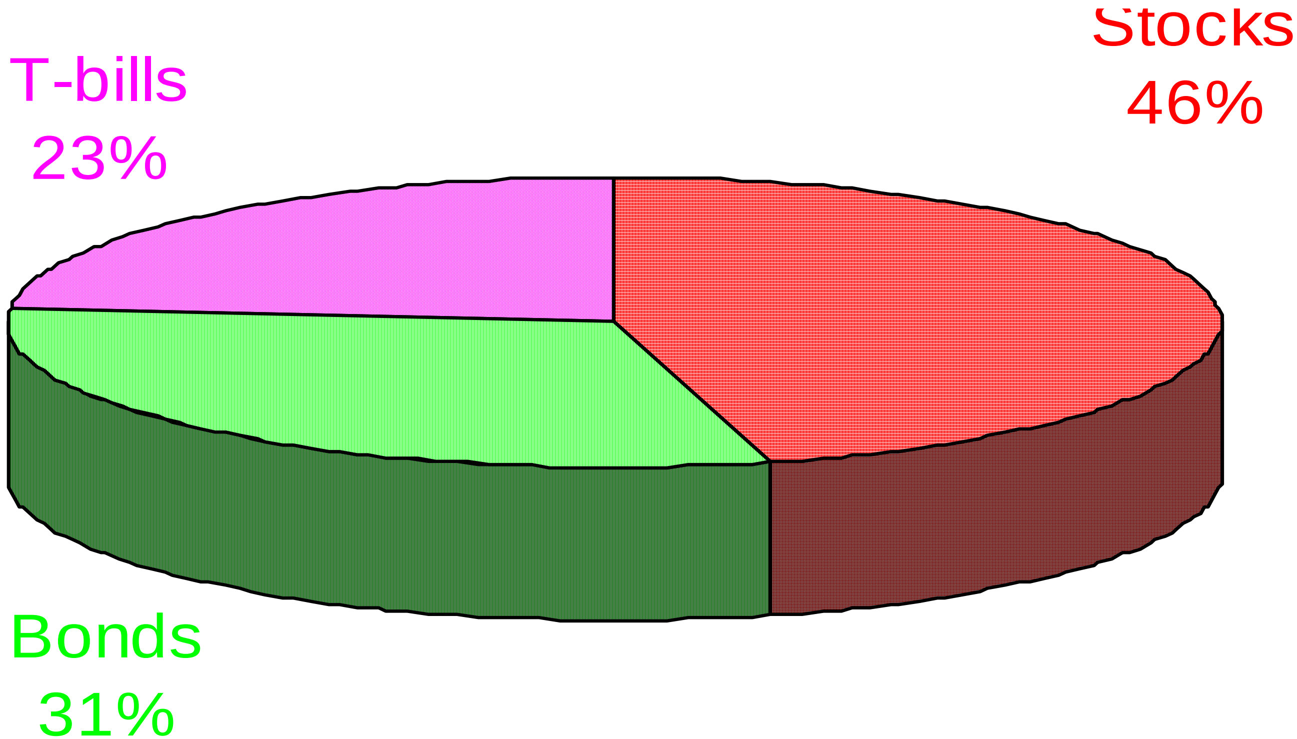
The Optimal Complete Portfolio

- Depends on *risk-aversion factor,* A
- Formula for proportion invested in risky portfolio P:
  - $y = [E(r_p) - r_f] / (0.01 \times A\sigma^2_p)$
- Remainder invested in T-bills

Optimal Complete Portfolio - Example

- Highly risk-averse: A = 4
  - r_f = 5%, $E(r_p)$ = 11%, σ_p = 14%
- We would invest:
  - y = [11 - 5] / (0.01 x 4 x 14^2) = 76.53% in the risky portfolio P
  - 23.47% in T-bills
- Make-up of Optimal Complete Portfolio:
  - 23.47% in T-bills
  - 76.53% x [60%stocks, 40% bonds]
    - 76.53% x 60% = 45.92% stocks
    - 76.53% x 40% = 30.61% bonds

Optimal Complete Portfolio - Example



Asset Allocation & Security Selection

- Why distinguish between two?
  - Process of constructing efficient frontier is identical
- Reason: asset classes so broad
  - Specialist expertise required
- In practice:
  - Optimize security selection for each asset class independently
  - Snr mgt. handles asset allocation

Security Selection

- From CAPM:
  - $r_i = r_f + \beta_i(r_M - r_f) + e_i + \alpha_i$
  - e_i is the firm-specific disturbance (zero mean)
  - α_i is the extra expected return (stock alpha)
- Focus on finding stocks α_i for which is > 0
- Select these stocks for an active portfolio A
- Then mix the active portfolio with the passive index portfolio, to create optimal portfolio P

Appraisal Ratio

- Sharpe Ratio for the Optimal Portfolio P
  - $S^2_P = S^2_M + [\alpha_A / \sigma(e_A)]^2$
    $= [(E(r_M) - r_f) / \sigma_M]^2 + [\alpha_A / \sigma(e_A)]^2$
- Appraisal Ratio
  - $[\alpha_A / \sigma(e_A)]^2 = \Sigma[\alpha_i / \sigma(e_i)]^2$
  - Appraisal Ratio = $\alpha_i / \sigma(e_i)$
  - Abnormal Return / Firm Specific Risk

Market Timing

- Definition:
  - Speculating on broad market moves, not security specific
- Merton's Example
  - Investor with $1,000 on Jan , 1927
  - Invests for 52 years, until Dec 31, 1978
  - Alternative Strategies:
    - All in 30 day commercial paper - $3,600
    - All in NYSE index (dividends reinvested) - $67,500

Market Timing - Merton's Example

- Suppose the investor could time the market perfectly:
  - Shifts all funds in cp or equities at the start of each period, depending on which will do better
- How much would s/he have made?
- $5.36 *billion*

Forecasting

- Key to market timing is forecasting:
  - Bull markets $r_M > r_f$
  - Bear markets $r_M < r_f$
- How do me measure forecasting accuracy?
  - If you predict cloud/rain in England you will be right 80% of time
  - Not evidence of forecasting ability!

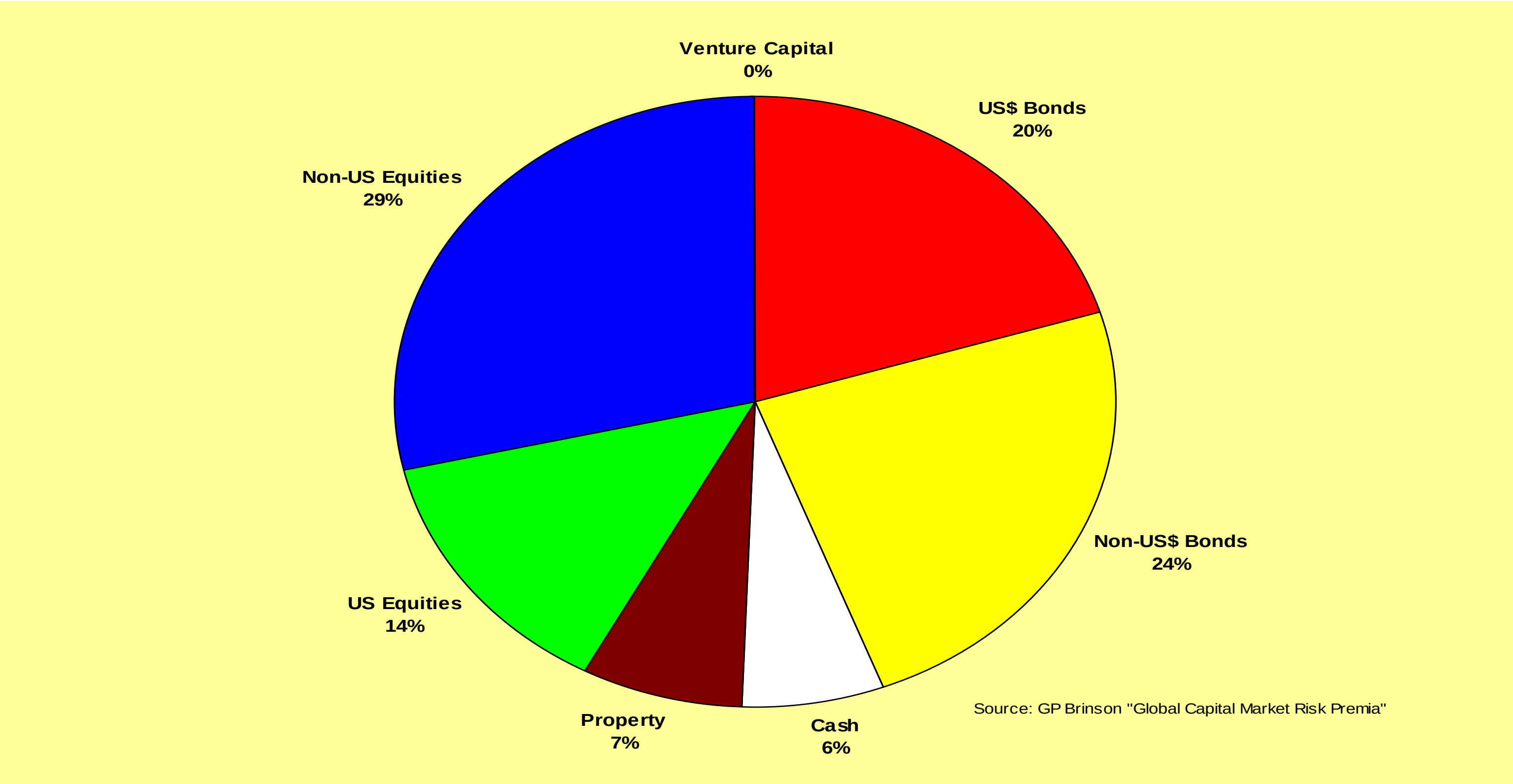
The Bear-Bull Statistic

- Measures forecasting ability
  - P_1 is percentage of correct bull market forecasts
  - P_2 is percentage of correct bear market forecasts
  - $B = P_1 + P_2 - 100\%$
- Example:
  - Investor who is always right on bull and bear calls:
    - $P_1 = P_2 = 100\%$; $B = 100\%$
  - Investor who calls all the bulls (but no bears)
    - $P_1 = 100\%$; $P_2 = 0\%$; $B = 0\%$

International Investment

- Global Wealth
- Global Capital Markets
- International Diversification
- The Global Efficient Frontier
- Risk in International Investment
- Passive & Active International Investment

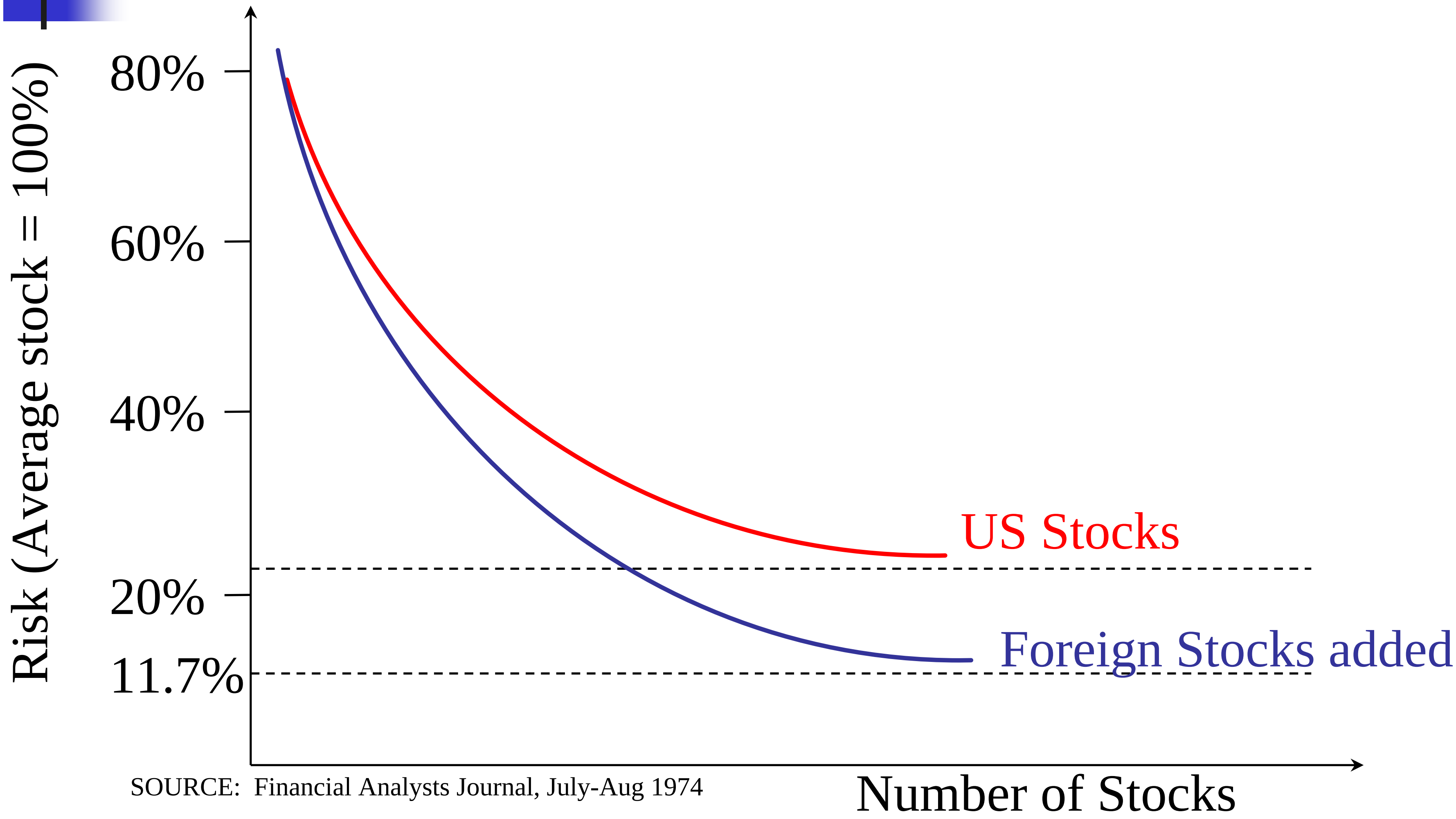
World Capital Markets



US vs. Global Investment

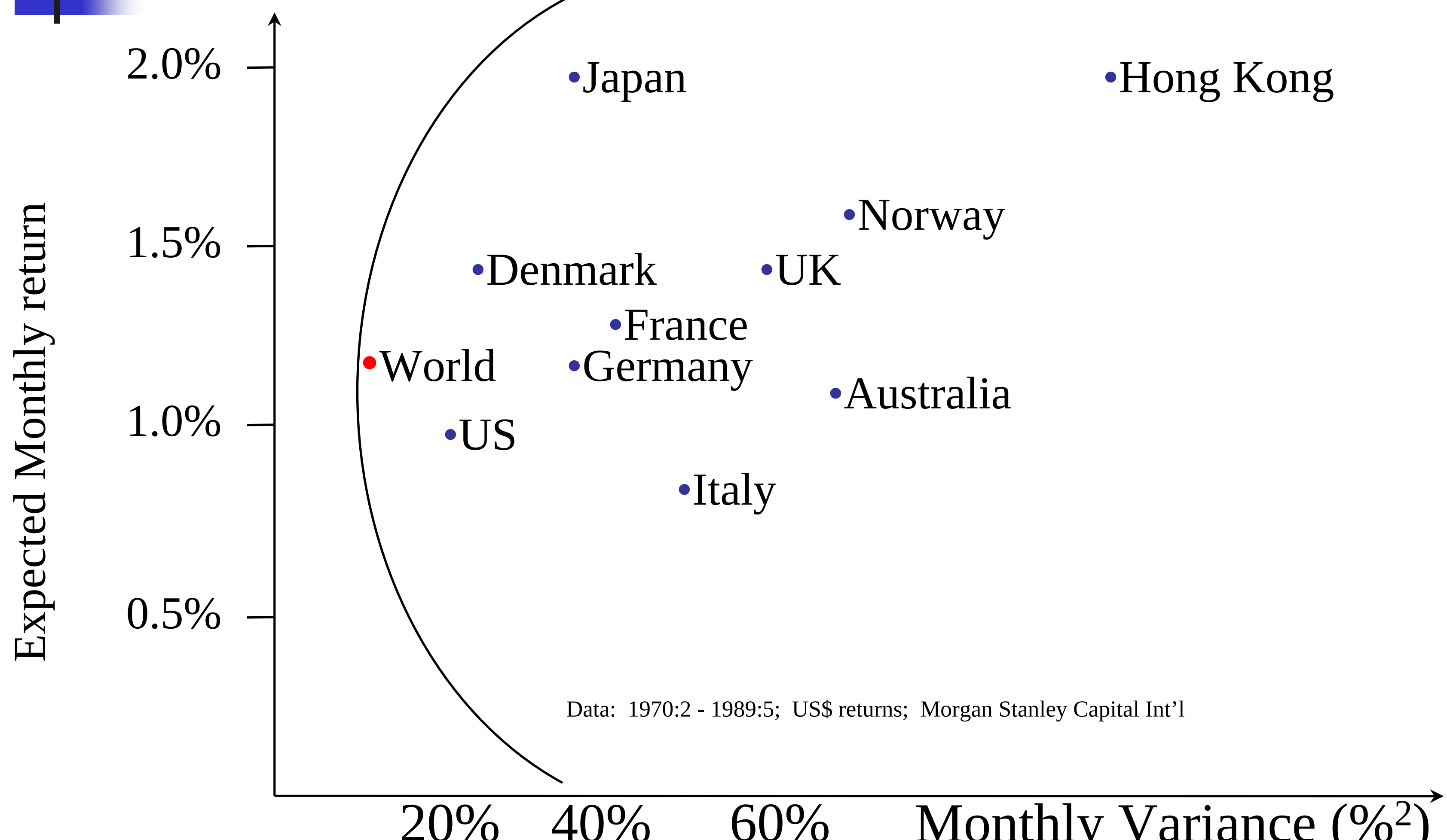
- Traditional US assets are only a fraction of potential universe of investments
- Foreign securities offer additional opportunities for diversification
  - Improves the risk-reward ratio
- International diversification cuts risk of a diversified portfolio
  - From approx. 21% (US stocks only)
  - To approx. 12% (US & foreign stocks)

International Diversification



SOURCE: Financial Analysts Journal, July-Aug 1974

Global Minimum Variance Frontier



SOURCE: Journal of Finance 46 (March 1991)

Techniques for International Investing

- American Depositary Receipts (ADR's)
  - (Claims on) foreign company stock traded on US exchanges
- International Mutual Funds
  - Single country funds
  - Foreign Index funds
  - Emerging market funds
  - Regional funds (European, Pacific Basin, etc.)
- Foreign Index options & futures
  - Nikkei, FTSE, DAX, CAC-40

Risk in International Investment

- Information Risk
  - Lack of available data for analysis
  - Different accounting conventions
- Political Risk
  - Tax policy
  - Appropriation
  - Exchange controls
- Currency Risk
  - Returns depend on exchange rate

Passive International Investing

- Benchmarks
  - EAFE (Morgan Stanley) - Europe, Australia & Far East Index
  - Others by Salomon, Goldman
- Weighting
  - Usually by capitalization
  - Some argue in favour of GDP weighting
- Cross-holdings
  - Equity investments made by one firm in another
  - Inflate the value of outstanding equity

Active International Investing

- Asset Allocation
  - Currency
  - Country
  - Stock/cash/bond
- Security Analysis
  - Major differences in accounting treatment of:
    - Depreciation (US dual system)
    - Reserves (US lower discretionary; also pensions)
    - Taxes (paid or accrued)
    - P/E ratios (Y/E shares vs. year avg. shares)