Portfolio Management – Risk and Return

Time Value of Money

- Simple vs compound interest
- Daycount methods
- Discounting principles

Time Value of Money

- Basic principle
  - Money received *today* is different from money received in the *future*
  - This difference in *value* is called the time value of money
  - When we borrow or lend, this difference is reflected by the *interest rate*

Time Value of Money

- Example:
  - I lend you 100 today but you have to pay me back 110 in one year
    - interest rate is 10%
  - Meaning:
    - 110 in one year has the same value as 100 today
    - or: the 1-year interest rate is 10%

Present and Futures Value

- 110 is the *future value* of 100 today
- 100 is the *present value* of 110 in 1 year's time
- Meaning:
  - 110 in one year has the same value as 100 today
  - or: the 1-year interest rate is 10%

Compound Interest Example

- Suppose interest rate = 10% and I have \$100 to invest
- What will I get in 1 year time?
  - Simple answer: \$110
    - \$100 x (1 + 0.1) = \$110
  - Complex answer: depends on how compute interest
    - By computing interest more frequently I can earn more than \$110

Compounding

- Suppose interest is calculated every 6 months
  - After 6 months, I get interest
    - how much: (1/2)($100 x 0.1) = $5
    - this is (1/2) a year's interest
    - now, my account balance is $105.
  - At the end of the year, I earn interest for the second half of the year *on $105*
    - how much: (1/2)($105 x 0.1) = $5.25
  - Now I have $110.25
    - I made $0.25 extra!

Compound Interest

- The extra bit is the “interest on the interest”
  - 10% applied for six months on $5
    - (1/2)($5*0.1) = $0.25
- This is called compounding
- If you are a lender, compounding more frequently is better
- If you are a borrower, you don’t like compounding

Compounding Frequency

- So you have to be careful to take account of how frequently interest is compounded
  - annually: r applied once
  - semi-annually: r/2 applied every 6 months
  - quarterly: r/4 applied every 3 months
  - daily: r/365 applied every day
  - "continuously": applied at every instant of time!
    - how does this work?

Compounding over Multiple Periods

- Initially invest P_0, at interest rate r, for n periods
- Compound by (1+r/n) each period:

P_0 → $P_0(1+r/n)$ → $P_0(1+r/n)(1+r/n)$

$= P_0(1+r/n)^2$

0 ——— 1 ——— 2

Compounding over Multiple Periods

Year	Investment	Compound Factor	Future Value
1	P_0 ↓	$(1+r/n)$	$P_0(1+r/n)$
2	$P_0(1+r/n)$	$(1+r/n)$	$P_0(1+r/n)^2$
.	.	.	.
.	.	.	.
.	.	.	.
n	$P_0(1+r/n)^{n-1}$	$(1+r/n)$	$P_0(1+r/n)^n$

Time Value of Money Equation

- If I invest $\$P_0$ today, what will be the value of my investment P_n after n periods?

- $P_0 \quad \times \quad (1 + r/n)^n \quad = \quad P_n$

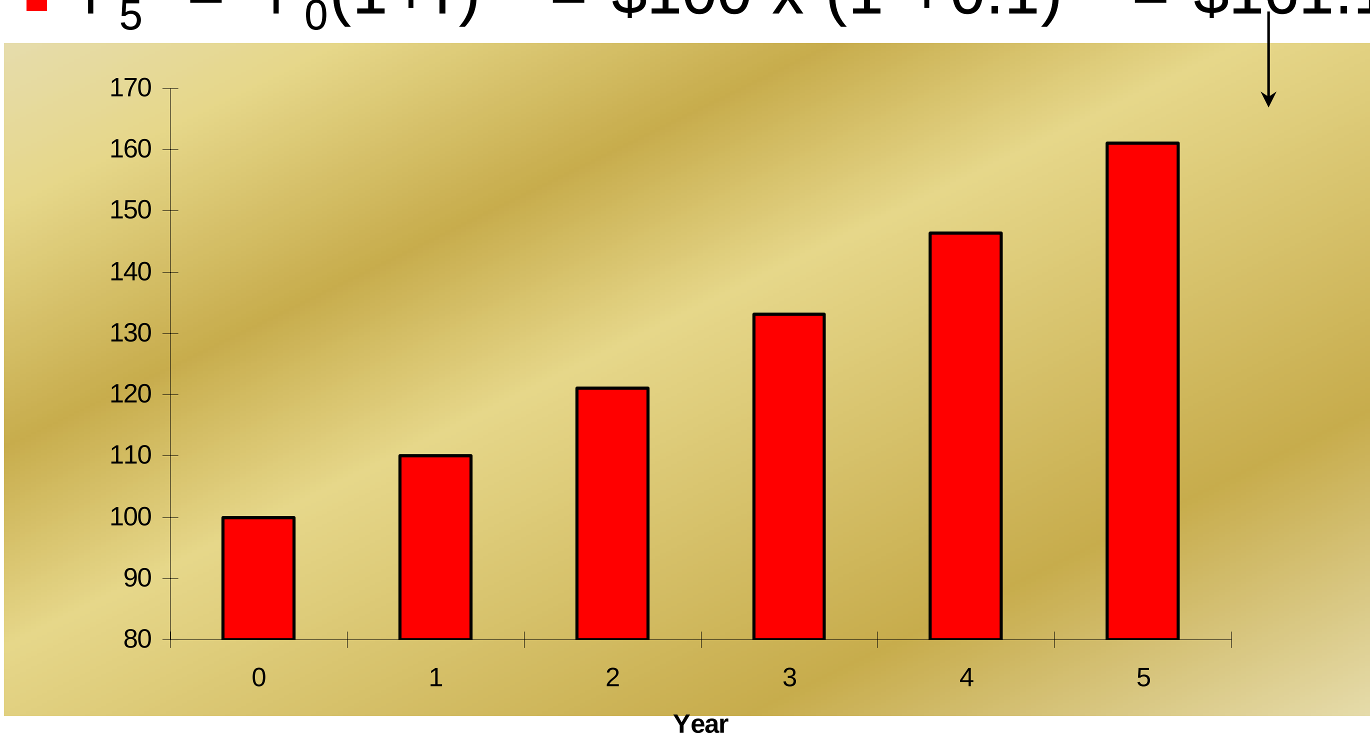
•Present Value
•Current Price
•Price at time 0

•Compound Factor
•Discount rate
•Internal rate of return
•Yield to maturity

•Future Value
•Ending Price
•Price at time N

Compounding Example

- Interest rate 10%, P = $100, compound annual:
  - $P_5 = P_0(1+r)^5 = \$100 \times (1+0.1)^5 = \$161.1$

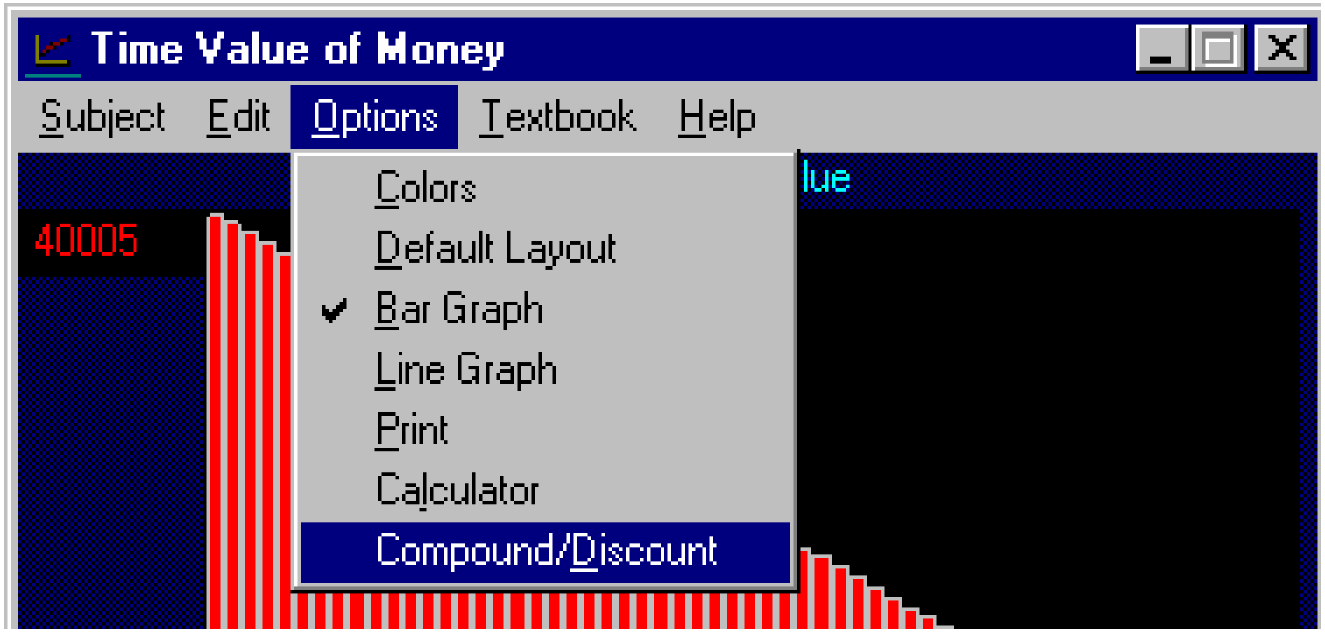


Compounding Factors

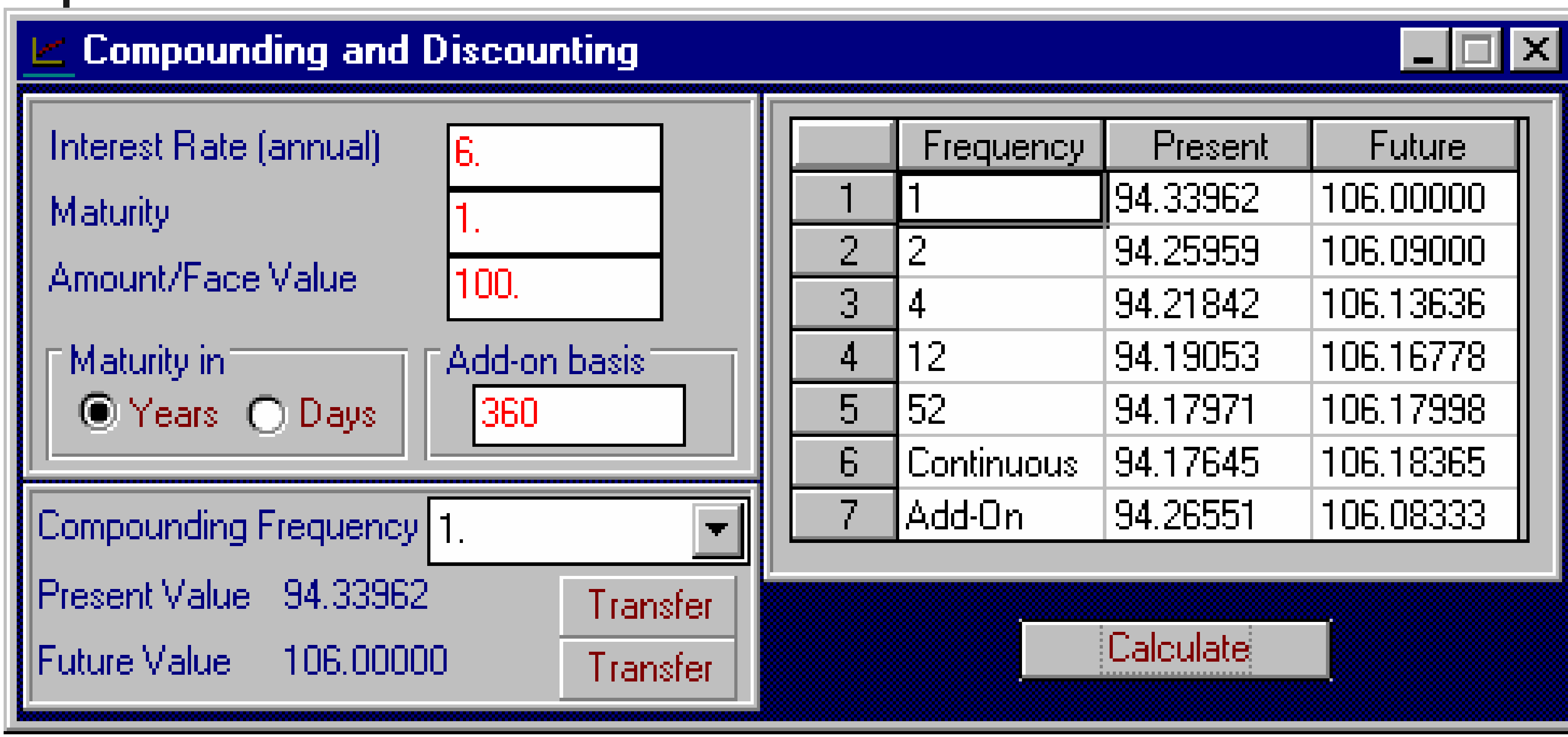
- Interest rates quoted on an annual basis
- Compounding Factors:
  - Annual: $(1+r)^n$, applied every year
  - Semi-annual: $(1+r/2)^{2n}$, applied every 6m
    - typically used for treasuries
  - Quarterly: $(1+r/4)^{4n}$, applied every qtr.
  - Daily: $(1+r/365)^{365n}$, applied every day.
  - n times a year: $(1+r/n)^{nt}$
  - Continuous: e^{rt}, limit as n increases infinitely

Quick Bond Tutor Exercises

- Select “Time Value of Money”
- Bring up “Compound/Discount”



Bond Tutor: Compound Interest



	Frequency	Present	Future
1	1	94.33962	106.00000
2	2	94.25959	106.09000
3	4	94.21842	106.13636
4	12	94.19053	106.16778
5	52	94.17971	106.17998
6	Continuous	94.17645	106.18365
7	Add-On	94.26551	106.08333

Bond Tutor: Compound Interest

- Look at how compounding changes the *future* value of 100
  - the frequency is the number of times a year the interest is applied

	Frequency	Future
1	1	106.00000
2	2	106.09000
3	4	106.13636
4	12	106.16778
5	52	106.17998
6	Continuous	106.18365

Simple Interest

- An old convention: pre-calculator
- Invest $100 for 90 days at 10%, simple interest
- Many markets: 360 day year
- After 90 days you have:
- $100 (1 + 10% x 90 / 360) = $102.50

Discounting

- Discounting is just the reverse of compounding:
- P_n in n periods is worth $P_0 = P_n / (1+r/n)^n$ today

$$P_0 \quad = \quad P_n \quad x \quad 1 / (1 + r/n)^n$$

•Present Value

•Current Price

•Price at time 0

•Future Value

•Ending Price

•Price at time n

Discount Factor

Discount Factors

- Always one or less
  - Cash today worth more than cash in future
- Always greater than zero
  - Cash is always worth having, no matter how far in the future
- Always decreasing
  - Cash gets less valuable the further away it is in time

Measuring Past Returns

- Holding Period Return
- Discounted Cash Flow
- Average Return
- Geometric Average Return

Holding Period Return

- HPR = (Ending Share Price - Beginning Price) + Cash Dividend / Beginning Price

  = Capital Gain + Dividend

- E.G.. Share price = $100, Ending price = $110, Dividend = $4

  HPR = ($110 - $100) + $4 / $100

  = 0.14, or 14%

Time Value Example

- Suppose I invest \$100 today, to get \$110 in 1 year
  - What is my rate of return?
  - Use compound interest model:
    \$100 x (1 + r) = \$110, so r = 0.1 or 10%
- If I invest \$100 today to get \$121 in 2 years
  - \$100 x $(1+r)^2$ = \$121, so again r = 10%

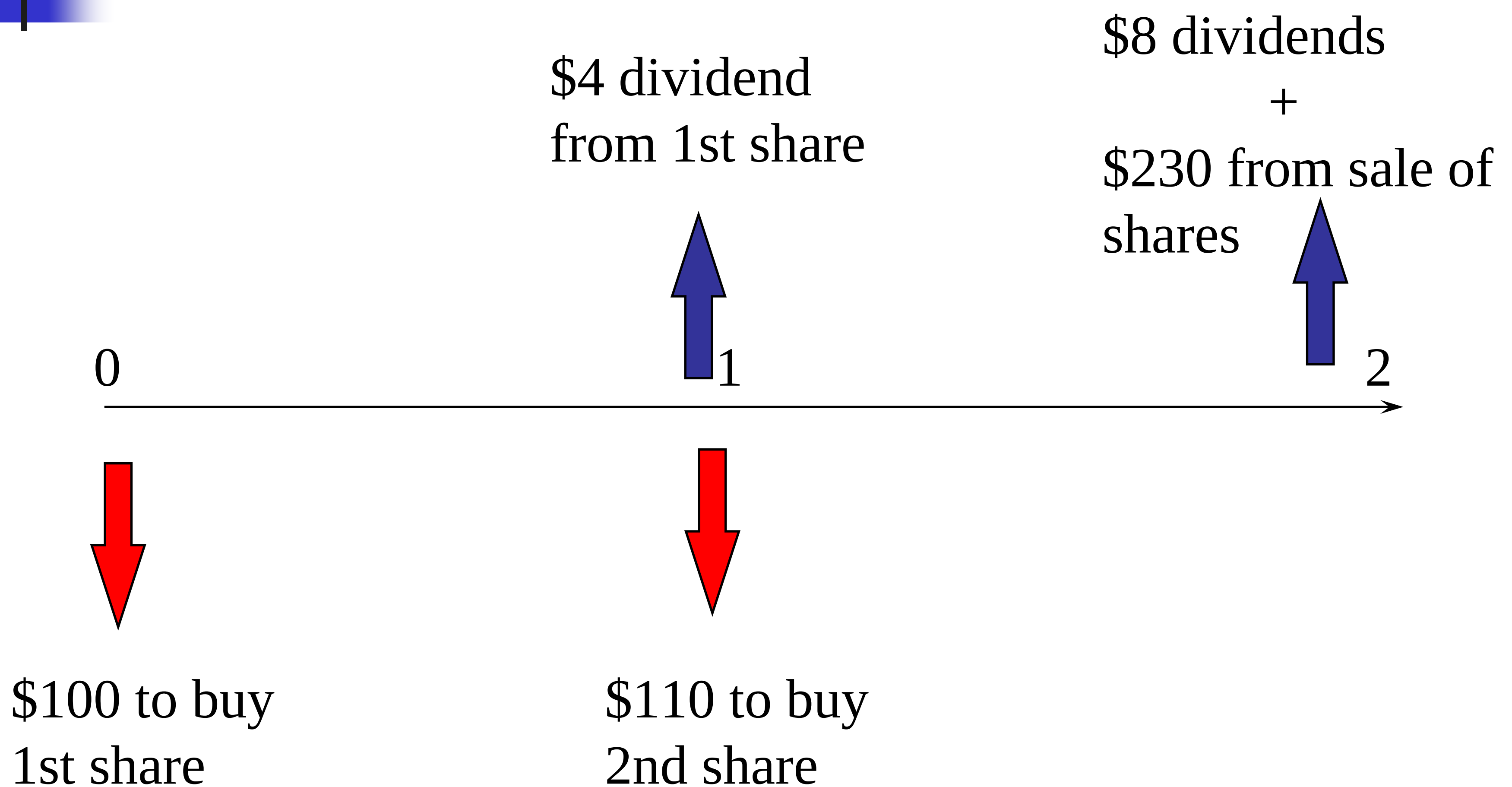
Time Value and HPR

- E.G. Share price = $100, Ending price = $110, Dividend = $4
- P0 = $100
- P1 = $110 + $4 = $114
- Hence $100 x (1 + r) = $114
- r = 14%
- So IRR is the same as the HPR, in this case

Multiple Period Returns

- Buy share at the start of year 1, as before
- Now purchase another share at end of year 1
- Hold both shares until end of year 2
- Sell both shares at the end of year 2 for $115 each

Cash Flows


$8 dividends
+
$230 from sale of
shares
$4 dividend
from 1st share
0
1
2
$100 to buy
1st share
$110 to buy
2nd share

Dollar Weighted Return

- Use DCF Approach:
- $$\$100 + \frac{\$110}{(1+r)} = \frac{\$4}{(1+r)} + \frac{\$238}{(1+r)^2}$$
- r = 10.12%
- This is the IRR or Dollar-Weighted Return
- Stock's performance in 2nd year has more influence as more dollars invested

Time Weighted Return

- Return in 1st Year:
  - $\frac{(\$110 - \$100) + \$4}{\$100} = 14\%$
- Return in 2nd Year
  - $\frac{(\$115 - \$110) + \$4}{\$110} = 8.18\%$
- Average Return over 2 Years:
  - $\frac{14\% + 8.18\%}{2} = 11.09\%$

Dollar vs Time Weighting

- Which to use?
- Money management industry uses *Time-Weighted Returns*
  - because money managers often have no control over timing or amount of investments
  - Example: pension fund manager

Geometric Average Returns

$$\xrightarrow{r_1} (1 + r_1) \xrightarrow{r_2} (1 + r_1) \times (1 + r_2)$$

$$\xrightarrow{r_G} (1 + r_G)^2$$

- R_G is the compound average growth rate
- In previous example:
  - $(1 + R_G)^2 = (1 + 0.14) \times (1 + 0.0818)$
  - $R_G = 11.05\%$

General Formulas Compared

Geometric Average:

$$R_G = [(1 + r_1) x (1 + r_2) \ x \ldots x (1 + r_N)]^{1/N}$$

Time-Weighted Average:

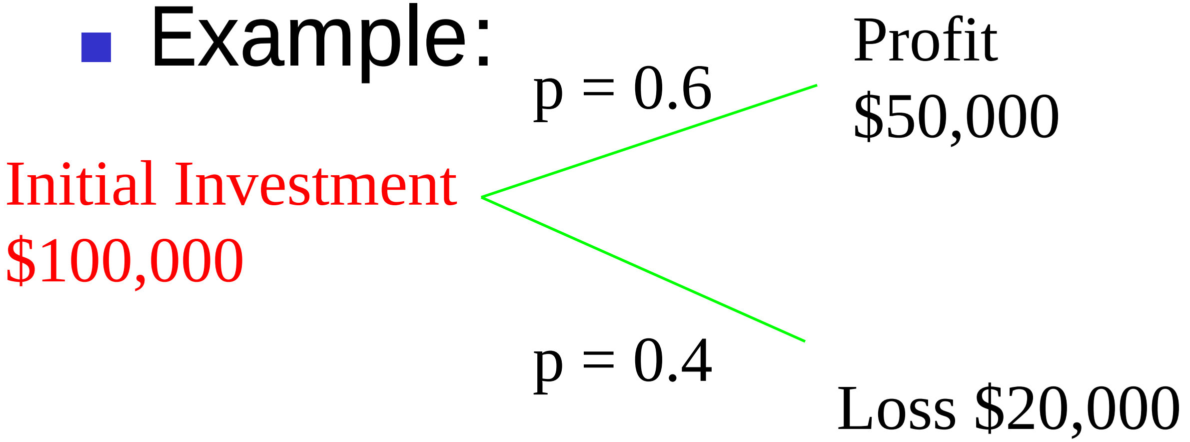
$$R_A = \frac{r_1 + r_2 + \ldots + r_N}{N}$$

Time Weighted vs. Geometric Average

- Which is better?
- Historic Returns:
  - *Geometric Average* gives exact constant rate of return which would have been needed to match actual historical performance
- Future Returns:
  - *Time weighted average* is better because it is an unbiased estimate of the portfolio's expected future return

Risk & Return

- Risk: *uncertain outcome*
  - When more than one outcome is possible
- Example:



Expected Profit

- $E(P) = pP_1 + (1\text{-}p)P_2$
- E(P) = 0.6 x $50,000 + 0.4 x (-$20,000)
- E(P) = $22,000

Standard Deviation as Measure of Risk

- *Variance* is the expected value of the squared deviations of each possible outcome from the mean
  - $\sigma^2 = p[P_1 - E(P)]^2 + (1-p)\,[P_2 - E(P)]^2$
  - $\sigma^2 = 0.6 \times [50{,}000 - 22{,}000]^2 + 0.4[-20{,}000 - 22{,}000]^2$
  - $\sigma^2 = 1{,}176{,}000{,}000$
- The *Standard Deviation* is the square root of the variance:
  - $\sigma = 34{,}292.86$
- This is a risky investment: Standard Deviation is much bigger than the Expected Profit

Risk Premium

- Suppose we could invest in an alternative riskless asset (e.g. T-bills) paying 5% p.a.
  - Yields a sure profit of $5,000
- The incremental profit, or *risk premium*, is:
  - $22,000 - $5,000 = $17,000
- <u>This risk premium is the compensation we receive for the risk of the investment</u>

Expected Returns

State of Economy	Probability	Ending Price	HPR
Boom	.25	$140	44%
Normal	.5	$110	14%
Recession	.25	$80	-16%

Expected Return $$E(r) = \sum_s p(s)r(s)$$

In this case: E(r) = (0.25 x 44%) + (0.5 x 14%) + ($0.25 x -16%)
= 14%

For historical data, the time-weighted average return is an unbiased estimate of the expected return

Standard Deviation

- The *standard deviation* of the rate of return is a measure of risk

$$\sigma = \sqrt{\sum_s p(s)[r(s) - E(r)]^2}$$

In this example: standard deviation = 21.21%

For historical data, the sample standard deviation is an unbiased estimate of the true standard deviation :

$$sd = \sqrt{\sum_t \frac{[r_t - \bar{r}]^2}{(N-1)}}$$

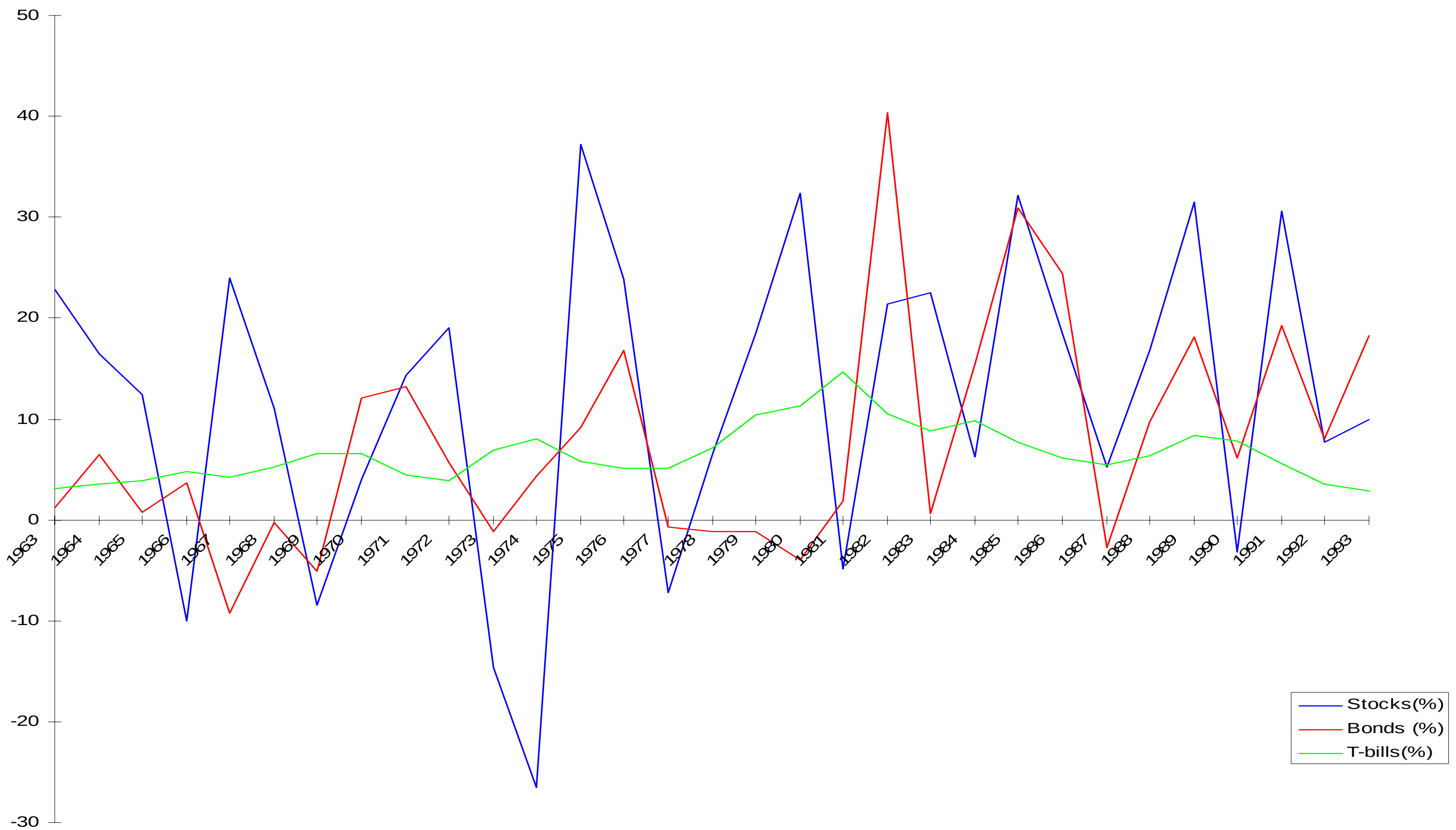
Risk-Free Rate & Risk Premium

- The *risk-free rate* r_f :
  - the rate you can earn on a riskless asset, T-bills
- The *risk premium*
  - Difference between the *expected* HPR on the portfolio and the risk-free rate
    - e.g. if r_f = 6%, and the portfolio expected HPR is 14% , the risk premium s 8%

Excess return

- Difference between the *actual* return on the portfolio and the risk-free rate
- So the risk premium is the expected excess return

Historical Rates of Return from Stocks, Bonds & Bills 1963-1993



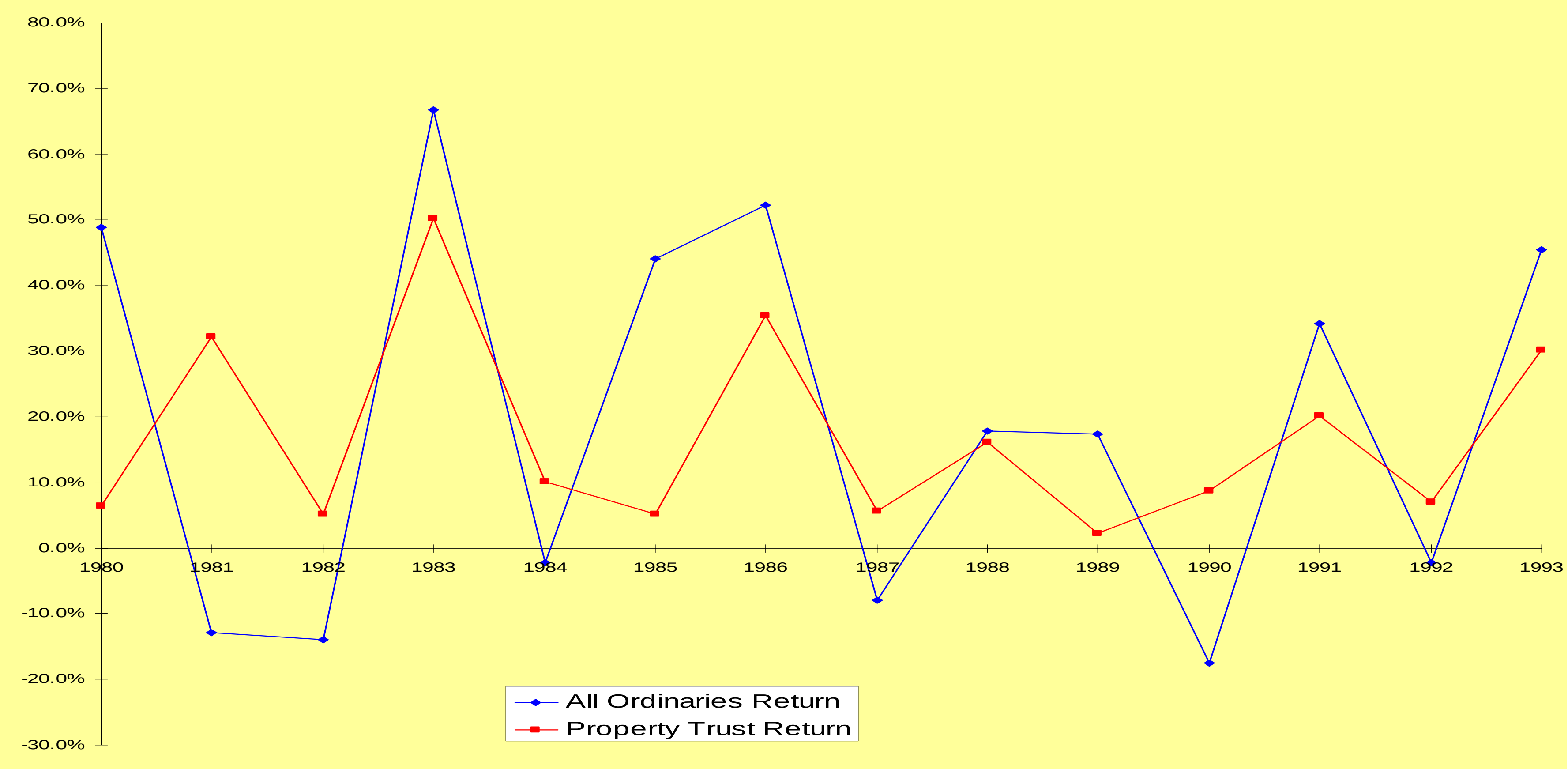
The Risk-Return Trade-Off Market Data from 1963-1993

Series	Average Return	Standard Deviation
T-Bills	6.57%	2.73%
Treasury Bonds	7.78%	11.12%
Stocks	11.94%	15.43%
Inflation	5.24%	3.22%

What to Invest in & When

Investment	Recession	Boom	---- Inflation ----- High	Low
Govt. Bonds	17%	4%	-1%	8%
Commodity index	1	- 6	15	- 5
Diamonds	- 4	8	79	15
Gold	- 8	- 9	105	19
Private home	4	6	6	5
Stocks	14	7	- 3	21
Stocks (Low Cap)	17	14	7	12
T-bills	6	5	7	3

Workshop: Australian Index Returns (1)



Next:

- Risk & Risk Preferences
- Utility
- Portfolio Theory

Market Efficiency

- Role of capital markets
- The Efficient Market Hypothesis
- Tests of the EMH
- Market Anomalies
- Alternative Hypotheses

Market Efficiency

- Kendall study in 1953:
  - No predictable patterns in stock prices
  - As likely to go up as down on any given day
  - Regardless of past performance

Market Predictability

- What would happen if prices were predictable?
  - If model predicted stock price would rise from $100 to $110 in 3 days time
  - Everyone would buy, no-one would sell below $110
  - Stock price would jump *immediately* to $110
- Conclusion:
  - any information that could be used to predict stock prices must already be reflected in current prices
  - this is what we mean by market efficiency

Random Walk

- Prices already incorporate *current* information
- Prices change in response to *new* information
- New information arrives unpredictably
  - If it was predictable, it would be part of today's information
- Hence stock prices must also change unpredictably
- Stock prices follow a random walk

Randomness and Rationality

- Stock price levels are rational
- Stock price *changes* are random
  - because new information arrives randomly
- The stock price changes to reflect “fair value” given the new information
- Doesn’t mean that prices always 100% ‘fair’:
  - They are, on average
  - Sometimes overvalued, sometimes undervalued
  - You can’t tell which!

The Efficient Market Hypothesis

- Weak Form
  - Stock prices reflect all historical data
- Semistrong Form
  - Stock prices reflect all past data
  - And, currently published data
- Strong Form
  - Prices reflect *all* information, including inside information

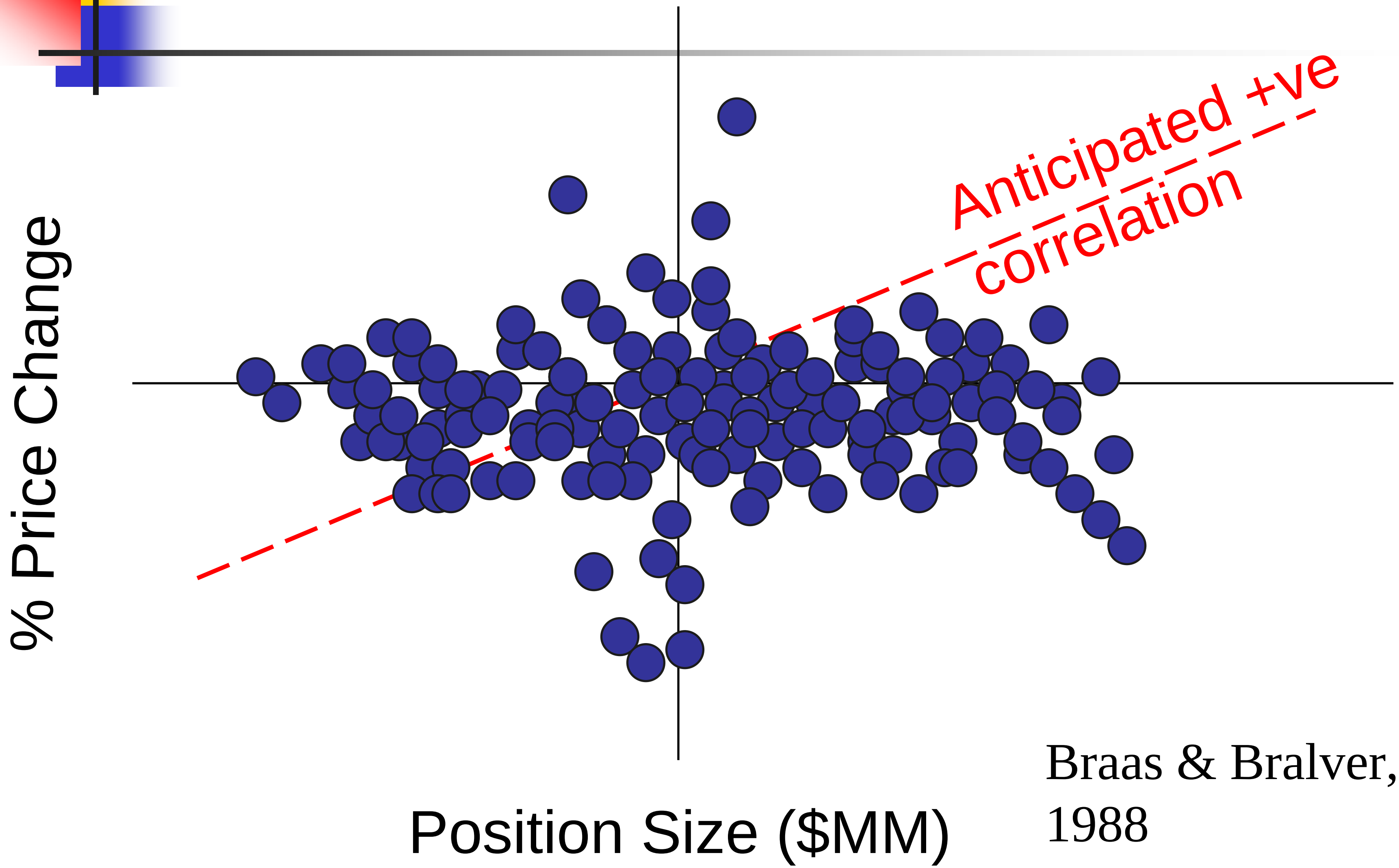
Implications of EMH

- Technical analysis is a waste of time
  - Based on analysis of historical data
- Fundamental analysis is mostly a waste of time
  - Prices already reflect published information
  - Can make money only if analysis is somehow superior
  - Or if a stock is somehow 'neglected'
- Active vs. Passive Portfolio Management
  - Stock picking is unlikely to pay off
  - Any stock mispricing will be too small to offset costs
  - Prefer a passive, buy & hold strategy, e.g. index fund
  - Keep costs to a minimum

Wall Street & the EMH

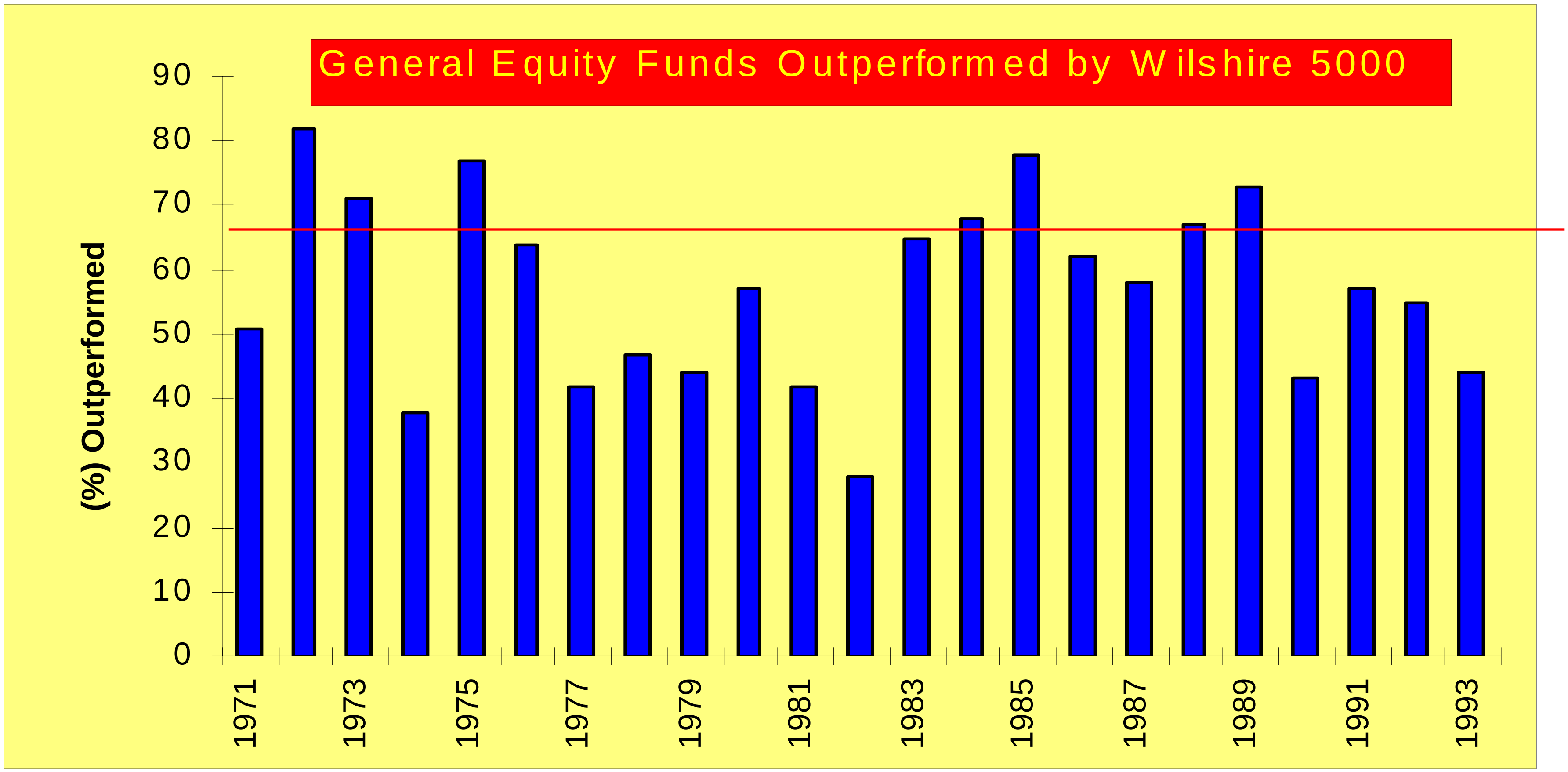
- Overwhelming evidence for EMH
- Widely disregarded by Wall Street - why?
  - runs counter to what it stands for!
- How does the Street make money?
  - Clients: fees, commissions, services etc.
  - Insider trading/privileged information
  - Relatively small amount from trading own capital

The Position Taking Myth



Braas & Bralver, 1988

Mutual Fund Performance



Other Findings

- Frank Russell Study:
  - Past performance of fund managers has little predictive power
- Blake, Elton, Gruber (1993)
  - Fixed income mutual funds underperform passive indices by an amount equal to expenses
- Cahart (1992)
  - There are consistent *underperformers* (due to expenses)

Implications for Private Investors

- Job 1: <u>minimize costs</u>
- Starting point:
  - T-Bills & inexpensive passive stock & bond funds
- Allocate capital according to risk & tax profile
- Only actively trade if you have:-
    - Privileged information
    - Evidence of a consistently large & tradable market anomaly
    - A new method of analysis which you can demonstrate is superior to Wall Street
    - Good luck and/or willingness to lose money for fun!

Market Efficiency - Conclusions

- Markets appear highly efficient most of the time
- Little evidence of consistently superior fund management performance
- Costs are important factor in overall performance
- Hence a spread of low-cost index funds is recommended

Risk Preferences

- Suppose I offer you:
  - Either: A sure profit of $1,000,000
  - Or: A profit of $2,000,000 if you toss a coin which turns up heads, $0 if it turns up tails
- Which alternative would you prefer?

Risk Preferences

- Alternatives Compared
  - The first alternative is riskless, the second is risky
  - Both alternatives offer the same expected return ($1,000,000)
  - So there is *zero risk premium*
- A prospect with a zero risk premium is called a *fair game*

Investor Risk Profiles

- *Risk-neutral* investors will accept fair games
  - they don't require a risk premium
- *Risk-lovers* will play a fair games
  - even *pay* a premium to take risk (gamblers)
- *Risk-averse* investors
  - will only consider risk-free investments
  - or risky investments which pay a risk premium

Utility

- We need a value system which incorporates *both* risk and return
- Utility Function: $U = E(r) - 0.005A\sigma^2$
  - The Expected Return is reduced by a factor depending on the risk
- Risk-averse investors will have $A > 0$
  - The greater A is, the greater the risk aversion

Utility Example:

- Investment Alternatives:
  - A portfolio has an E(r) of 20% and s.d. of 20%
  - T-Bills pay 7%
- If the Utility factor A is 4:
  - $U = 20\% - 0.005 \times 4 \times (20\%)^2 = 12\%$
  - Utility of T-bills is 7%
  - So prefer the portfolio
- If A is 8?

Portfolio Returns

- A portfolio is a diverse collection of assets $A_1, A_2, \ldots, A_N$
- We invest a proportion w_i in asset A_i
- The w_i are called *weights* and sum to 1.
- The Expected Return on the portfolio is

$$W_1E(r_1) + W_2E(r_2)$$

  - $E(r_1)$ is the expected return on asset A_1

Portfolio Returns - Example

- Suppose we have a two asset portfolio
  - A_1 is a stock with a 15% expected return
  - A_2 is a riskless T-bill with expected return 6%
- Assume we invest 50% in the stock and 50% in T-bills
  - The $W_1 = W_2 = 0.5$
- Expected portfolio return is
  - $0.5(15\%) + 0.5(6\%) = 10.5\%$

Covariance - ASX Example

- Excel Workbook: IF_Labs.xls
  - Select Chart ASX Returns from main screen
- Look at chart of ASX returns
  - When the All Ordinaries index rises so does the Property Trust index
  - The indices move together, or *co-vary*.

Covariance Defined

- Define *covariance* of two random variables:

$$\sigma(x, y) = \sum_s \Pr(s)[x(s) - E(x)][y(s) - E(y)]$$

- The unbiased sample covariance is

$$Cov(x, y) = \frac{1}{N} \sum_t [x_t - \bar{x}][y_t - \bar{y}]$$

Correlation

- Covariance is difficult to interpret
  - its size depends on units of x and y
- Instead, use *correlation coefficient*
  - always between +1 and -1

$$\rho(x,y) = \frac{\sigma(x,y)}{\sigma_x \sigma_y}$$

- Estimate using sample correlation:

$$r(x,y) = \frac{\text{cov}(x,y)}{Sd(x)Sd(y)}$$

Portfolio Risk

- For a two asset portfolio:

$$\sigma_p = \sqrt{[w_1^2\sigma_1^2 + w_2^2\sigma_2^2 + 2w_1w_2\sigma_{12}]}$$

  - where σ_{12} is the covariance between assets 1 and 2

- Estimate portfolio standard deviation using:

$$Sd_p = \sqrt{w_1^2 Sd_1^2 + w_2^2 Sd_2^2 + 2w_1w_2\operatorname{cov}(1,2)}$$

Portfolio Risk - Example

- Suppose you invest 50% in each of two stocks
  - the Sd of the two stocks is 20% and 10%
  - the covariance between the two stocks is .01
- Then the portfolio Sd is:
  - $[(0.5)^2 (0.2)^2 + (0.5)^2 (0.1)^2 + 2(0.5)(0.5)(0.01)]^{1/2}$
  - 13.2%

Lab - ASX Portfolios

- Excel Workbook: Investment Math
- Select ASX from Labs Menu
  - Select Worksheet1
  - For solution select Solution 1
- Do ASX Lab Parts 1 & 2
  - ASX - Part 1
  - Complete ASX Worksheet 1
  - ASX - Part 2
  - Complete ASX Worksheet 2
- See printed Notes & Solution

Questions about Correlation

- What is the correlation between A and A?
- What is the correlation between A and -A?
- What is the standard deviation of the risk-free asset?
- What is the correlation between the risk-free asset and any other asset?

Combining Risky and Risk-Free Assets

- The standard deviation of the risk-free asset is zero
- The correlation (covariance) between the risk free asset and any other asset is zero
- Suppose you have:
  - a risky asset with expected return 15% and standard deviation 20%
  - a riskless asset with expected return 6% (Sd of zero)
  - a portfolio consisting of 50% invested in each asset
    - the expected return is 0.5 x 15% + 0.5 x 6% = 10.5%
    - the portfolio Sd is 0.5 x 20% = 10%

Risk Reduction & Diversification

- Risk Reduction
  - You can reduce risk by investing a greater proportion of your wealth in the risk-free asset
  - You can eliminate risk altogether by investing 100% in the risk-free asset
  - The “price” you pay is a lower expected return
- Diversification
  - Offers the potential to reduce risk while maintaining return

The Best Candy Example

Best Candy	Normal Year for Sugar: Bullish Market	Normal Year for Sugar: Bearish Market	Abnormal Year: Sugar Crisis
Probability	.5	.3	.2
Return	25%	10%	-25%

Best Candy Risk & Return

Best Candy	Normal Year for Sugar: Bullish Market	Normal Year for Sugar: Bearish	Abnormal Year: Sugar Crisis
Probability	.5	.3	.2
Return	25%	10%	-25%

Expected Return $$E(r) = \sum_s p(s) r(s)$$

$$= 0.5(25\%) + .3(10\%) + .2(-25\%) = 10.5\%$$

Standard Deviation $$\sigma = \sqrt{\sum_s p(s)[r(s) - E(r)]^2}$$

$$=[.5(25\%-10.5\%)^2 + .3(10\%-10.5\%)^2 + .2(-25\%-10.5\%)^2]^{1/2} = 18.90\%$$

SugarKane

Sugar Kane	Normal Year for Sugar: Bullish Market	Normal Year for Sugar: Bearish Market	Abnormal Year: Sugar Crisis
Probability	.5	.3	.2
Return	1%	-5%	35%

Lab: Best Candy Portfolio

- Excel Workbook: Investment Math
  - Select Best Candy from Labs menu
  - Select Best Candy Worksheet
  - See also: Best Candy Solution
- Complete worksheet
  - Compute SugarKane expected return
  - Compute SugarKane Sd.
  - Later: Portfolio returns
- See printed Notes & Solution

Solution: SugarKane Risk & Return

Sugar Kane	Normal Year for Sugar: Bullish Market	Normal Year for Sugar: Bearish Market	Abnormal Year: Sugar Crisis
Probability	.5	.3	.2
Return	1%	-5%	35%

Expected Return $$E(r) = \sum_s p(s) r(s)$$

$$= 0.5(1\%) + .3(-5\%) + .2(35\%) = 6\%$$

Standard Deviation $$\sigma = \sqrt{\sum_s p(s)[r(s) - E(r)]^2}$$

$$=[.5(1\%-6\%)^2 + .3(-5\%-6\%)^2 + .2(35\%-6\%)^2]^{1/2} = 14.73\%$$

Asset Risks & Returns

Asset	Expected Return	Standard Deviation
Best Candy	10.5%	18.90%
SugarKane	6%	14.73%
T-bills	5%	0%

Lab: Best Candy Portfolio Risks & Returns

Portfolio	Expected Return	Standard Deviation
100% Best Candy	10.5%	18.90%
50% Best Candy 50% TBills	?	?
50% Best Candy 50% SugarKane	?	?

Solution: Best Candy Portfolio Risks & Returns

Portfolio	Expected Return	Standard Deviation
100% Best Candy	10.5%	18.90%
50% Best Candy 50% TBills	7.75%	9.45%
50% Best Candy 50% SugarKane	8.25%	4.83%

Nb. Correlation between Best Candy returns and SugarKane returns is -0.864

Conclusions about Diversification

- Diversification
  - can reduce risk
  - without necessarily sacrificing returns
  - by using imperfectly correlated assets as a hedge
- *even if an asset appear unattractive in its own right, it may be attractive to you as a hedge*

Next Question: How should we diversify?

- How much allocated to each asset?
  - Does its matter?
- Is there a limit to the benefit we can achieve?