

Yield Curve Modeling

Modeling The Tax-Specific Yield Curve

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Investment Analytics

Modeling the Tax-Specific Yield Curve

- The impact of taxes
- Implication for bond valuation
- Bond Efficiency
- Tax Arbitrage
- Building an After-Tax Yield Curve

Tax Effects

- Tax effects in most bond markets
- Tax effects are important even for tax-exempt investors
- Tax effects typically create arbitrage opportunities, depending on:
 - ✓ frictions (spreads)
 - ✓ short-selling constraints
 - ✓ asymmetric tax treatment of long and short positions

Summary of Analysis Without Taxes

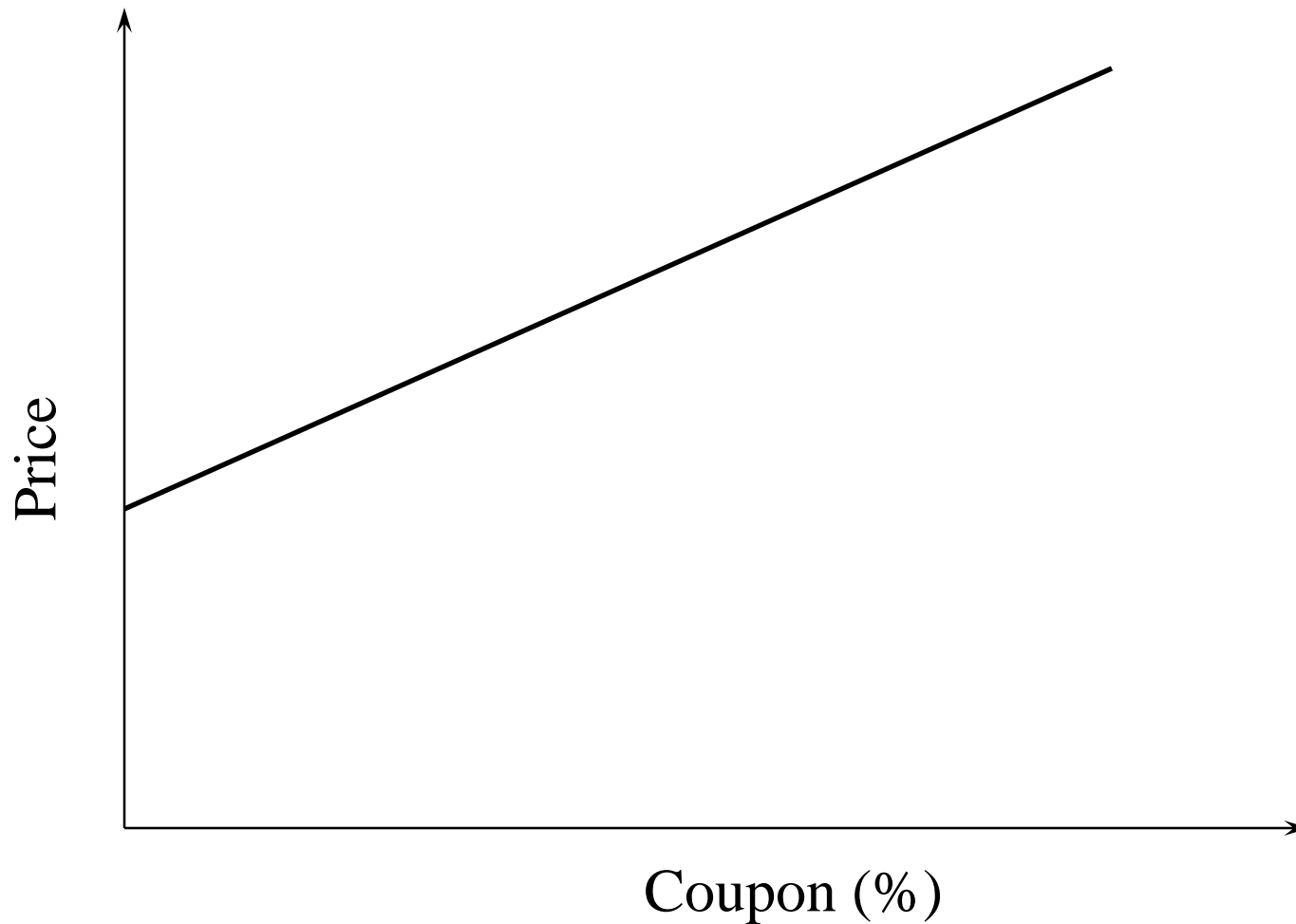
- Price = Present value for each bond
 - ✓ All bonds, all investors)
- Each investor prepared to hold any bond
- PRICE = COUPON X ANNUITY FACTOR
+ 100 X DISCOUNT FACTOR

$$P = \sum_1^n C \times D_i + 100 \times D_n$$

$$P = C \times \sum_1^n D_i + 100 \times D_n$$

$$P = C \times AF + 100 \times D_n$$

The Price-Coupon Relationship with No Taxes



The Problem of Tax Effects

- Investors in different tax brackets will not agree on relative value of bonds

Coupon	After Tax Cash Flows		
	T = 0%	T = 30%	T = 60%
3%	103	102.1	101.2
9%	109	106.3	103.6
15%	115	110.5	106
V15/V3	115.0/103.0 = 1.117	110.5/102.1 = 1.082	106.0/101.2 = 1.047

Tax Analysis of Bonds

- Suppose we live a world with 3 coupon bonds
 - All three are 1 year bonds
 - Coupons are 3%, 9% and 15%
 - The 1 yr. spot rate is 10%
- Suppose we have 3 groups of tax-payers
 - 0%, 30% and 60%
- What are the bond prices?
 - What are the post-tax cash flows?
 - What are the post-tax yields?
- Use worksheet: Bonds & Taxes

Solution: Tax Analysis of Bonds



	BOND		
	1	2	3
Coupon	3%	9%	15%
Price	93.64	99.09	104.55
After-tax cash flows			
Tax rate 0%	103.0	109.0	115.0
Tax rate 30%	102.1	106.3	110.5
Tax rate 60%	101.2	103.6	106.0
After-tax yield			
Tax rate 0%	10%	10%	10%
Tax rate 30%	9.0%	7.3%	5.7%
Tax rate 60%	8.1%	4.6%	1.4%

Implications of Tax Analysis

➤ A Zero Tax Payer

- All the bonds offer the same yield
- Hence, s/he will hold any of the bonds

➤ A Tax Payer

- Will receive higher yield on low coupon bond
- Hence, will prefer a low coupon bond to high coupon bond

Efficient Bonds

- Bond Price $>$ NPV of cash flows for some investors
- An Efficient Bond:
 - Price = NPV
- Example:
 - The 3% coupon bond is efficient
 - For both 30% and 60% taxpayers
- Next Issue: What is the spot rate?

Post-Tax Spot Rate

- Post-Tax Yields: Non-Taxpayer
 - For the 0% tax payer, the yield on all bonds is 10%
 - Hence pre-tax spot rate = post-tax spot rate = 10%
- Post-Tax Yields: Taxpayers
 - Yields vary for each bond
 - Which yield is the post-tax spot rate?
- The Post-Tax Spot Rate: the *highest* yield
 - The yield on the *efficient* bond
- Example:
 - For 30% taxpayers, post-tax spot rate is 9%
 - For 60% taxpayers, post-tax sport rate is 8.1%

Tax Specific Yield Curves

- Investors in different tax brackets will see the same price for different after-tax cashflows
- No one investor will set prices for all bonds
- Different investors will therefore have different after-tax discount factors and yield curves
- Investors in different tax brackets may or may not agree to hold the same bond

Post-Tax Bond Valuation

- Spot rates are different for different tax brackets
- So they will not agree on bond values
- Example: and 8% coupon 1 year bond
 - Use the Bonds & taxes worksheet
 - Use the post-tax spot rates
 - Find the NPV of the post-tax cash flows

Example: Bond Valuation

➤ Value of an 8% 1-year Bond

➤ Tax Rate	Post-Tax Spot rate	Post-Tax Cash Flow	Present Value
0%	10.0%	108.0	98.18
30%	9.0%	105.6	96.88
60%	8.1%	103.2	95.47

Tax Clientele Hypothesis

➤ No Tax Effects

- All bonds priced so they can be held optimally by a 0% tax payer
- Price coupon relationship linear
- No arbitrage

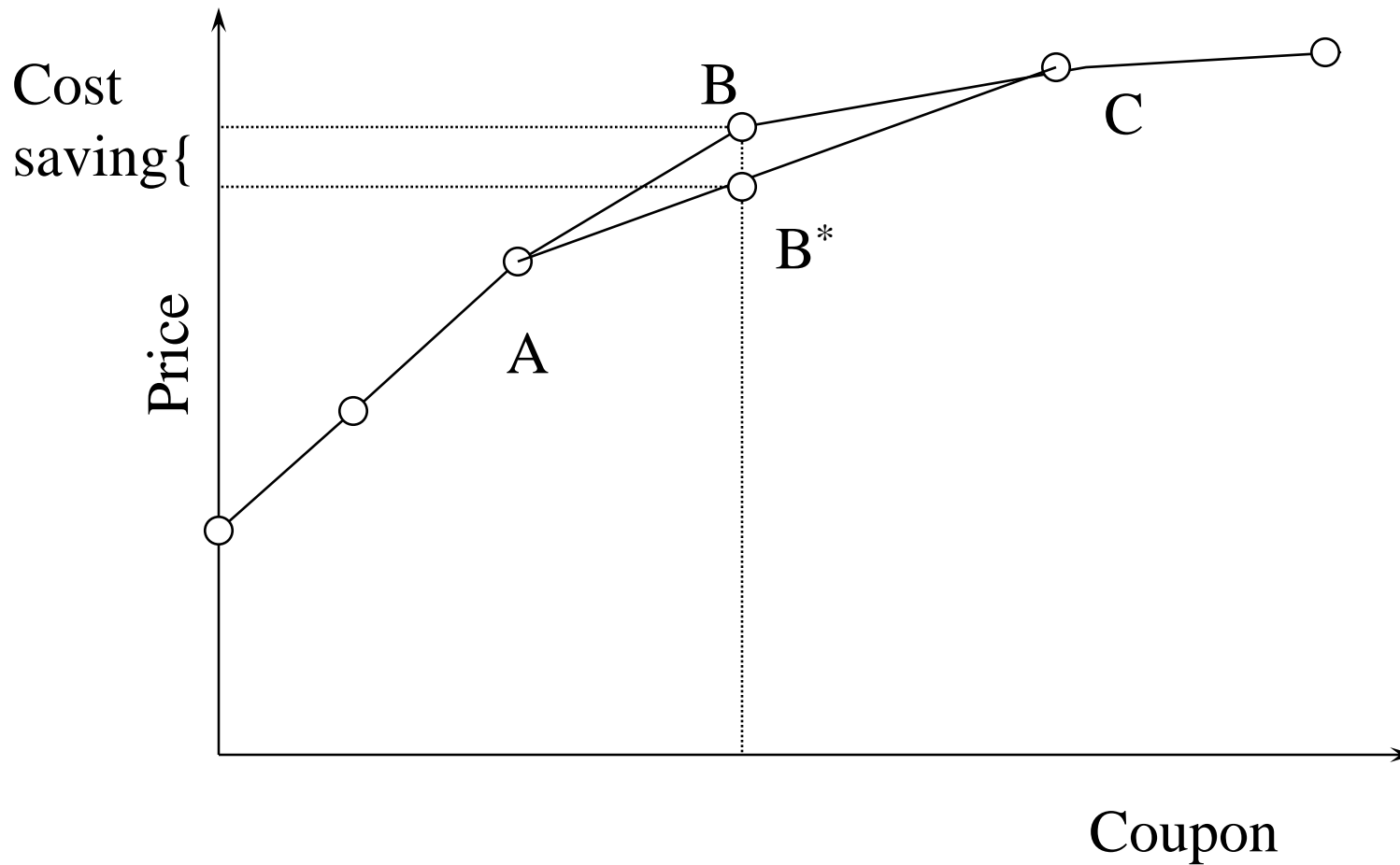
➤ Clientele Effects

- Different bonds prices so they can be held optimally by different tax-clienteles of investors
- Price coupon relationship non-linear
- Arbitrage

Tax Implication

- Assume different tax rates
- Bonds with different coupons
- Then there will be a tax arbitrage for at least one tax bracket
 - Note: Green & Oedegaard reject the no-tax hypothesis in formal test of US market

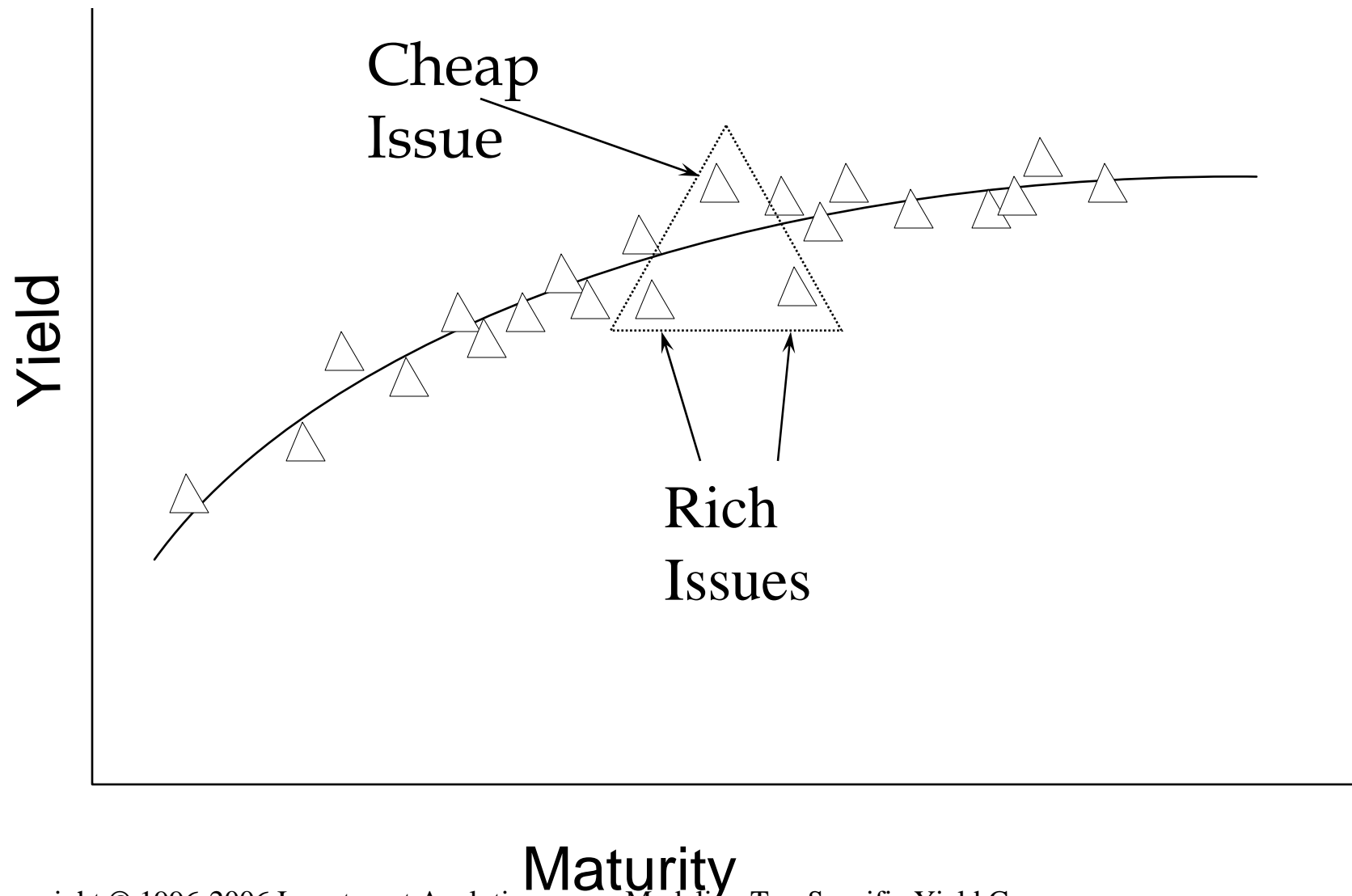
Tax Arbitrage: Non-linear Price-Coupon Curve



Rich/Cheap Analysis & Relative Value Trading

- Select the appropriate tax-rate
- Identify the tax-efficient bonds
- Plot the spot tax-yield curve using the efficient bonds
- Identify the issues which are low yield ('rich') or high yield ('cheap') relative to the curve
- Initiate duration-weighted trade

Rich/Cheap Graphical Analysis



Example: Bond Replication

➤ Bond	Coupon	Cash Flows	Price
A	3%	103	93.64
B	5%	105	96.00
C	8%	108	98.18

➤ Replicate Bond B, using Bonds A & C:

- Create $B^* = W_1 A + W_2 C$
- Require W_1 and W_2 so that
- $W_1 (103) + W_2 (108) = 105$
- $W_1 + W_2 = 1$

Example: Bond Replication

➤ Create replicating Bond B*

- $B^* = 3/5 A + 2/5 C$

➤ Check cash flows:

- $3/5(103) + 2/5(108) = 105$

➤ Cost Saving

• Price of Bond B	96.00
Cost of B* = $3/5(93.64) + 2/5(98.18)$	<u>95.45</u>
Cost Saving	0.55

➤ Arbitrage Trade

- Sell 10 x Bond B
- Buy 6 x Bond A and 4 x Bond C
- Riskless profit of \$5.50

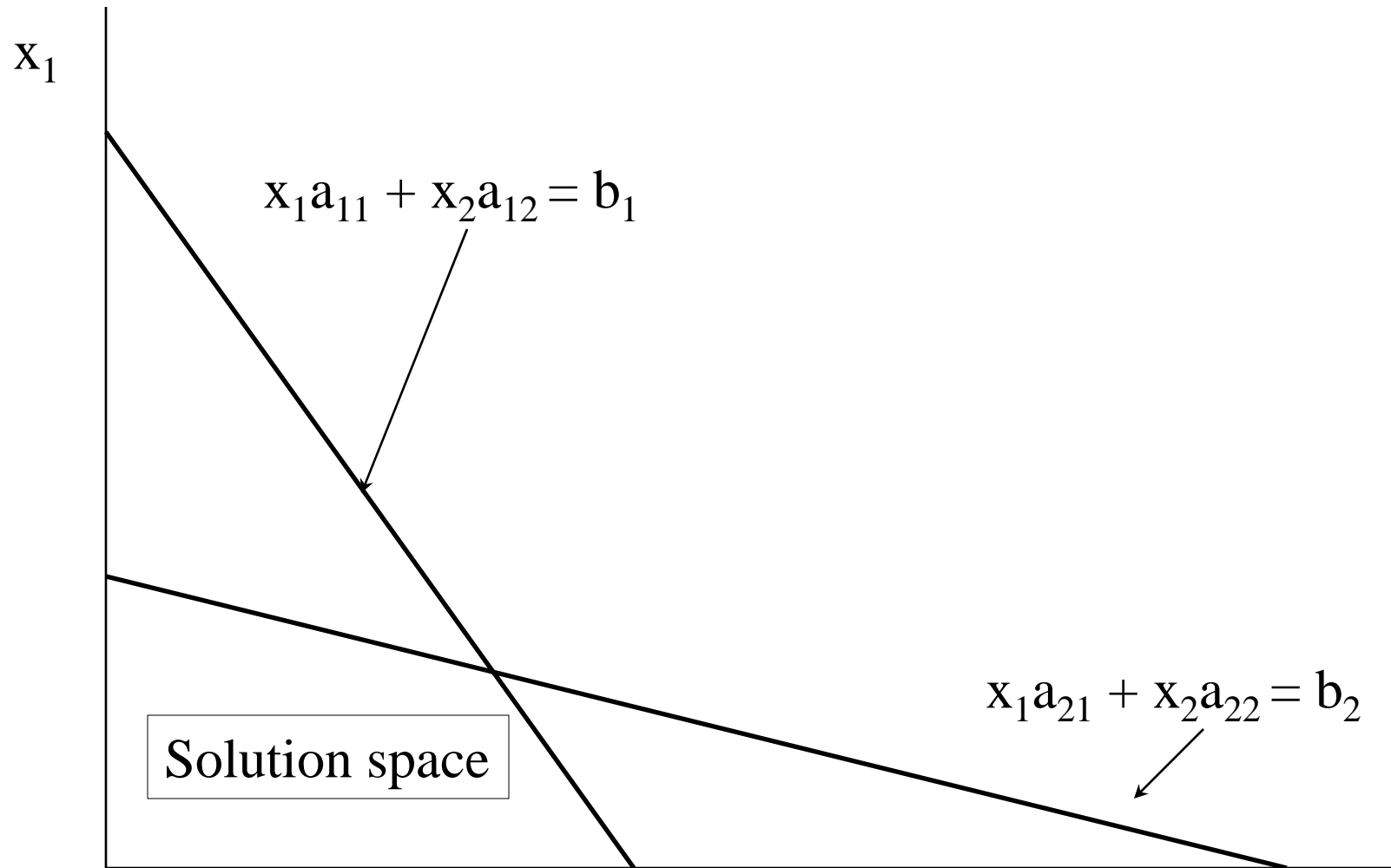
Tax Arbitrage: Basic Idea

- Use LP to find bond portfolio which replicates CF's of target bond, more cheaply.
 - ✓ Will identify & use efficient bonds
- Pick a target bond & replicate test for each tax rate.
- Restrictions on arbitrage:
 - ✓ Transaction costs
 - ✓ Short sale restrictions

Linear Programming

- Maximize (or minimize) an objective function
 - ✓ Subject to a set of constraints
- Objective function and constraints are linear
 - ✓ Maximize $x_1p_1 + x_2p_2$
 - ✓ Subject to:
 - $x_1a_{11} + x_2a_{12} \leq b_1$
 - $x_1a_{21} + x_2a_{22} \leq b_2$

Linear Programming



Summary: Tax Effects

- Taxes important for **both tax paying and tax-exempt** investors
- Strong evidence of **tax-clienteles**
- Taxes will influence investors **choice** of bonds
- A non-linear price-coupon relationship implies **arbitrage** opportunities
 - Studies in Germany, Japan, UK & USA all show that prices reflect non-zero income and capital gains taxes

Building After-Tax Yield Curves

- Use LP to find efficient bonds:
 - ✓ Bonds that minimize the cost of a given cash flows
- Build a yield curve from the efficient bonds
 - ✓ Use regression and basis splines
- Curve will be different for each tax bracket
- NB- Assumptions:
 - ✓ No short sales allowed
 - ✓ No transaction costs

LP Formulation for Cost Minimization

➤ Optimal portfolio:

- ✓ Provides cash flows at a minimum cost
- ✓ Minimize cost of a given set of cashflows from a portfolio of bonds

➤ Notation:

- $j = 1, \dots, T$ Periods in time
- $I = 1, \dots, m$ Bonds in portfolio
- x : Bond holding
- p : Bond price
- s : Cash flows from portfolio
- d : Discount factor
- a : After tax payment from bond

$$\text{Min} \sum_{i=1}^m x_i p_i$$

Subject to :

$$\sum_i a_{ij} x_i \geq s_j$$

$$x_i \geq 0$$

LP Formulation for Cashflow Maximization

- Maximize discounted cashflows from given portfolio
 - ✓ Subject to a yield curve generated from efficient bonds

- Notation:

- $j = 1, \dots, T$ Periods in time
- $I = 1, \dots, m$ Bonds in portfolio
- x : Bond holding
- p : Bond price
- s : Cash flows from portfolio
- d : Discount factor
- a : After tax payment from bond

$$Max \sum_{j=1}^T s_j d_j$$

Subject to:

$$\sum_{j=1}^T a_{ij} d_j \leq p_i$$

$$d_j \geq 0$$

LP & Basis Splines

- Use weighted sum of basis splines to represent discount function:

$$d(t_j) = \sum_1^L \alpha_l f_l(t_j)$$

$$\text{Max} \sum_{l=1}^L \omega_l \alpha_l \quad \omega_l = \sum_{j=1}^T s_j f_l(t_j)$$

Subject to:

$$\sum_{l=1}^L \alpha_l \sum_{j=1}^T a_{ij} f_l(t_j) \leq p_i$$

Portfolio Cashflows

- Simple version:
 - ✓ Set all cashflows = 1
- More advanced:
 - ✓ Make objective function equally sensitive to changes in yield curve at all points
 - Set cashflows: $S_j = [jd(t_j)]^{-1}$
 - Iterate to compute the values of s_j and identify efficient bonds.

Curve Building Procedure

- Use LP to select the efficient bonds for given tax bracket
 - ✓ This will select a small number of bonds
- Include some slightly less efficient bonds to fill out the curve
 - ✓ Use bonds which are “relatively efficient”
- Apply basis splines & fit regression model

Relative Efficiency

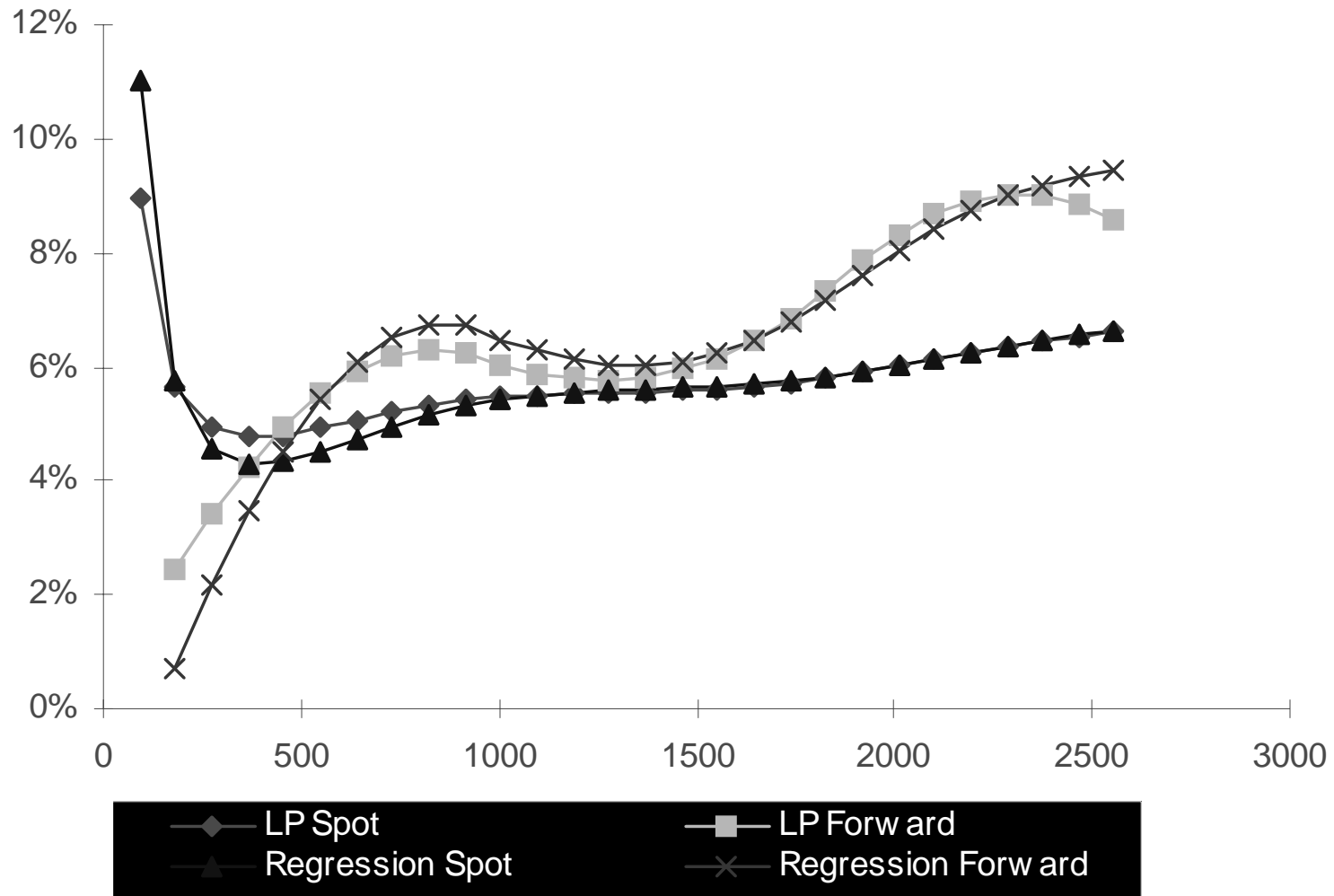
- Define the Relative Efficiency of a bond for a given tax bracket as:
 - ✓ NPV of Cash flows / Bond Price
- Define a tolerance level (e.g. 99%)
- Include bonds with:
Relative Efficiency > Tolerance

Lab: Pencoia Fund Management

- Scenario:
 - ✓ Running private client bond fund
 - ✓ Customers in different tax brackets (0%, 25%, 50%)
- Analyze market and recommend suitable purchases for different tax clienteles
- Worksheet: Pencoia Fund Management
 - ✓ See Lab and Solution Notes

Solution: Pencoia Fund Management

25% tax rate



Summary: Modeling the Tax-Specific Yield Curve

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