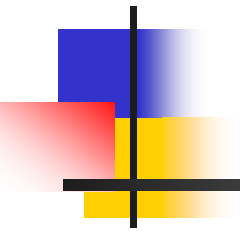


Portfolio Insurance

Portfolio Insurance

- What is it?
- Why use it?
- How to apply in practice

Portfolio Insurance: Rationale

- Portfolio
  - 1000 shares stocks A, 1000 shares stock B
  - Volatility of each stock 30%
  - Current stocks price of A and B is $50
  - T-Bill rate is 8%
- Scenario:
  - Expect FED to tighten in next three months
  - Portfolio value must not fall below $90,000
- Calculate all-in costs of protection:
  - Use the option calculator
  - Worksheet: Portfolio Insurance Intro

Cost of Protection

- Cost of 3m put option with $45 strike is $0.79
- Need 2,000 options
- Hence all-in cost is $0.79 x 2000 = $1,586

Hedging a Portfolio

- Next, assume A and B are uncorrelated: Corr(A,B) = 0
- Compute the volatility of the portfolio
- Calculate the all-in cost of hedging the portfolio
- Reminder:

$$\sigma_p = \sqrt{[w_1^2\sigma_1^2 + w_2^2\sigma_2^2 + 2w_1w_2\rho\sigma_1\sigma_2]}$$

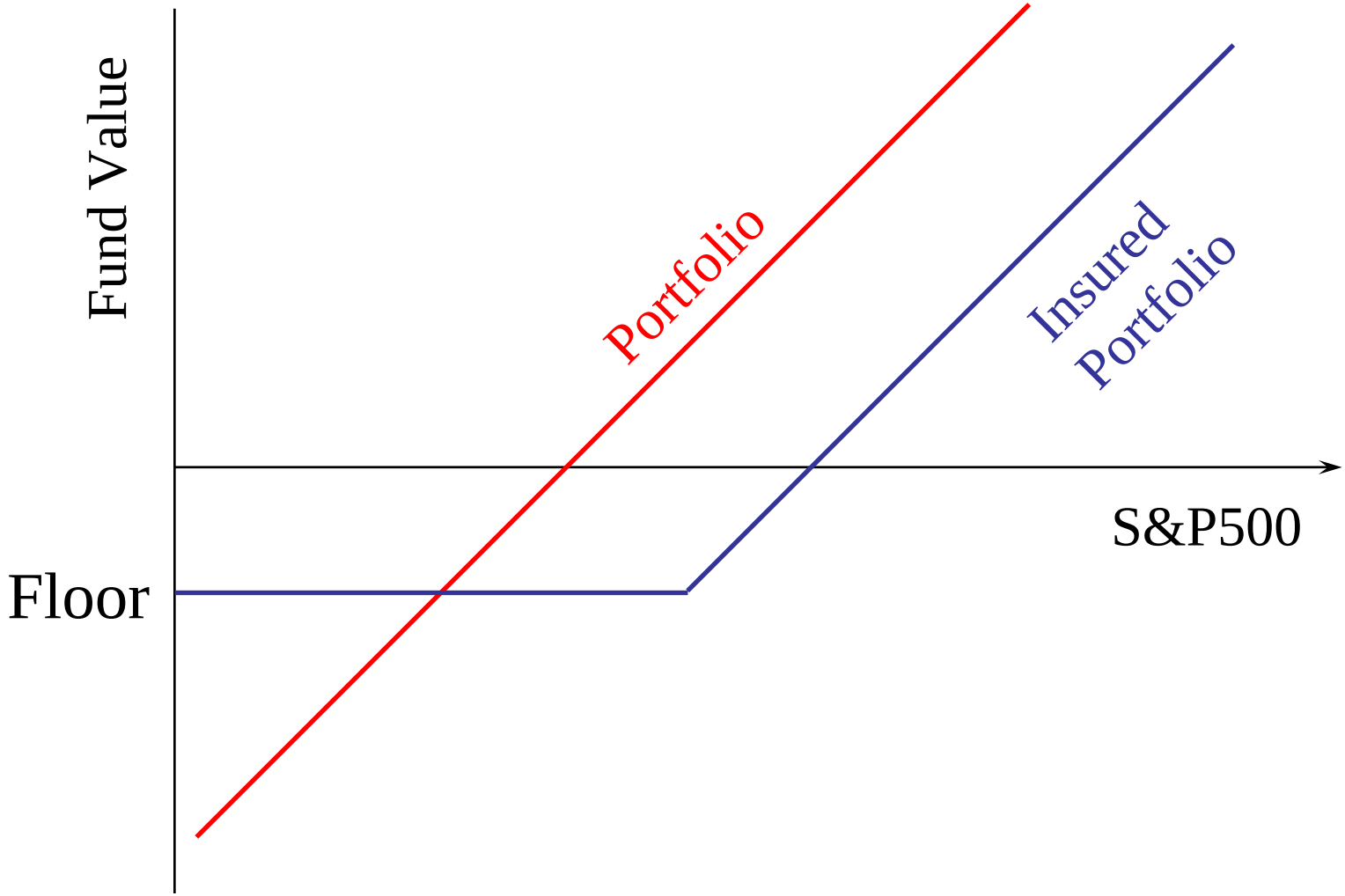
Cost of Hedging Portfolio

- Hedging Cost:
  - SD of Portfolio 21.21%
  - Black Scholes option price $0.29
  - Hence cost of portfolio hedge:
    2,000 x $0.29 = $579
  - Cost saving = $1,007
- Finding:
  Cost of option on portfolio is less than cost of portfolio of options

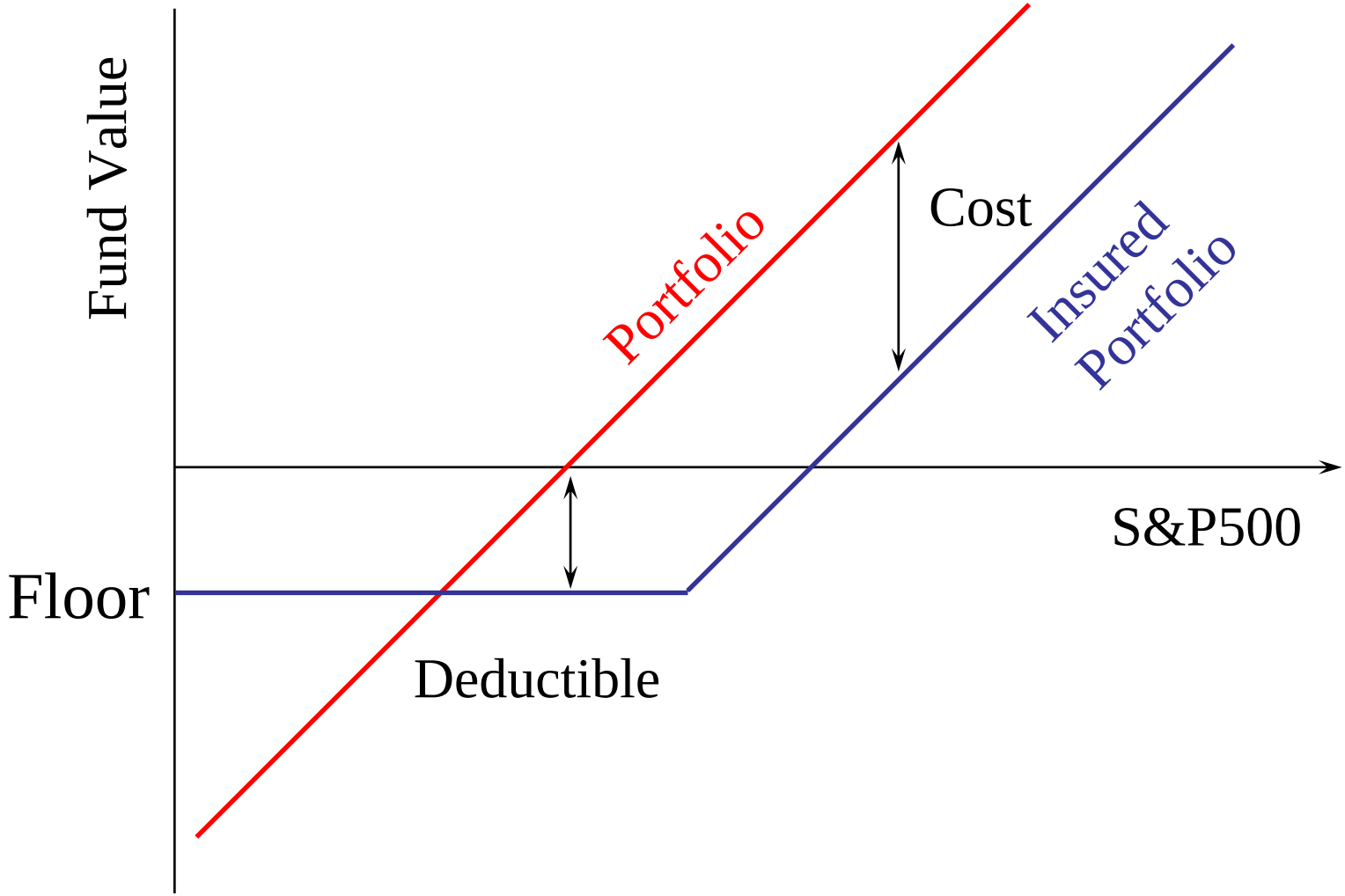
Evolution of Portfolio Insurance

- PROBLEM: option on portfolio doesn't exist
- Hence create put option synthetically
- Replicate hedged portfolio
- Dynamic Hedging Strategy
- Leyland, O'Brien, Rubenstein (LOR)- 1980's

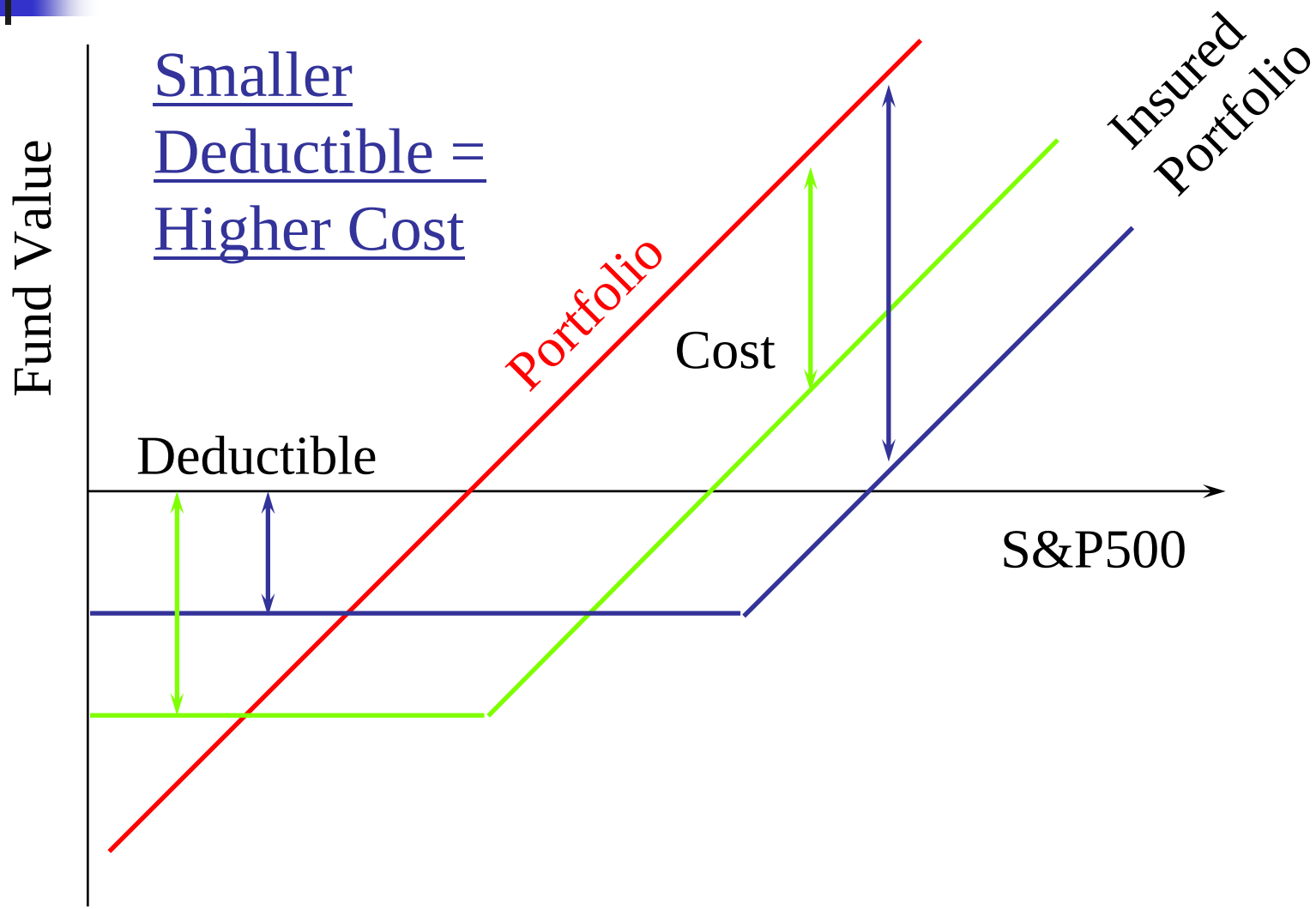
Portfolio Insurance Illustrated



Portfolio Insurance Illustrated

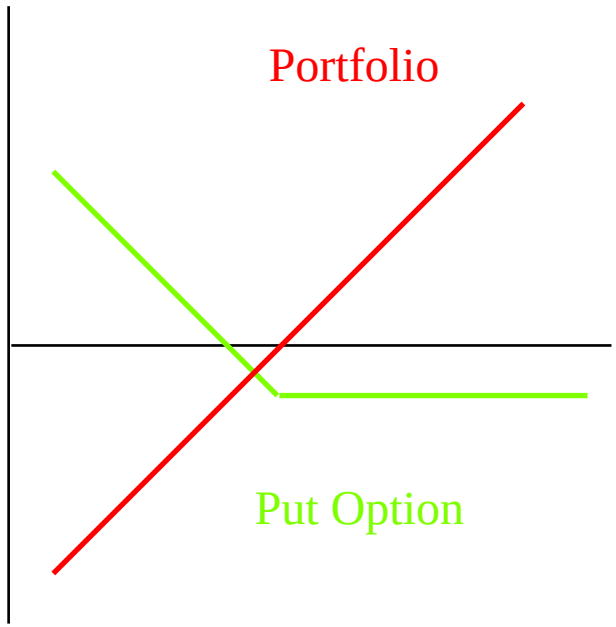


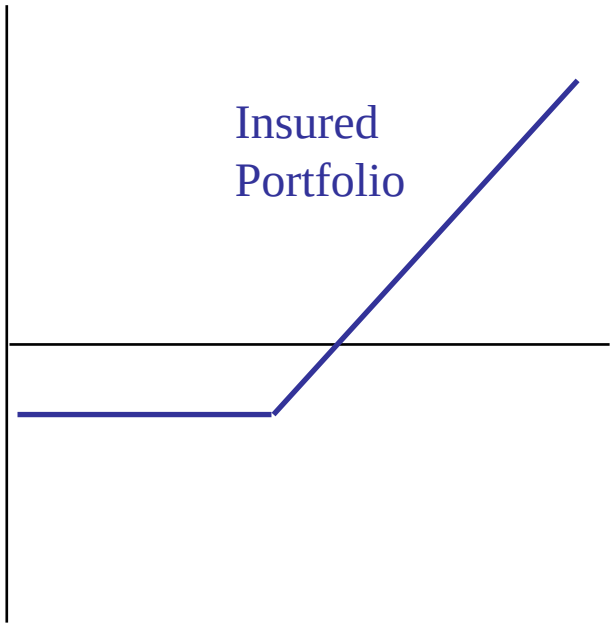
Insurance


Smaller
Deductible =
Higher Cost
Fund Value
Portfolio
Cost
Insured
Portfolio
Deductible
S&P500

Portfolio Insurance as Call Option

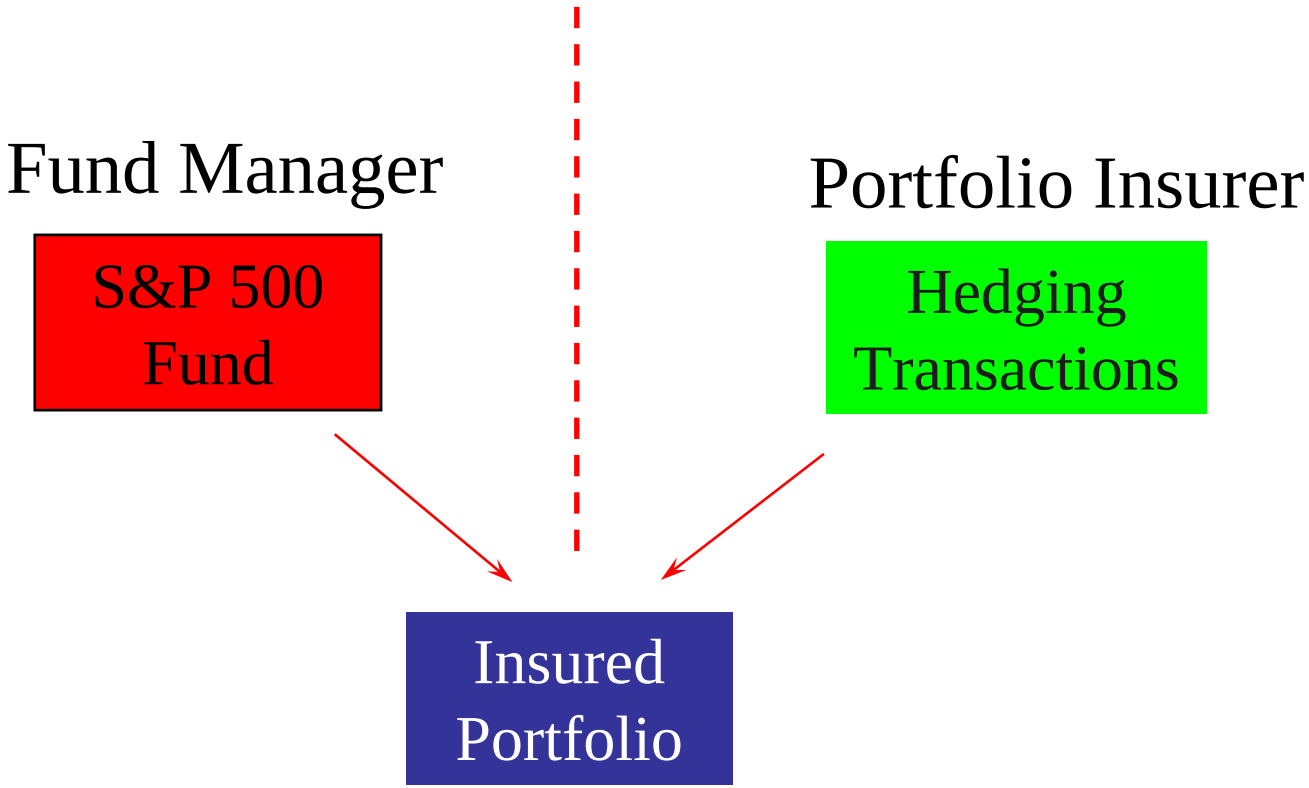
- Put-Call Parity: Put + Stock = Call + Cash





Role of Portfolio Insurer

- Portfolio can be insured separately:



Lab: S&P500 Tracking Portfolio

- Client has $100MM S&P500 Tracking Fund
- Wants to ensure fund value > $95MM at year-end
- Engages you as insurer
- Background:
  - S&P500 volatility 15%
  - Tracking error volatility 2.5%
- How do you proceed?

Portfolio Insurance Steps

- Estimate volatility of portfolio
- Determine option insurance required
  - Put/call
  - Strike
- Use Option Calculator to price option
- Delta gives us proportion we need to invest in risky assets (the portfolio)

Constructing Portfolio /1

- We need a put option with a $95 strike
- From Black Scholes: Put cost = $1.53MM
- Problem: Have $100MM fully invested
  - Can't afford another $1.53MM for option
- Scale down portfolio size from $100MM so that:
  - Portfolio value + option cost = $100MM

Next step:

- Experiment with various portfolio values (below $100MM)
  - Use option calculator to price option
  - Add portfolio value to option price
  - If total <>$100 then repeat!

Constructing Portfolio /2

- Solution: Fund $98.12MM, Put option $1.88MM
- But: we want to replicate option, not buy it
- How do we replicate it?
- Steps:
  - Use Option Calculator to derive option delta
  - Proportion to invest in portfolio: (1 - delta)

Constructing Portfolio /3

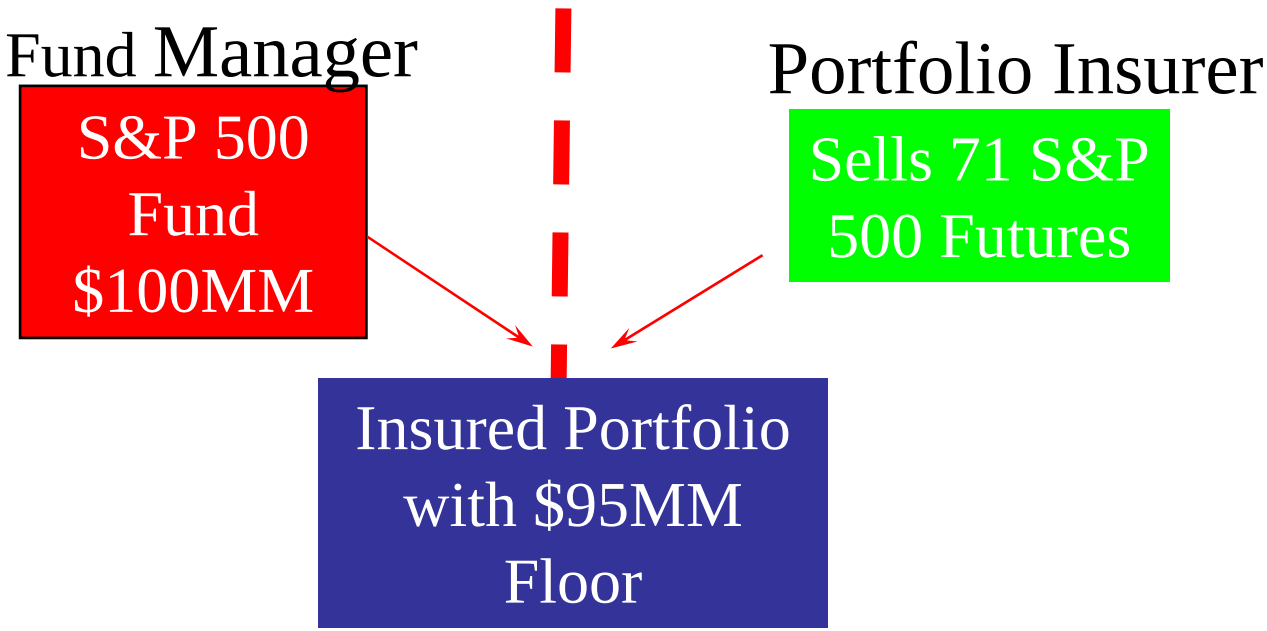
- Option delta is -0.208
- So invest (1 - 0.208) x $98.12MM =
  $77.7MM in portfolio
  $22.3MM in T-Bills
  TOTAL $100MM

Constructing Portfolio /4

- Need equiv. of $77.7MM in portfolio, $22.3 in T-Bills
- BUT: already have $100MM in portfolio
- So use S&P500 futures to create equiv. position
- STEPS:
  - Need to sell equiv. $22.3MM of portfolio
  - Current S&P500 index level is 630
  - Value of 1 S&P contract?
  - How many contracts to sell?

Constructing Portfolio /5

- Value of 1 S&P500 future is $630x 500 = $315,000
- So, sell $22.3MM / $315,000 = 71 contracts



Lab: Checking Insurance Policy

- What happens if S&P index falls to 617?
- Work out new value of portfolio:
  - Original fund value $100MM
  - How many S&P futures is this equiv to?
  - Multiply this no. by 600 to get new fund value
- Work out gain on short S&P500 futures
- Don’t forget T-Bill interest!

Checking Insurance

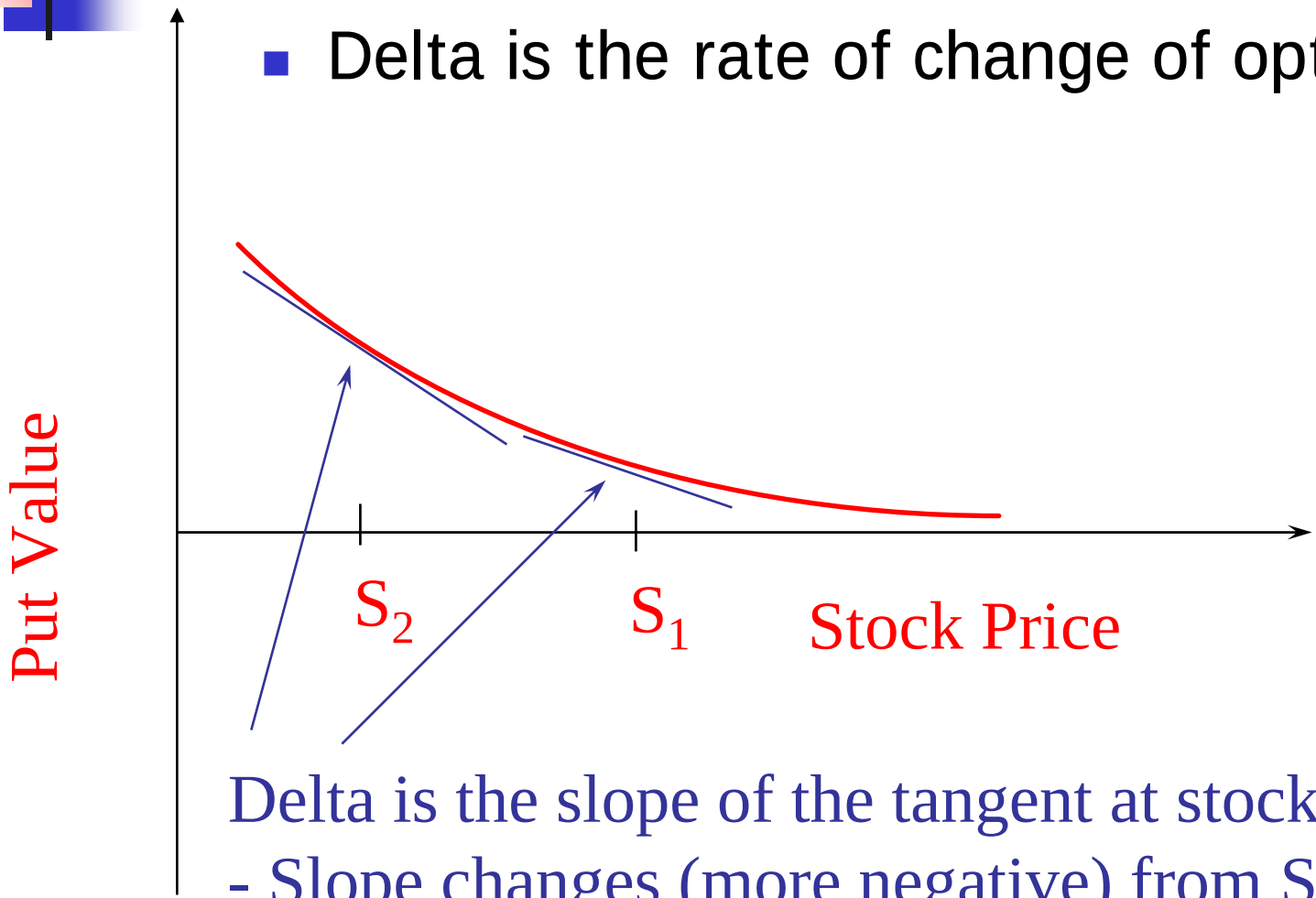
- Fund Value $100MM
  - Was equiv to $100MM / (630 x 500) = 317 Futures
  - Value Now: 617 x 317 x $500 = 97.79MM
- Gain on Short Futures
  - 71 x (630-617) x $500 = $0.46 MM
- Interest on T-Bills
  - $22.3MM x 8% = $1.78MM
- TOTAL VALUE
  - $97.79MM + $0.46MM + $1.78MM = $100.04MM

Limits to Insurance:

- What happens if S&P500 index falls to 550?
  - Portfolio Value is 550x317x$500 = $87.18MM
  - Gain on Short Futures Hedge = 71x(631-550)x$500 = $2.83MM
  - Interest = $1.78MM
- Total Value is $91.79MM
- Floor was supposed to be $95MM
- What went wrong?

Delta Revisited

- Delta is the rate of change of option value



Delta is the slope of the tangent at stock price
- Slope changes (more negative) from S_1 to S_2

Rebalancing

- The Delta of a put option changes as the S&P500 index moves
- We need to reflect this by changing the proportions held in T-bills & risky asset
- As index falls, delta becomes more negative
- So we need to sell more stock as stock falls, buy more stock when it rises
- Buy High, Sell Low

Lab: Rebalancing

- Suppose after 6 months, S&P index falls to 610
- Calculate the new value of the fund
- Use the Option Calculator to find the new delta
- Compute the amount to be held in the Fund (and the amount in T-Bills)
- How many S&P Futures do we need to sell?

Solution: Rebalancing

- New Fund Value :
  - 317 x 610 x $500 = $96.69MM
- Delta is now 0.278
  - Hence need only $96.69MM x (1-0.278) = $70.87MM in Fund
- Need to sell an equiv. of:
  - ($77.74-$70.87) = $6.87MM of Fund
- Current Futures Value is $500 x 610 = $305,000
- Hence sell $6.87MM/0.305MM = 23 contracts

Rebalancing Filter Rules

- Continuous rebalancing too expensive
- In practice, rebalance:
  - at specified time intervals
  - when portfolio value changes by specified amount
  - by using band around correct hedge ratio

1987 and All That

- Bull market was well into 5th year
- Equity values at historical extremes
  - Many P/Es over 20, dividend yield below 3%
- Investors regarded portfolio insurance as cash substitute:
  - why sacrifice upside?
  - Fund under management increased 4x in 87
- Over-estimated liquidity of stock market

October 19, 1987

- Market fell by over 20%
- $1trillion wiped off market value
- As prices fell, insurers sold index futures
- Futures were at massive discount to cash
- More selling in the cash market by program traders
- Not enough liquidity in cash or futures markets
- Hedging became impossible

Portfolio Insurance Today

- Portfolio Insurance was blamed by investors
- Brady Commission said PI contributed to severity
- Much less popular today, although still very widely used