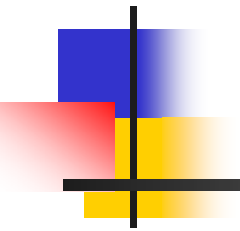


Option Risk Management

Agenda

- Option sensitivity factors
- Delta
  - Delta hedging
- Option time value
- Gamma and leverage
- Volatility sensitivity
- Gamma and Vega hedging

Option Sensitivity Factor

- What affects the price of an option
  - the asset price, S
  - the volatility, σ
  - the interest rate, r
  - the time to maturity, t
  - the strike price, X

Option Greeks

- Delta (“price sensitivity”)
  - change in option price due to change in stock price
- Gamma (“leverage”)
  - change in delta due to change in stock price
- Vega (“volatility sensitivity”)
  - change in option price due to change in volatility
- Theta (“time decay”)
  - change in option value due to change in time to maturity

Option Delta

- Key sensitivity
  - Change in option value for $1 change in underlying stock
  - Range –1 to +1
- Option Delta
  - Put options: negative delta
  - Call options: positive delta

Delta Example

- Call option with Delta 0.5
- Delta tells us how the call price changes
  - If stock moves up by $10, call price increases by $5
  - If stock drops by $10, call price decreases by $5

Delta Position

- Delta Position
  - The call ‘behaves’ like 0.5 units of stock
  - If we hold 10 calls, our position behaves like 5 units of stock
  - we say we are “long 5 deltas”

Position Deltas

- Call options
  - have positive delta
  - like being long stock
- Put options
  - have negative delta
  - like being short stock
- Stock - has a delta of 1!
- Bonds - have a delta of zero
- Combinations
  - may have positive, negative or zero delta

Delta Hedging

- If you know the position's delta, you can hedge it
- Example, 10 calls, each of delta 0.5
- How to hedge: Sell Delta units of stock
  - This creates a portfolio of 10 calls plus -5 units of stock
  - The combined position has a delta of +5 - 5 = 0
  - Like being long 5 stock & short 5 stock = net zero stock
  - The value of the portfolio will be unchanged, no matter whether the stock moves up or down

Delta Neutral Positions

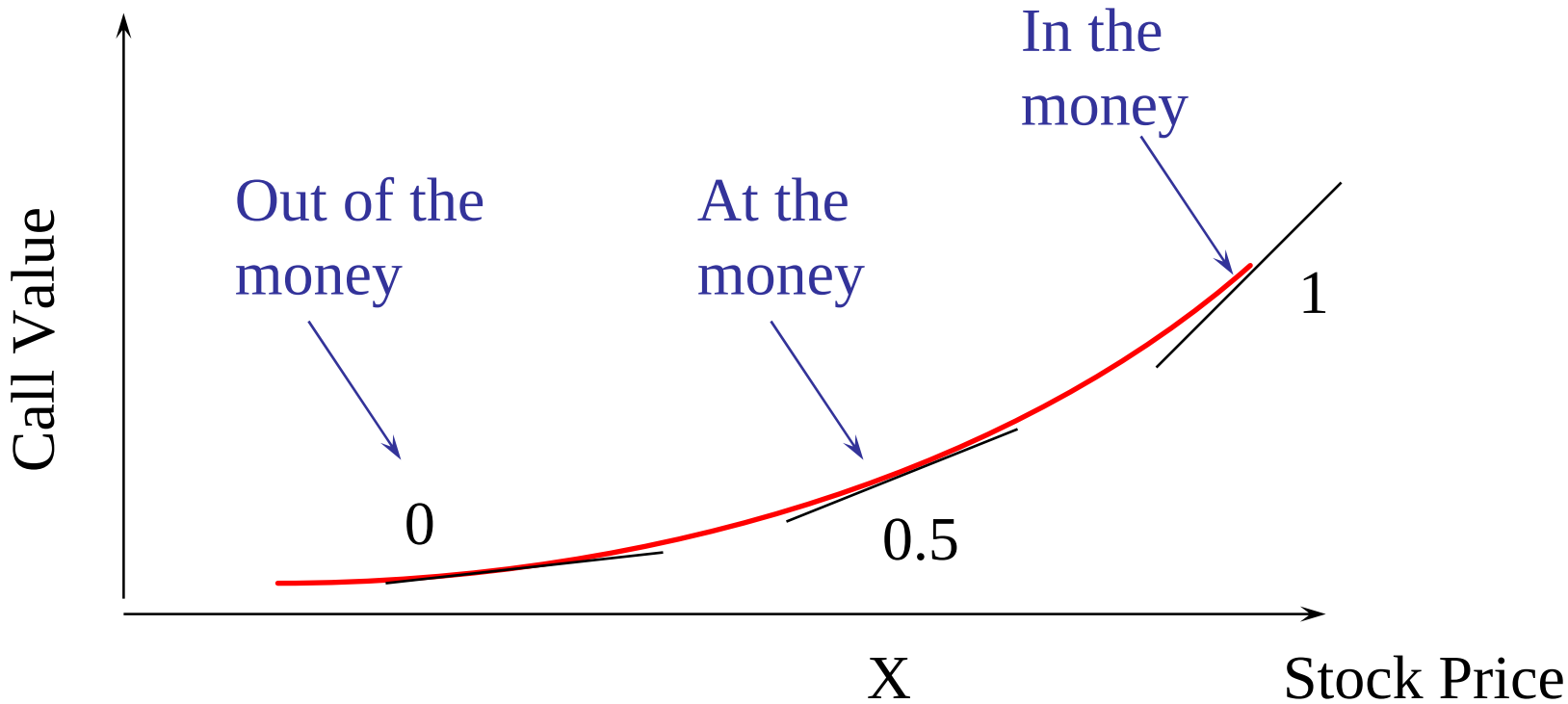
- A portfolio is *hedged* when its position delta is zero
- We say we have a *delta-neutral* position
- Examples: assume call delta 0.5, put delta -0.5
  - Long 10 calls, short 5 stock
  - Long 10 calls, long 10 puts
  - Short 10 calls, long 5 stock
  - Short 10 puts, short 5 stock
  - Short 10 puts, short 8 calls, short 1 stock

Delta Neutral Strategies

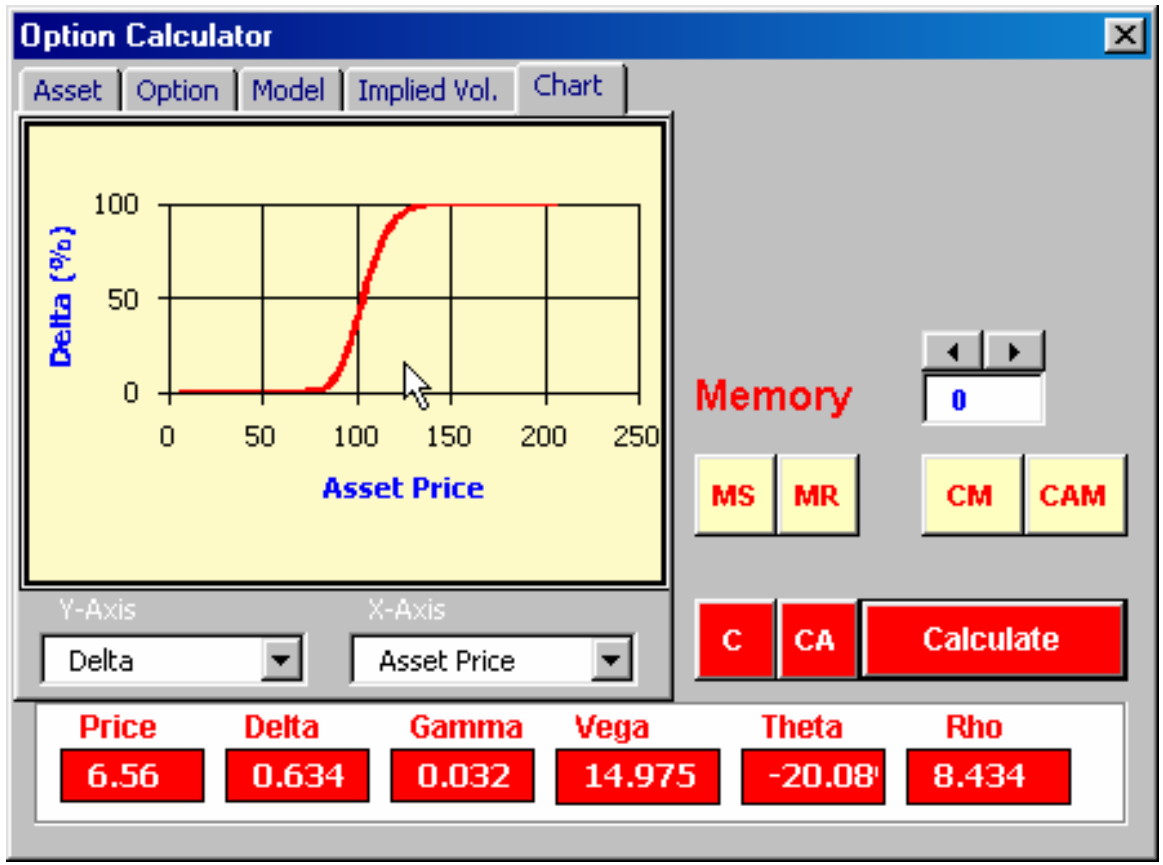
- *Delta neutral strategies* are non-directional
  - Often make money while market doesn't move
- Examples:
  - Butterflies, straddles, strangles are typically delta neutral
- *Directional* strategies are typically not delta-neutral
  - Call spreads have +ve delta
  - Put spreads have -ve delta

The Delta of a Call Option

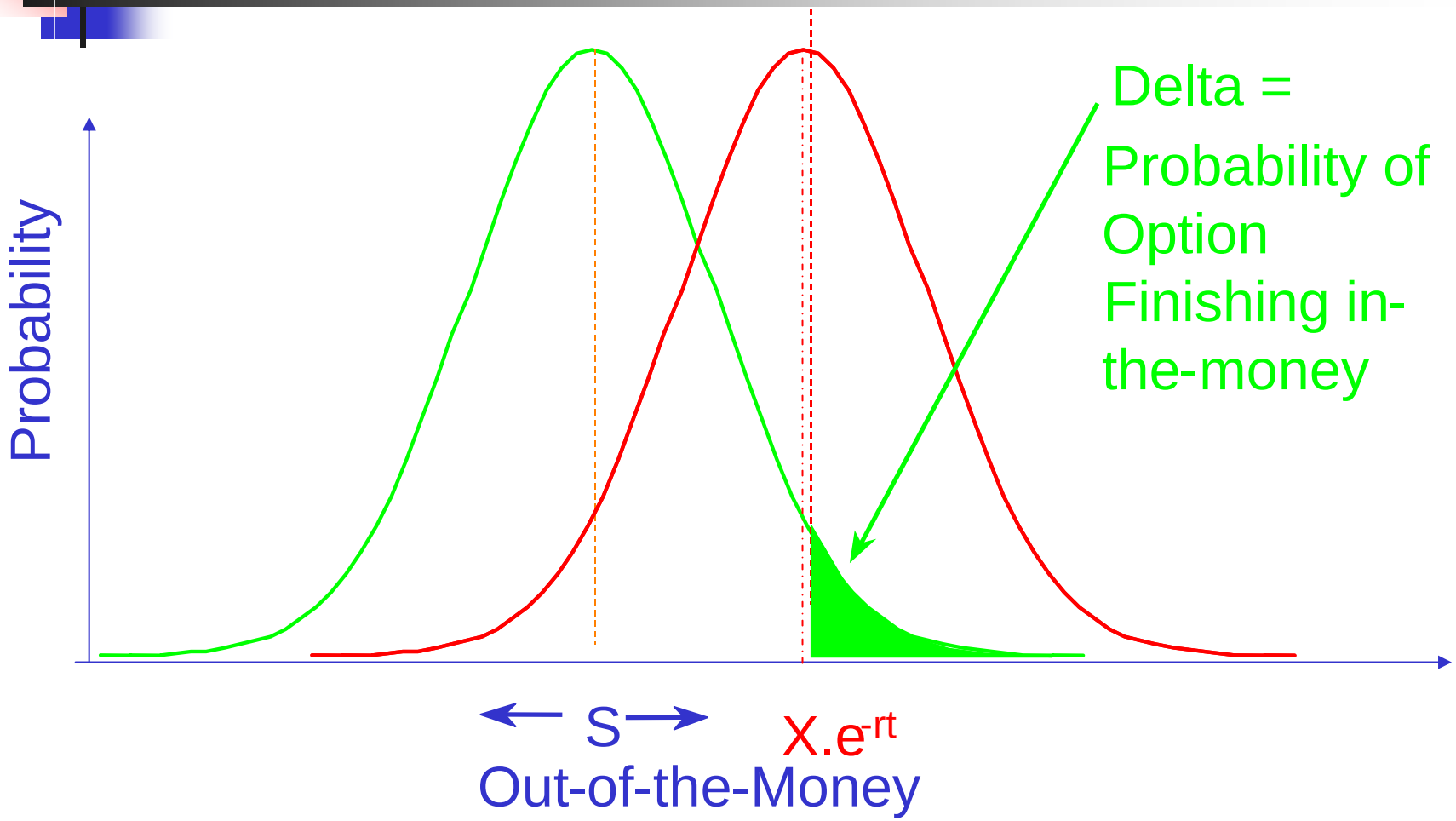
- Delta changes as stock price changes
  - *Gamma* measures rate of change of delta



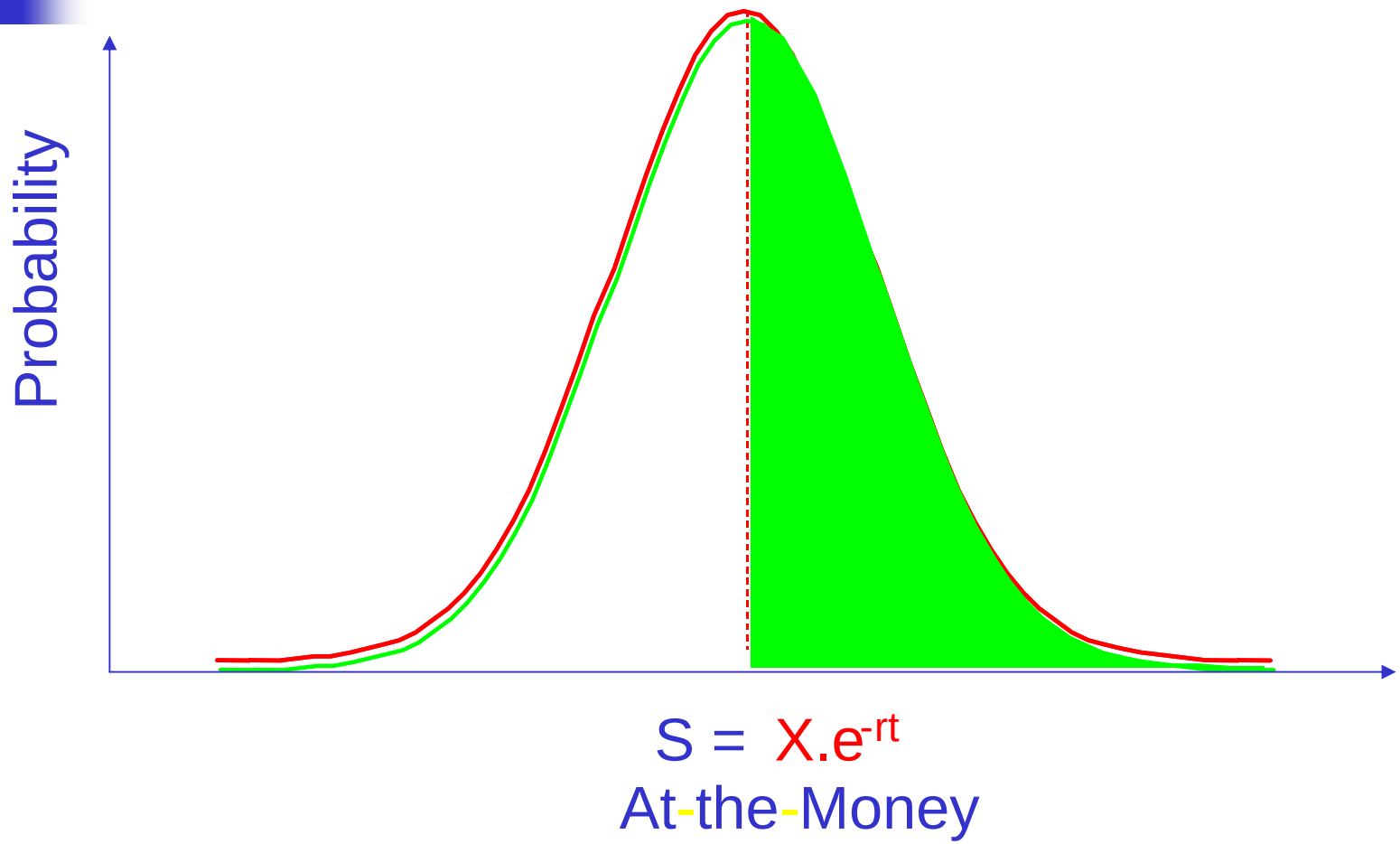
IBM Option Delta



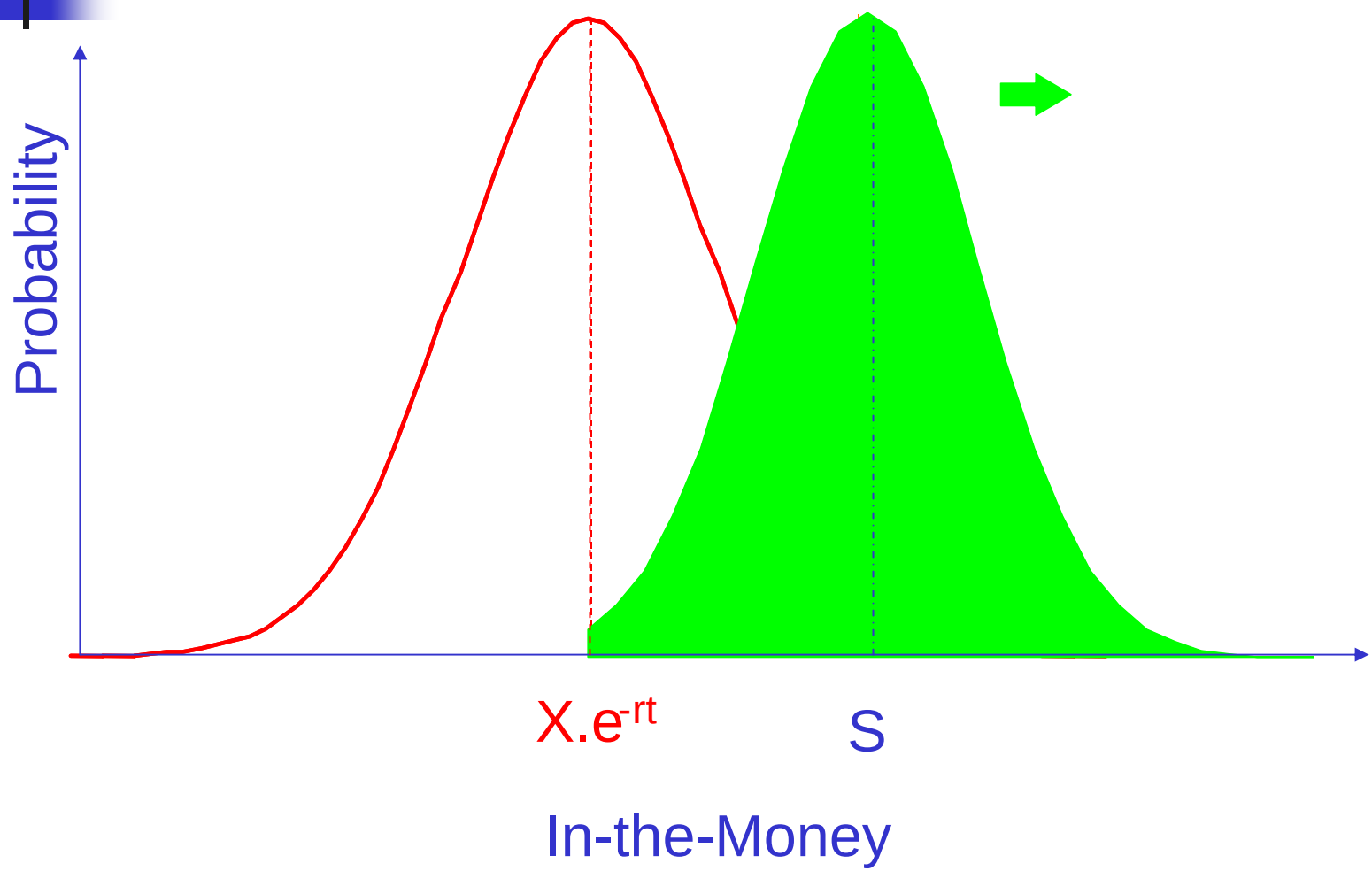
A New Look at Delta



Delta: At-the-Money



Delta: In-The-Money



Delta: Conclusions

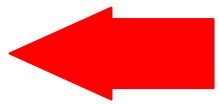
- Delta “measures probability of finishing in the money”
- A deep out-of-the money option has a low delta
- An at-the-money has a delta of around 0.5 as it approaches expiry
  - i.e. a 50-50 chance of ending up in the money
- As an option moves deep into the money, its delta rapidly approaches 1
  - Gamma measures rate of change of Delta

Option Value

- Intrinsic Value

  - **Asset Price**
  - **Strike Price**
  - **Interest rates**

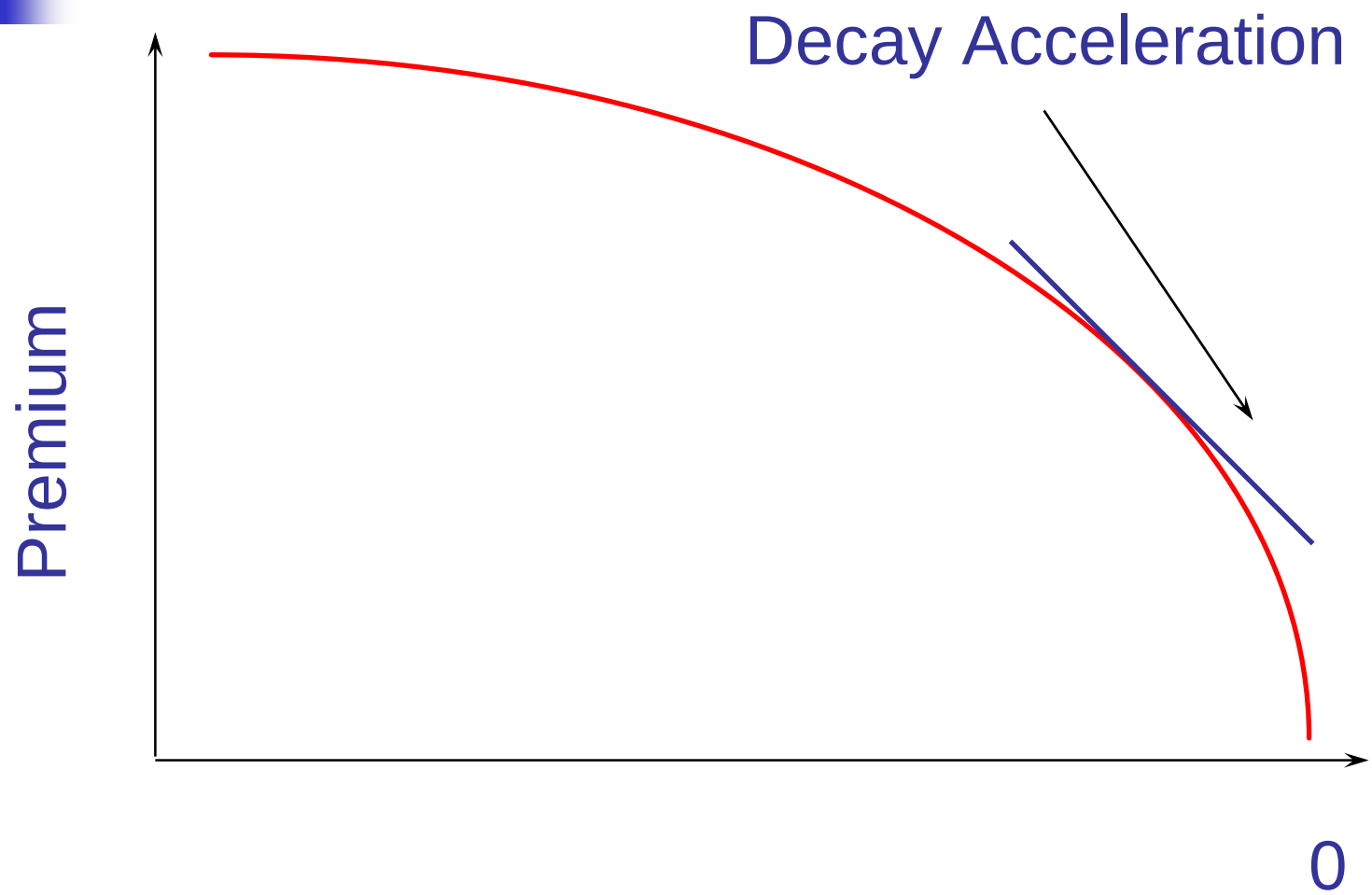
- Time Value



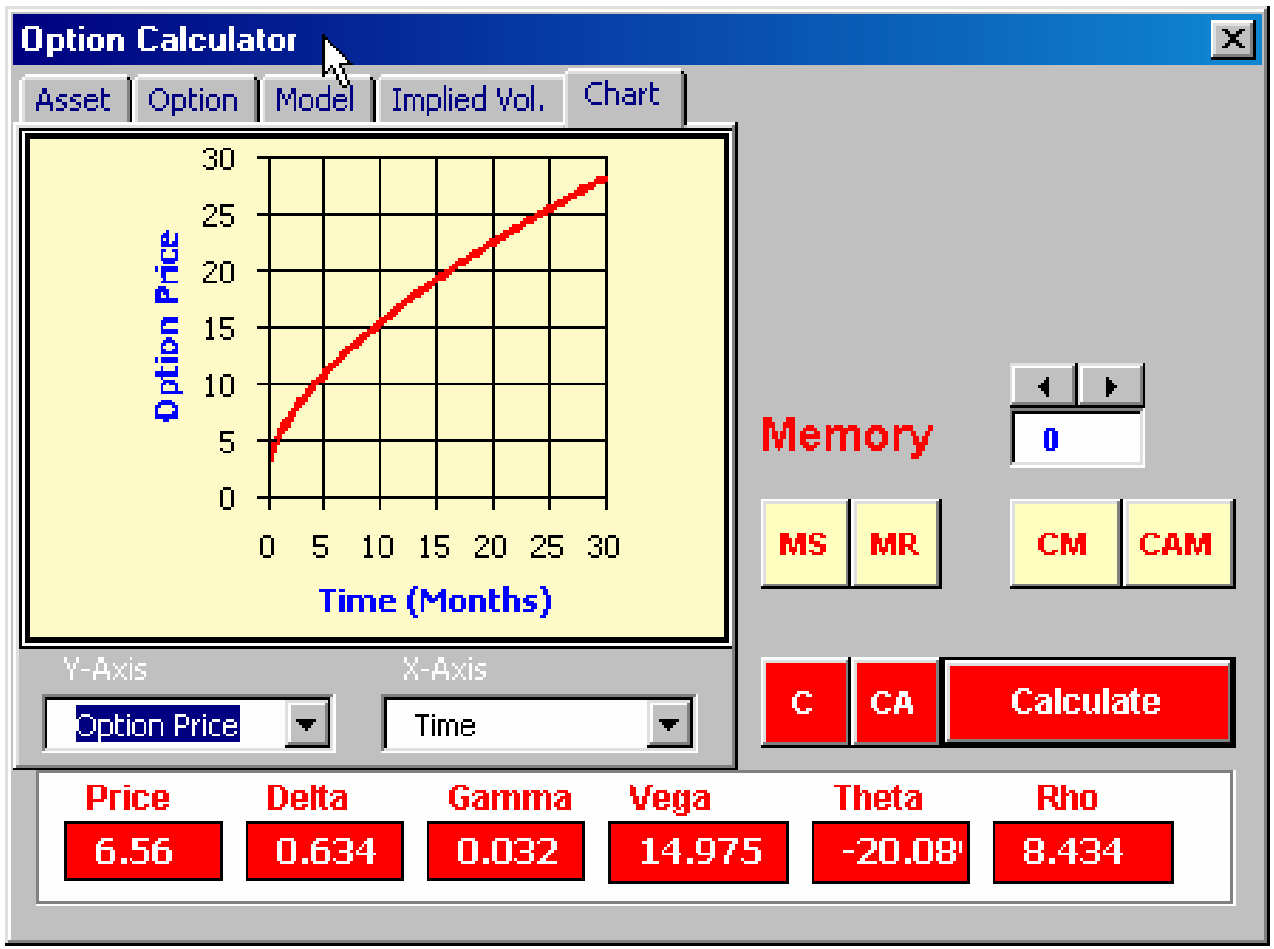
  - **Time to Expiration**
  - **Volatility**

How Time Value Decays

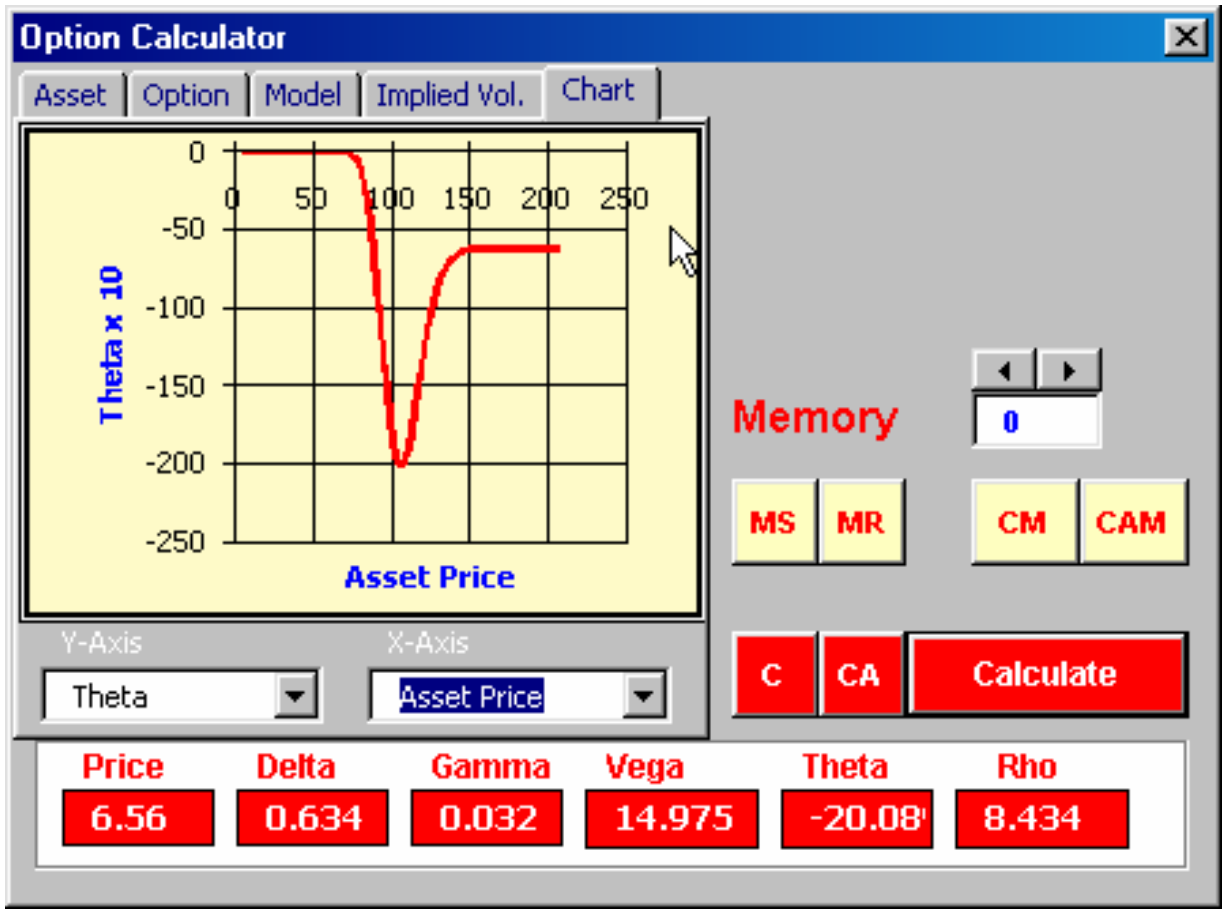
Time Decay (Theta)



Time Value of IBM Option



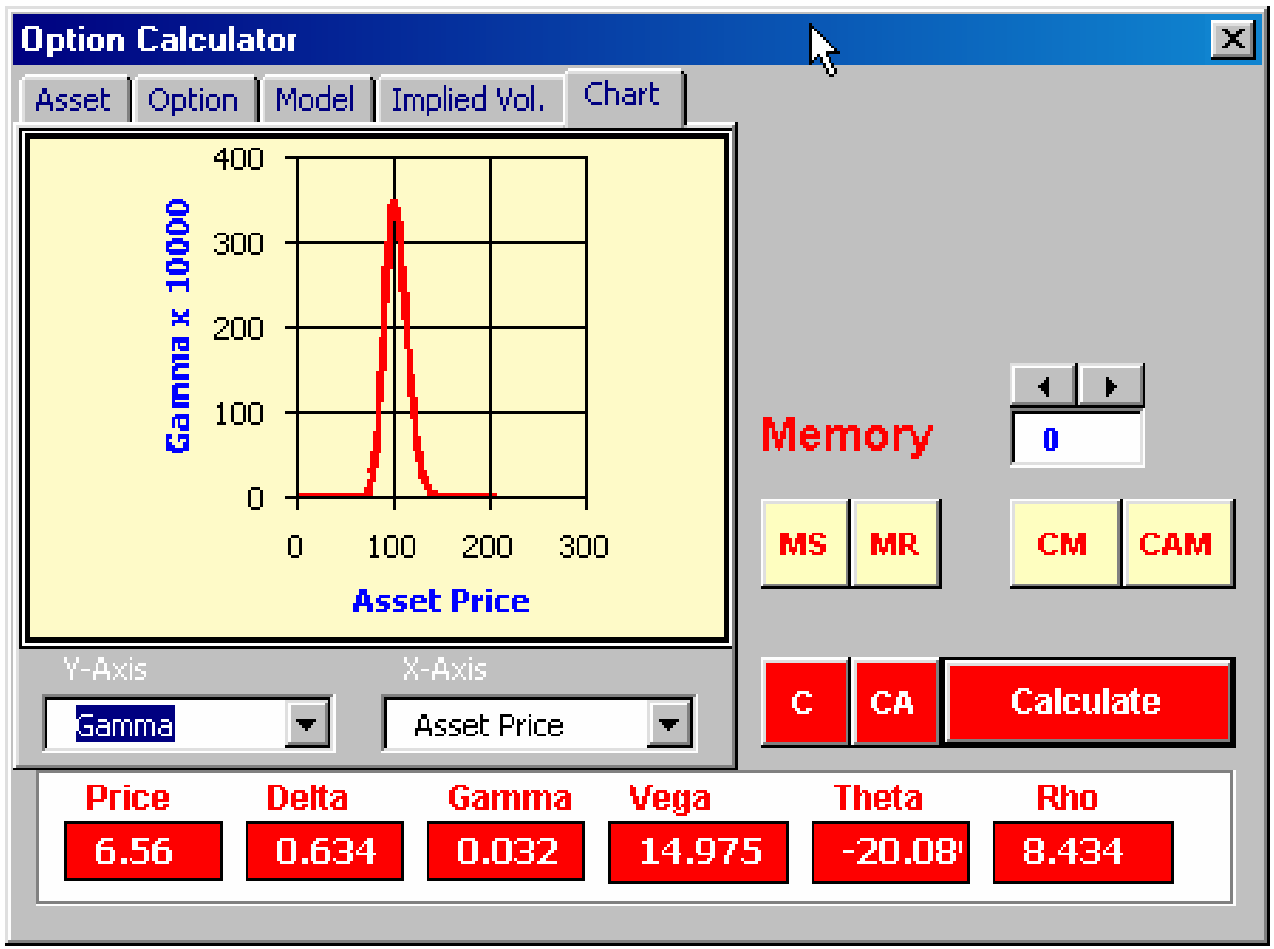
IBM Option Theta



Option Gamma

- Measures change of Delta for $1 change in stock
- “Rate of Acceleration” of option value with stock price
- Call and put options have positive gamma
  - Option delta becomes larger as stock appreciates
  - Becomes smaller (more negative) as stock declines
- Gamma changes as underlying stock moves
  - Highest for ATM options

IBM Option Gamma



Lab: Leverage

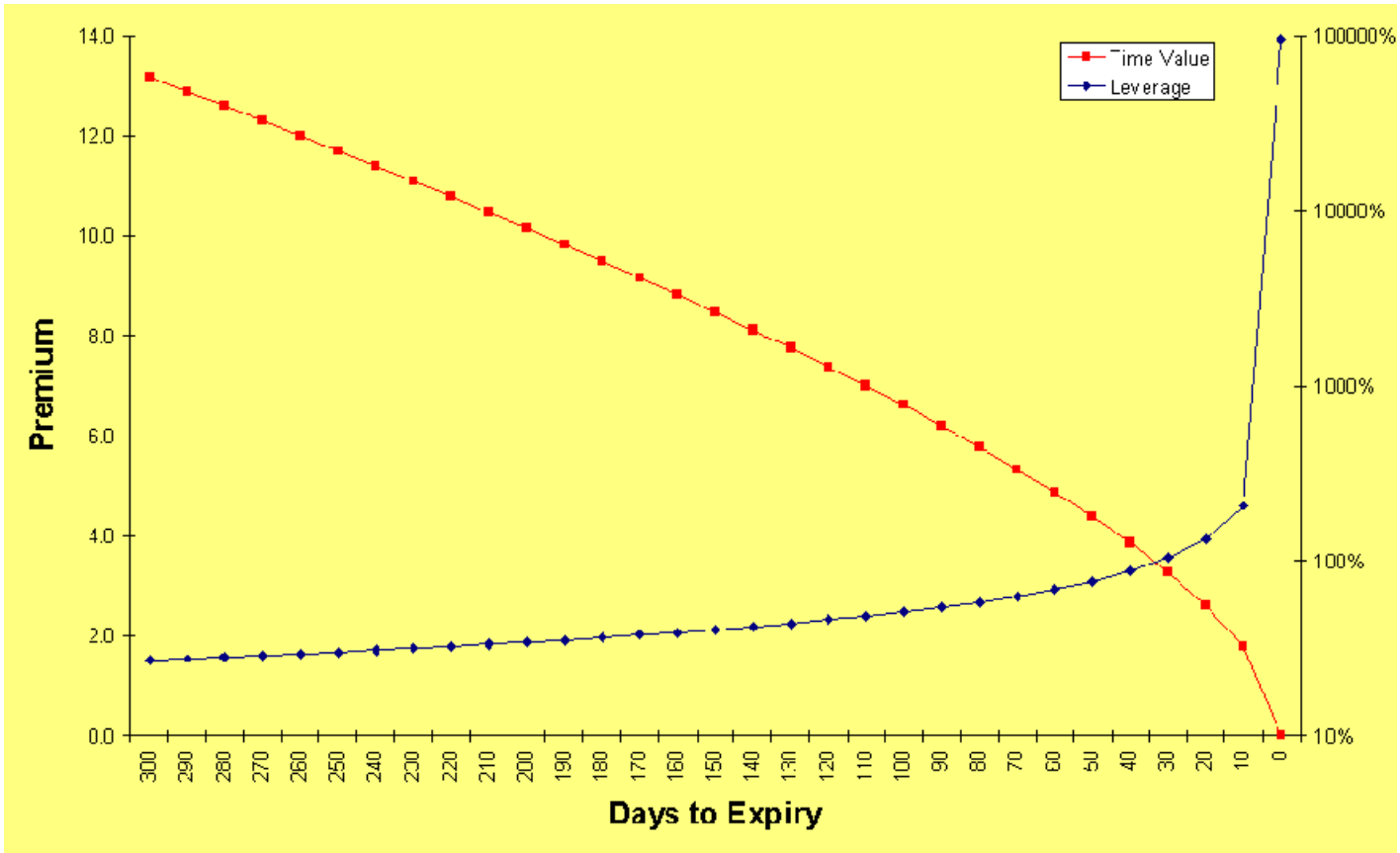
- An Experiment:
  - Assume stock $100
  - Risk free rate 10%, volatility 25%
  - Call option, strike price 100 (at the money)
- Leverage:
  - If stock moves by $5, how much does option value change?

Solution: Leverage

	Call Prices		Return	
Maturity	S=100	S=105	(%)	Gamma
1.00	12.34	15.66	27%	0.015
0.50	8.26	11.48	39%	0.022
0.25	5.60	8.80	57%	0.032
0.10	3.40	6.69	97%	0.050
0.01	1.02	5.07	396%	0.160

NOTES:
Col 2 is option value with stock price = 100
Col 3 is option value with stock price = 105
Col 4 is 'Leverage': (C1 - C2)/C1

Time vs Leverage

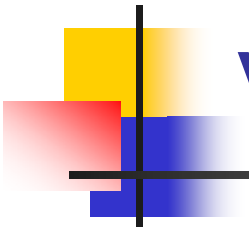


Conclusions: Time Value & Leverage

- Gamma is a measure of leverage
- An option with higher gamma gives “more bang for the buck”
- Gamma measures how option value “accelerates”
- Gamma /Leverage increase as time to expiry falls
- Gamma and Theta are “opposites”:
  - High leverage means rapid time decay

Volatility Greeks

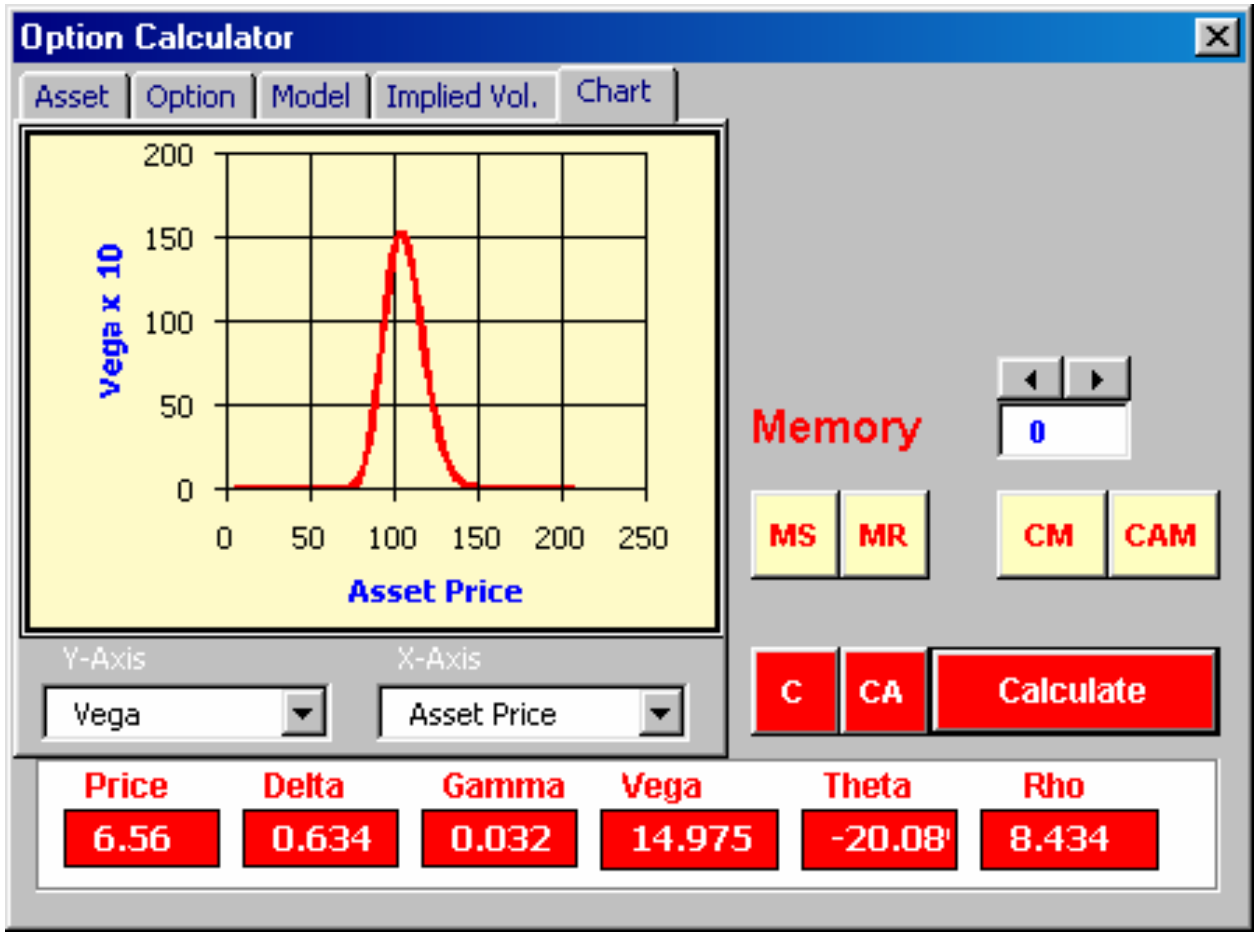
- Vega - sensitivity to implied volatility
- Gamma - sensitivity to actual volatility
- Example: Weather
  - People carrying umbrellas (implied risk of rain) = Vega
  - Rain (the wet stuff) = Gamma
- Implied volatility
  - estimated s.d. implied by option prices by B-S model
  - “market’s” estimate of current volatility

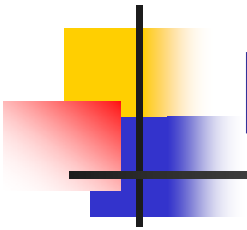


Vega

- Measures change in option value for 1% change in implied volatility
- Call and put options have positive Vega
  - Increase in value as implied volatility increases
  - Option positions may have positive or negative Vega
- Vega changes with underlying stock
  - Highest for ATM options
    - Most sensitive to changes in implied volatility

IBM Option Vega



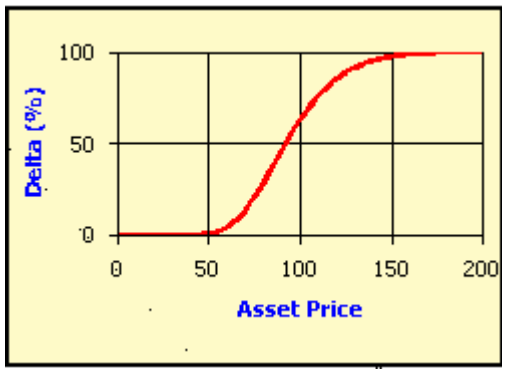


Lab: Option Sensitivities

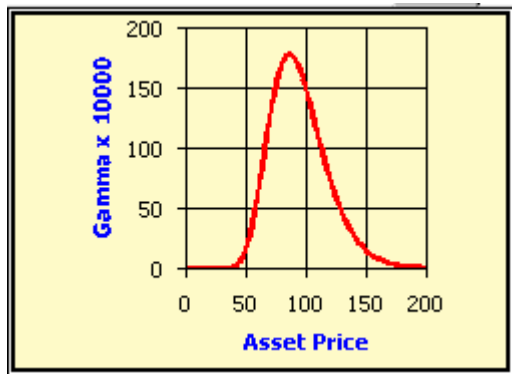
- Examine Delta, Gamma, Vega & Theta
  - Characteristic functions
- How do they vary over time?
- How do they vary with volatility?

Solution: Option Sensitivities

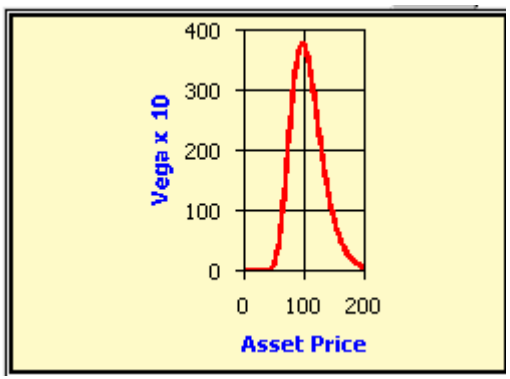
- Delta



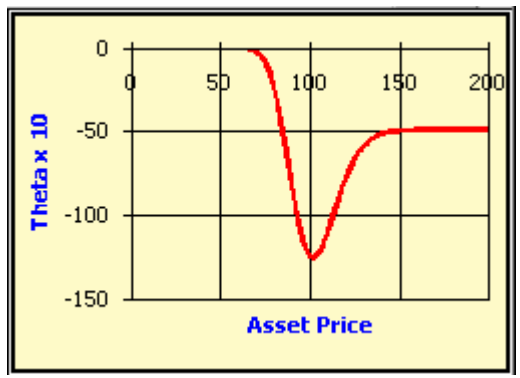
- Gamma

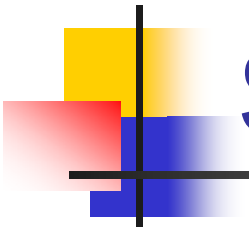


- Vega



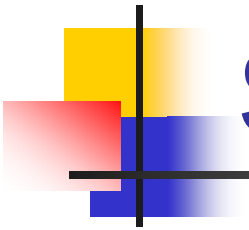
- Theta





Solution: Option Sensitivities

- Delta
  - Short-dated, ATM options on less volatile stock are more sensitive
- Gamma
  - Greatest for short-dated ATM options on less volatile stock



Solution: Option Sensitivities

- Vega
  - Long-dated, ATM options on less volatile stock more volatility sensitive
- Theta
  - Short-dated ATM options on more volatile stock experience greatest rate of decay

Volatility Types

- Historical
- Implied
- Forecast
- Seasonal

Historical Volatility

- Fama & French study (1965) of US stock prices, tested:
  - A: Volatility between consecutive trading days
  - B: Volatility over weekend (close Friday to close Monday)
  - Expected B = 3 x A
  - Found B was only 20% higher
- Volatility is much higher on trading days
- Use daily data from trading days (252 per year)

Estimating Historical Volatility

- Standard Deviation

$$X_i = Ln(P_{i+1} / P_i)$$

$$\sigma = \sqrt{\sum (X_i - \overline{X})^2 / (N-1)}$$

- Parkinson (5x times more efficient)

$$\sigma = \frac{1}{2N\sqrt{Ln(2)}} \sum Ln(H_i / L_i)$$

Gamma Hedging

- Suppose unhedged position = one call
  - You can make it delta neutral
  - But this only works for small changes
  - Need to eliminate gamma risk too
- How do you make it Delta & Gamma neutral?
  - Can't use stock: gamma is zero
  - Must use other options, O_1 and O_2
- Solve:

$$m\delta_1 + n\delta_2 = 0$$
$$m\Gamma_1 + n\Gamma_2 = 0$$

Vega Hedging

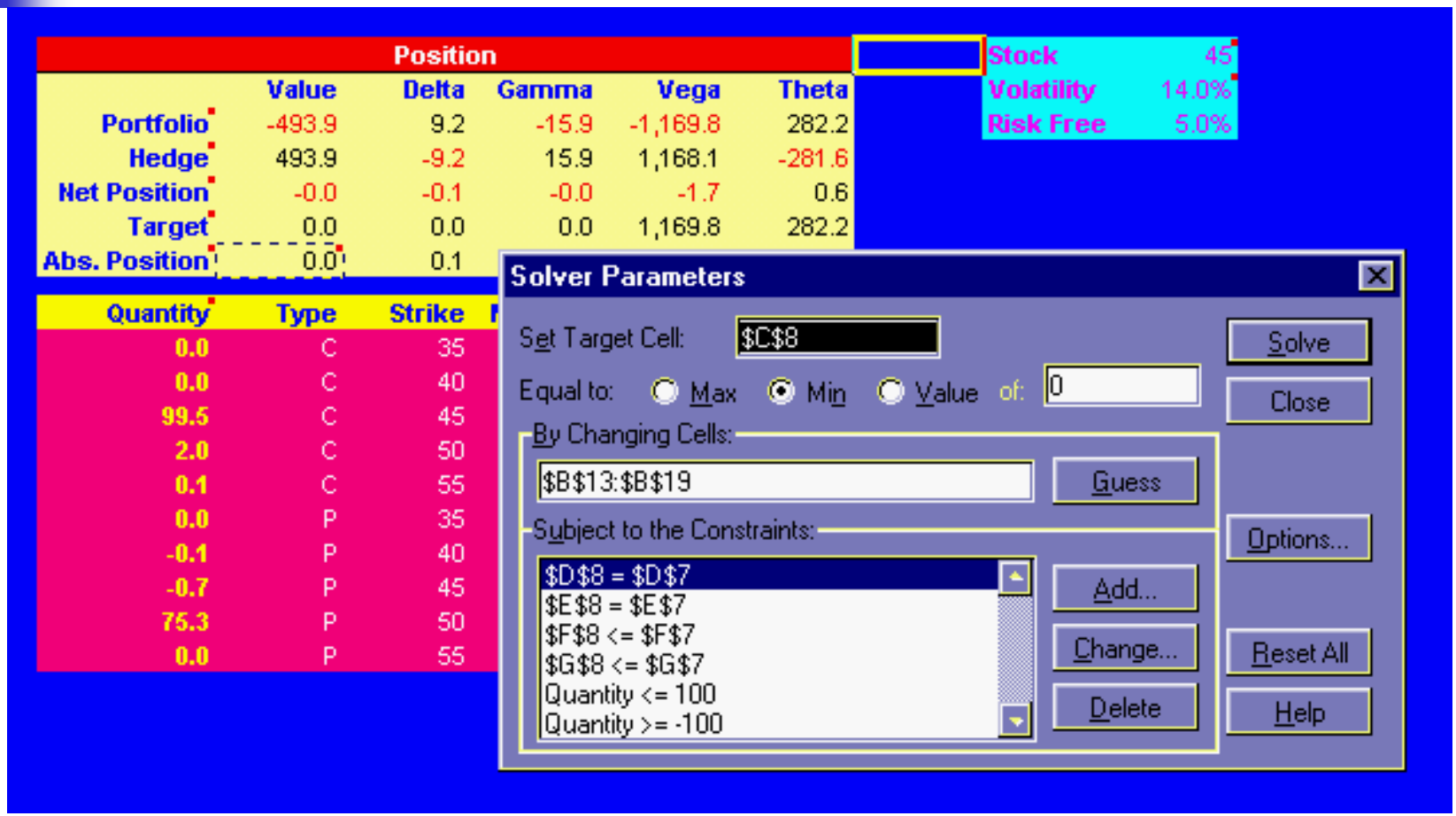
- Make a portfolio Delta-Vega neutral
- Again, use other options
- Solve:

$$m\delta_1 + n\delta_2 = 0$$
$$mV_1 + nV_2 = 0$$

Lab: Greek Hedging

- Excel: Workbook
  - Worksheet: Greek hedging
- Sensitivity:
  - Check how position value, delta changes with stock price
  - Check how position value changes with volatility
- Hedging:
  - Gamma hedge the given portfolio
  - Check sensitivity of position to stock price
- Use Solver

Greek Hedging – Using SOLVER



Solution: Greek Hedging

Position	Value	Delta	Gamma	Vega	Theta
Portfolio	-493.9	9.2	-15.9	-1,169.8	282.2
Hedge	493.9	-9.2	15.9	1,168.1	-281.6
Net Position	-0.0	-0.1	-0.0	-1.7	0.6
Target	0.0	0.0	0.0	1,169.8	282.2
Abs. Position	0.0	0.1	0.0	1.7	0.6

Stock	45
Volatility	14.0%
Risk Free	5.0%

Quantity	Type	Strike	Maturity	Price	Delta	Gamma	Vega	Theta
0.0	C	35	0.25	10.44	1.00	0.000	0.01	-1.73
0.0	C	40	0.25	5.53	0.97	0.020	1.49	-2.33
99.5	C	45	0.25	1.55	0.59	0.119	8.77	-3.70
2.0	C	50	0.25	0.14	0.10	0.053	3.90	-1.31
0.1	C	55	0.25	0.00	0.00	0.004	0.27	-0.08
0.0	P	35	0.25	0.00	0.00	0.000	0.01	0.00
-0.1	P	40	0.25	0.04	-0.03	0.020	1.49	-0.35
-0.7	P	45	0.25	0.99	-0.42	0.119	8.77	-1.47
75.3	P	50	0.25	4.52	-0.90	0.053	3.90	1.16
0.0	P	55	0.25	9.32	-1.00	0.004	0.27	2.63

Summary: Option Risk Management

- Option Greeks
  - Measures of option sensitivity
  - Delta hedging
  - Gamma & Vega hedging