Risk Management

Interest Rate Risk

Agenda

- Basic Concepts
  - Bond Values & Interest Rate Risk
- Interest Rate Risk Measurement
  - Duration
  - Immunization
  - Convexity
  - Multi-factor Duration Models
- Advanced Interest Rate Risk Modeling
  - Index Rate Duration
  - Interest Rate Options
  - Deterministic & Simulation Analysis

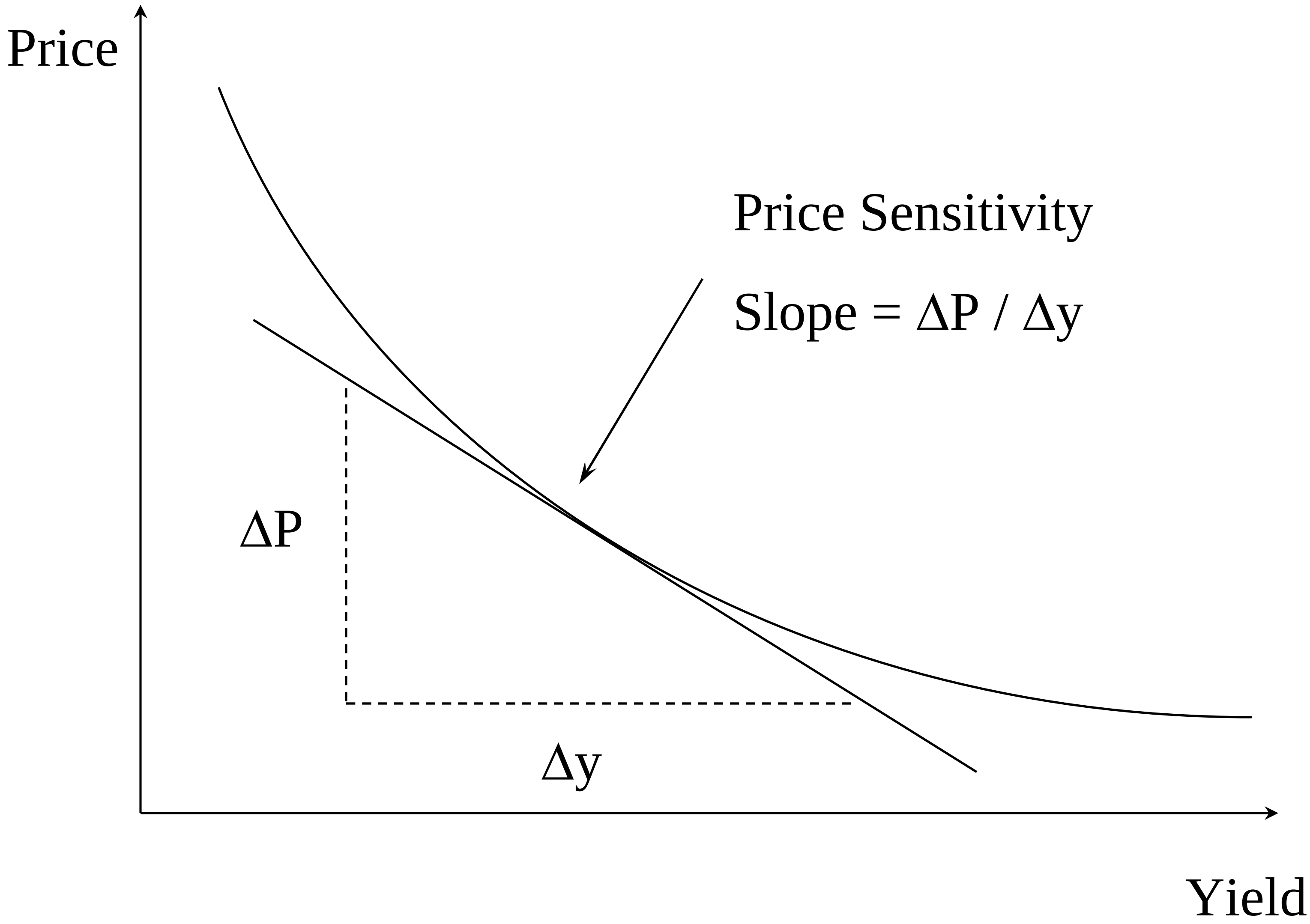
Bond Values and Interest Rates

- What is the relationship between a bond's price and interest rates?
- How does this sensitivity depend on the maturity of the bond?
- Are coupon bonds more sensitive to interest rates than zero coupon bonds?

Interest Rate Risk - Example

- Interest rate changes cause bond prices to fluctuate:
- Example: 8% coupon bond
  - If rates are at 8%, it will sell at par
  - If rates rise to 9% , price must fall below par
    - no-one will want to hold the bond at par value, so price will fall
    - must have expected capital gain to compensate for coupon below market rate
  - If rates fall to 7%, price will rise above par
    - everyone will bid for bond paying above market rate
    - forces price up & builds in expected capital loss to offset coupon above current market rate

The Price-Yield Relationship



Worked Exercise:
Bond Values & Interest Rates

- Start Bond Tutor
- Subject: Bond Values & Interest Rates
- Follow worked exercise

Factors Affecting Interest Rate Sensitivity

- Term
  - Long term bonds are more sensitive than short term bonds
- Coupon
  - Low (Zero) coupon bonds are more sensitive than high coupon bonds
- Yield
  - bonds at lower yields are more sensitive than at higher yields

Duration

- The further away cash flows are, the more their PV is affected by interest rates:
  - $PV = C/(1 + r)^t$
- *Duration* measures weighted average maturity of cash flows:
  - $D = \Sigma t \times W_t$
    - $W_t = \dfrac{CF_t / (1 + y)^t}{PV}$
    - y is yield to maturity
- Higher duration means greater risk

Duration & Risk

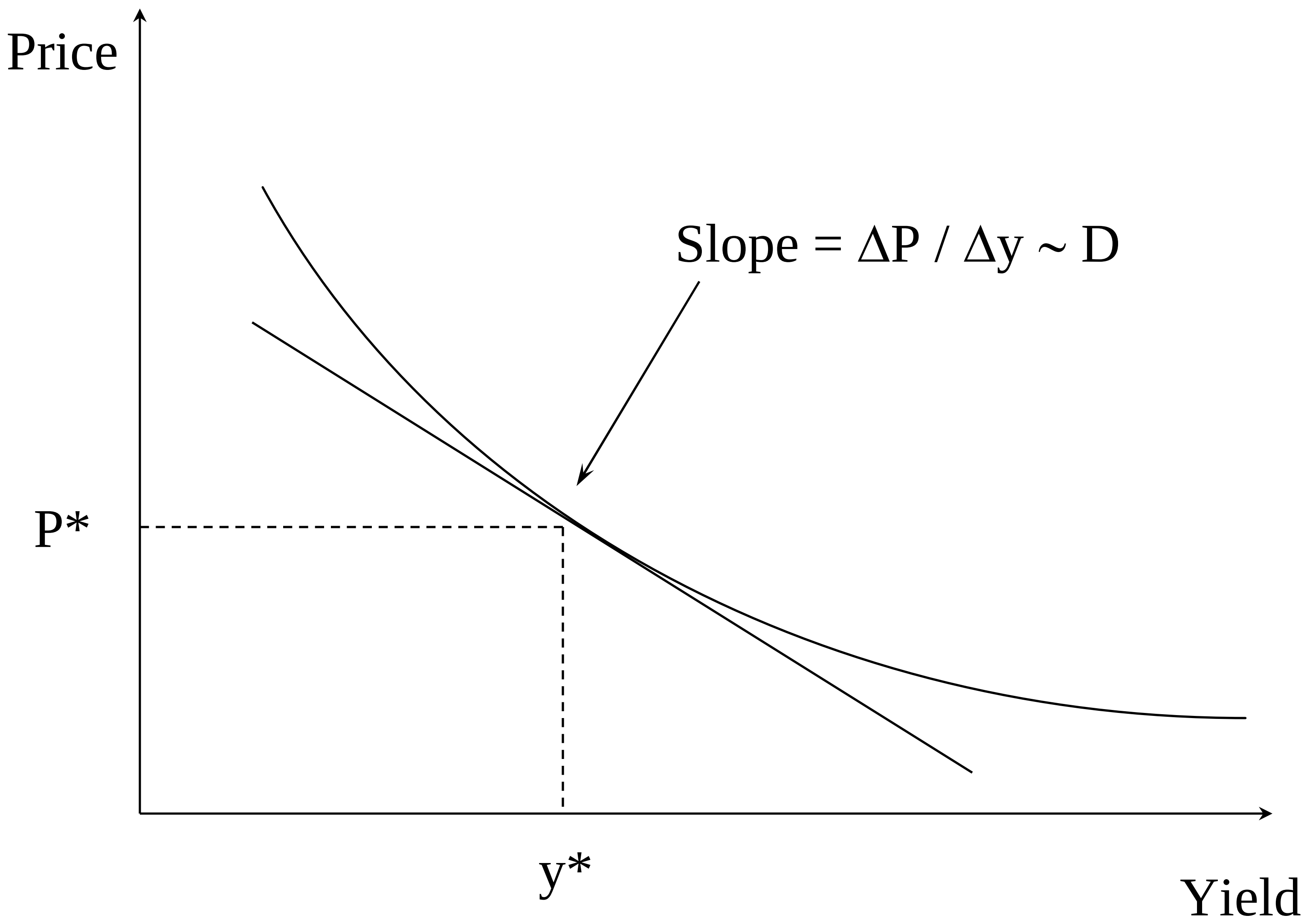
- Impact of changes in YTM:
  - $\Delta P = -[D / (1 + y)] \times P \times \Delta y$
  - $D / (1 + y)$ is known as *modified duration D**
  - $D^* = [\Delta P / P] \times (1 / \Delta y)$
  - Percentage price change $[\Delta P / P] = D^* \times \Delta y$
- Limitations:
  - Small changes in y
  - Parallel changes in yield curve

Example of Duration Calculation: Interest Rate = 10%

Time	Cash Flow	Discount Factor	PV of Cash Flow	PV Weight	PV Weight x Time
1	100	0.9091	90.91	0.2398	0.2398
2	100	0.8264	82.64	0.2180	0.4360
3	100	0.7513	75.13	0.1982	0.5946
4	100	0.6830	68.30	0.1802	0.7207
5	100	0.6209	62.09	0.1638	0.8190
TOTAL			**379.07**	**1.0000**	**2.8101**

- Duration = 2.81 Years
- Modified Duration = 2.81 / 1.1 = 2.55 years

Duration & Price-Yield Relationship



Two Ways to Think About Duration

- Weighted Average Time to Maturity
  - Weight the time of each cashflow by proportion of total NPV it represents

- As the *sensitivity* of a security's PV to change in interest rates
  - Sensitivity = $\delta P / \delta y = -\Sigma t\, [CF_t / (1 + y)^t] \times 1/P$

Immunization

- If
  - duration of assets = duration of liabilities
  - value of assets = value of liabilities
- Portfolio is "immunized"
- Portfolio value will be unchanged
  - for small, parallel changes in yield

Worked Exercise on Duration

- Start Bond Tutor
- Subject: Duration
- Follow worked exercise

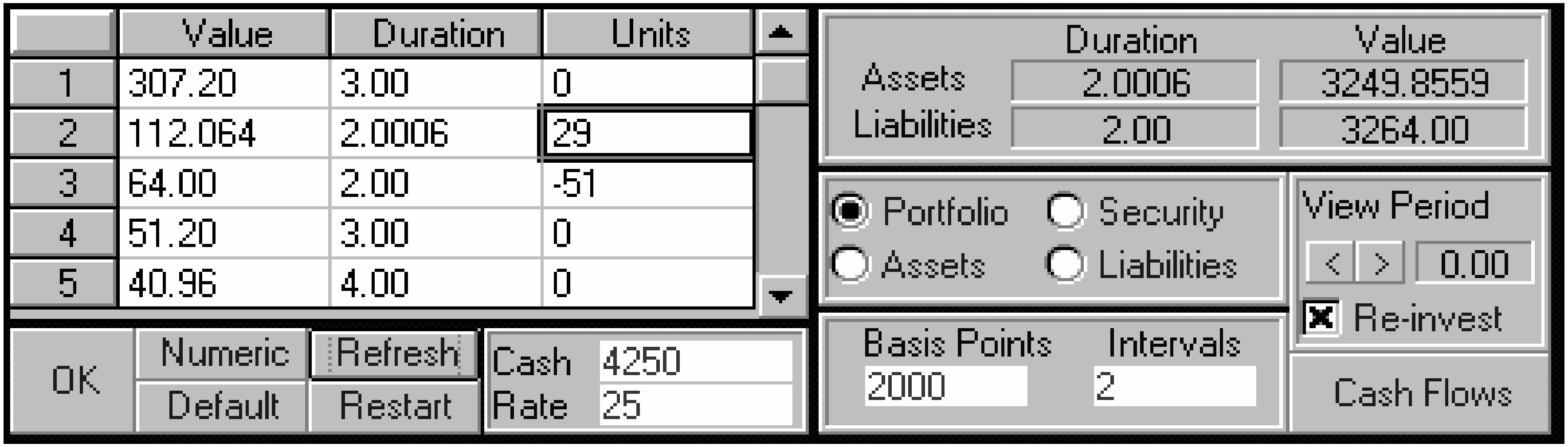
Trading Case B04

- Flat yield curve 25%
- Can move to: 5% to 45%
- You have a liability/asset which you cannot trade
- Must try and preserve value of portfolio

Analysis of Case B04

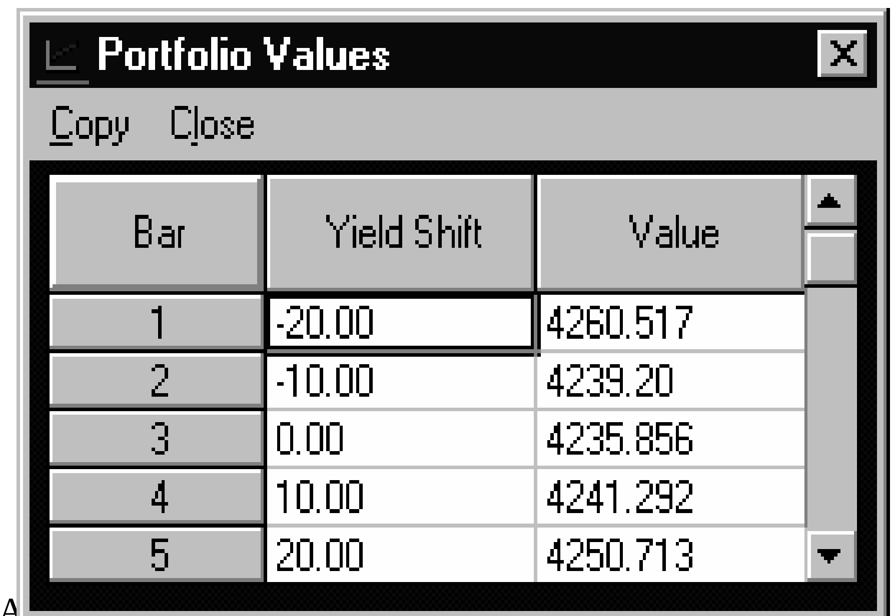
- Position 1
  - 3200 cash
  - 14 of sec worth 307
  - -51 of sec worth 64
- <u>What should you do</u>
  - Sell 14 @ 307.2
  - Buy 29 @ 112.064
  - Why 29?:
    - asset value = 29 * 112 = 3250
    - liability value = 51 * 64 = 3264

Analysis of Case B04



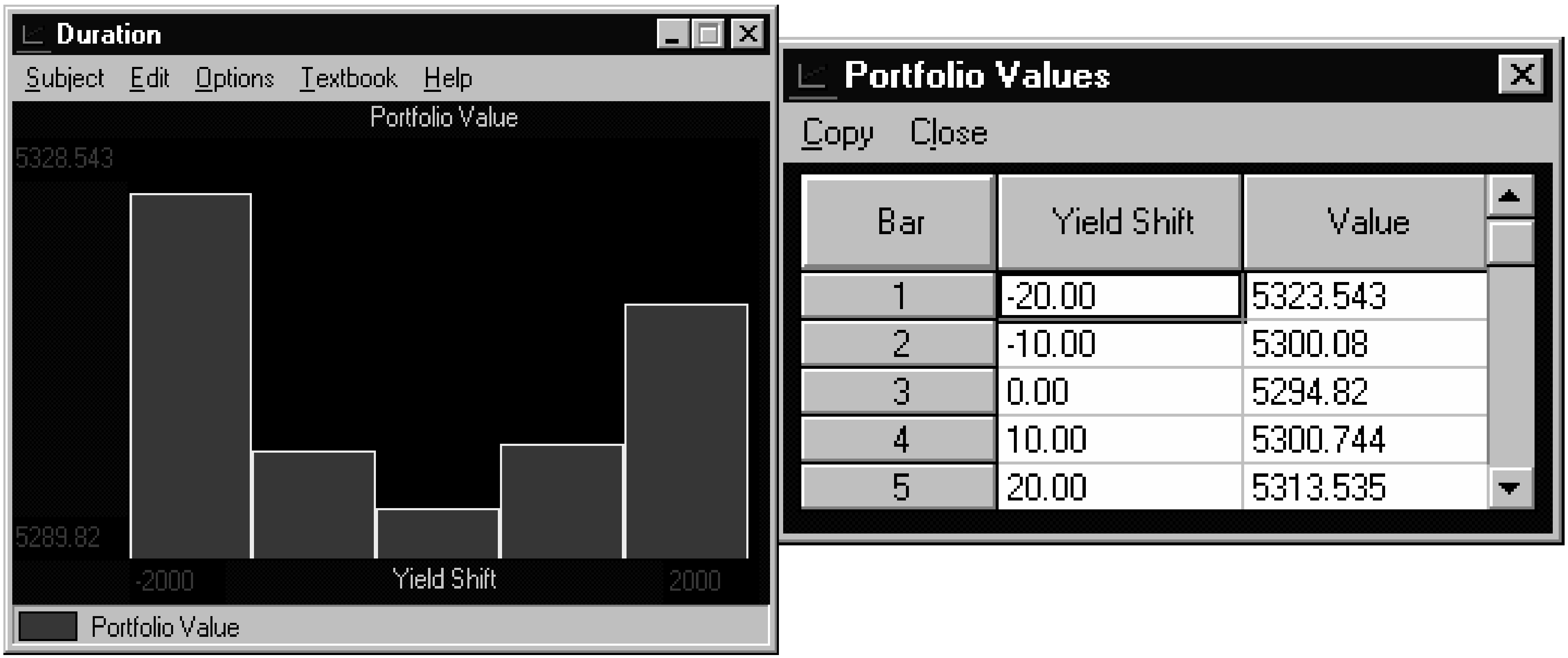
Note: cash = 4250 after trade

"instantaneous exposure"



Analysis of case B04

Exposure at end of period

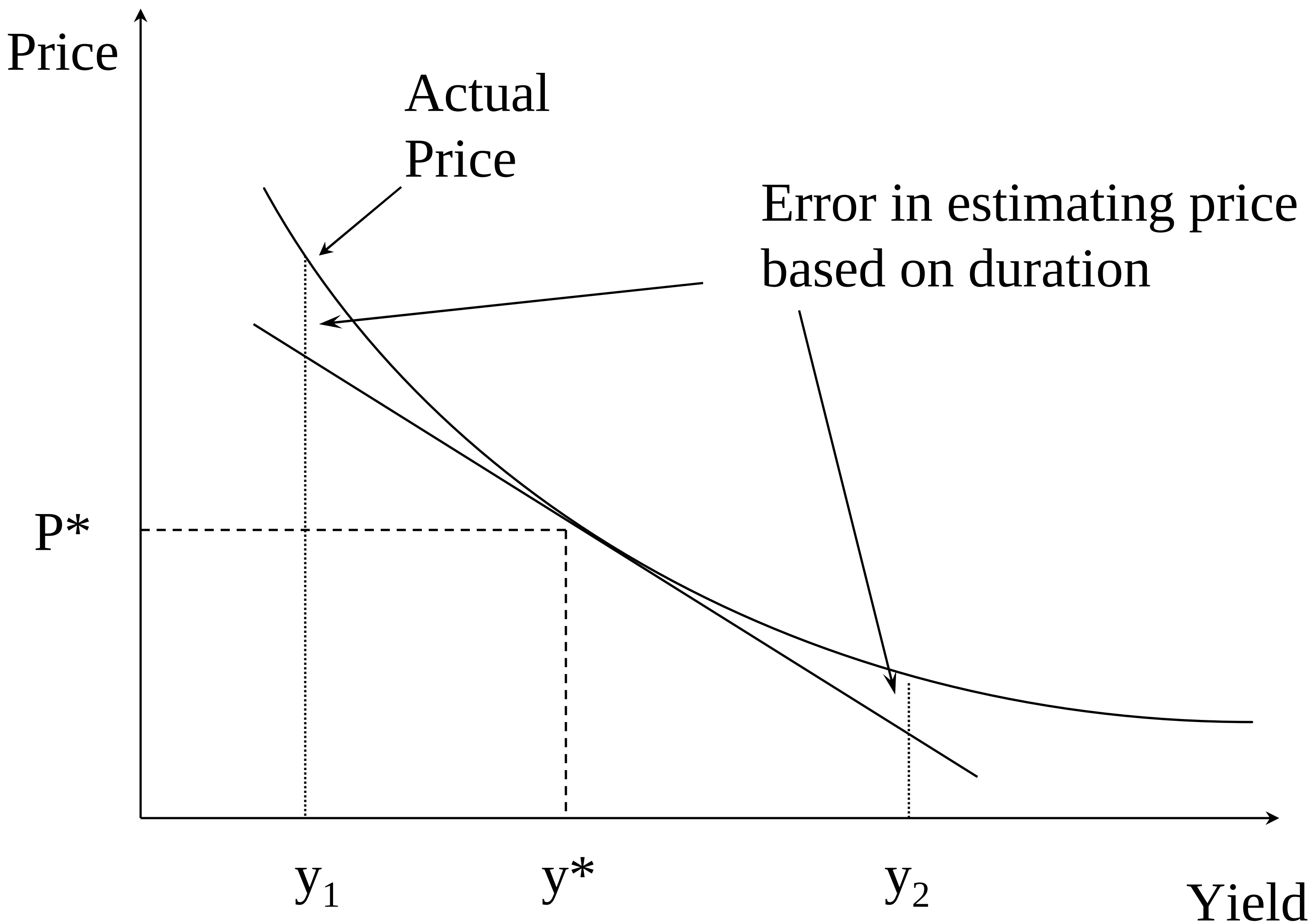


Bar	Yield Shift	Value
1	-20.00	5323.543
2	-10.00	5300.08
3	0.00	5294.82
4	10.00	5300.744
5	20.00	5313.535

Problems with Conventional Immunization

Assumption	Empirical Evidence
➢ Yield curve shifts are parallel	Short rates move more than long rates
➢ Yield curve changes perfectly correlated along the curve	Correlation between short and long rates much less than 1.0

Price Approximation Using Duration



Convexity

- Duration assumes linear price-yield relationship
  - Duration proportional to the slope of the tangent line
  - Accurate for small changes in yield

- Convexity recognizes that price-yield relationship is curvilinear
  - Important for large changes in yield

Convexity Formula

- Dollar Convexity:
  - $\delta^2 P / \delta y^2 = \Sigma CF_t \times t(t+1) / (1 + y)^{t+2}$
  - Price change due to convexity:
    - $\Delta P = \text{Dollar Convexity} \times (\Delta y)^2$

- Convexity = $[\delta^2 P / \delta y^2] \times (1 / P)$
  - Percentage price change due to convexity:
    - $\Delta P / P = 0.5 \times \text{Convexity} \times (\Delta y)^2$

Convexity Adjustment: Example

- Straight Bond
  - 6% coupon, 25yr, yield 9%
  - Modified Duration =10.62
  - Convexity = 182.92
- % Price Change:

Yield Move	Duration $(D^* \Delta y)$	Convexity $0.5 \times C (\Delta y)^2$	Total
+200bp	-21.24%	3.66%	-17.58%
-200bp	+21.24%	3.66%	+24.90%

Summary: Interest Rates & Risk

- How interest rates affect bond prices
- Duration
- Immunization
- Convexity

A Two Factor Model of Yield Curve Changes

$$\text{Change in spot rate} = A_t \times \text{Change in short rate} + B_t \times \text{Change in long rate}$$

$$= \alpha_t \times \text{Change in spread} + \beta_t \times \text{Change in long rate}$$

- Spread: (Long rate - Short rate)
- Two factor Model:

α_T : sensitivity of T-period spot rate to changes in spread

β_T: sensitivity of T-period spot rate to changes in long rate

Immunization with Two Factor Model

- Factors
  - Long rate
  - Spread = long rate - short rate

- Durations: each asset has two durations
  - Long Duration: sensitivity to change in long rate
  - Spread Duration: sensitivity to change in spread

Computing Two Factor Durations

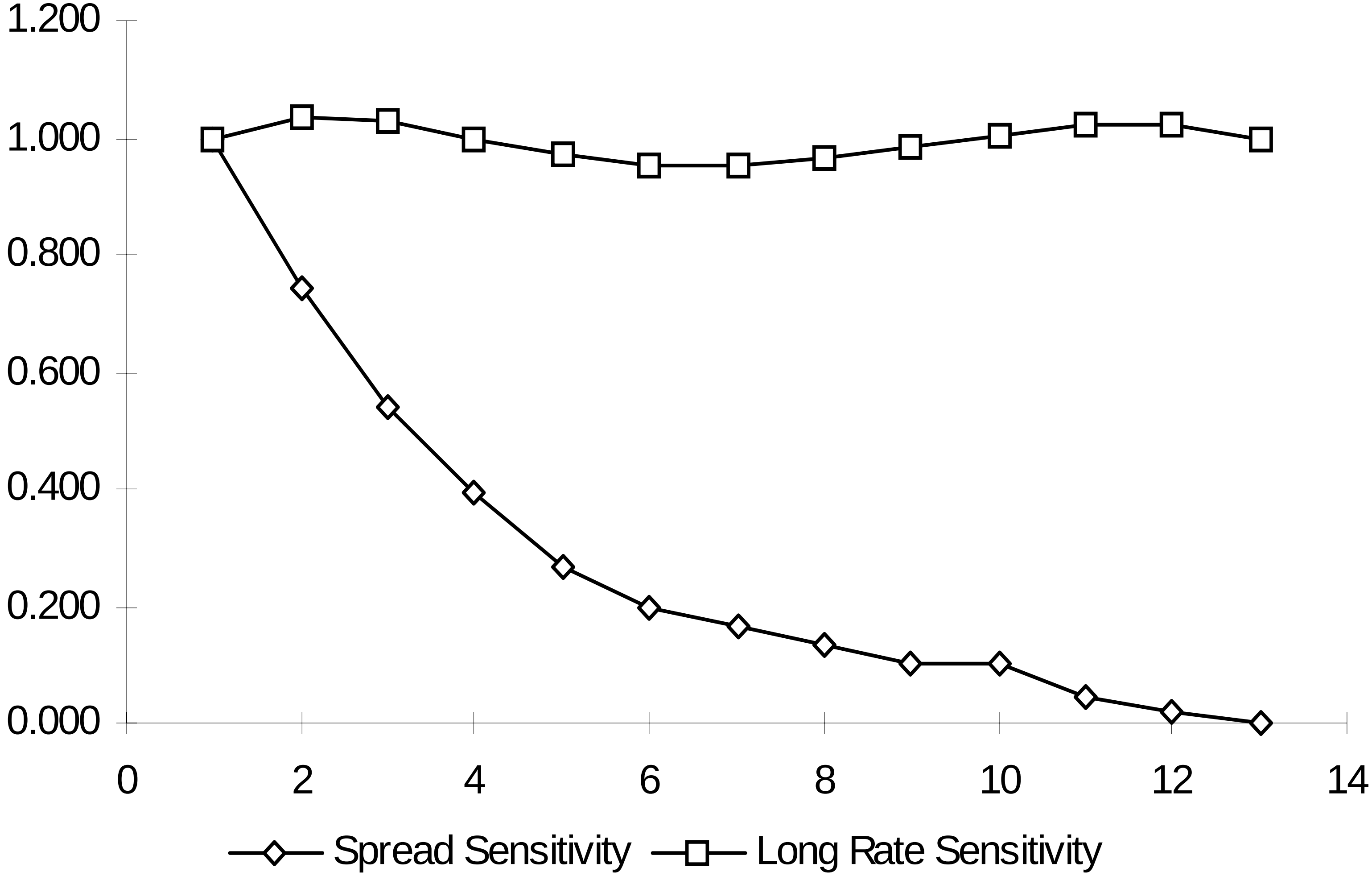
- Duration formula:
  - $D_S = -\Sigma T_i \, \alpha_{Ti}[c_i e^{-RTi}/PV]$
  - $D_L = -\Sigma T_i \, \beta_{Ti}[c_i e^{-RTi}/PV]$
- Regression Analysis

$$\Delta R_T = A_T + \alpha_T \Delta S + \beta_T \Delta L + \varepsilon_T$$

Estimated Long Rate & Spread Sensitivities (Nelson/Schaefer)

Maturity (Years)	Spread Sensitivity	Long Rate Sensitivity

Spread & Long Rate Sensitivities



Implied Spot Rates: Relative Importance of Factors

Maturity	Total Variance Explained	% of Total Explained Variance Accounted for by Factor 1	Factor 2	Factor 3
6 Months	99.5	79.5	17.2	3.3
1 year	99.4	89.7	10.1	0.2
2 years	98.2	93.4	2.4	4.2
5 years	98.8	98.2	1.1	0.7
8 years	98.7	95.4	4.6	0.0
10 years	98.8	92.9	6.9	0.2
14 years	98.4	86.2	11.5	2.2
18 years	93.5	80.5	14.3	5.2
Average	**98.4**	**89.5**	**8.5**	**2.0**

Source: Journal of Fixed Income, “Volatility and the Yield Curve”, Litterman, Scheinkman & Weiss

Example: Calculating Spread Duration

- 8% 4-year bond
- Spot rates 10% flat

Time	Cash Flow	DF	PV	Spread Sensitivity	Time x PV x Spread Sensitivity
1	8	0.9091	7.27	1.000	7.27
2	8	0.8264	6.61	0.743	9.82
3	8	0.7513	6.01	0.542	9.77
4	108	0.6830	73.77	0.391	115.37
TOTAL			93.66		142.24

Spread Duration = 142.24 / 93.66 = 1.52

Immunization Conditions

- Portfolio Weights add to One
- Match Spread Duration
  - Weighted average of spread duration of assets = spread duration of liabilities
- Match Long Duration
  - Weighted average of long duration of assets = long duration of liabilities
- Equations
  - $w_1 + w_2 + w_3 = 1$
  - $w_1 D_{1S} + w_2 D_{2S} + w_3 D_{3S} = D_S$
  - $w_1 D_{1L} + w_2 D_{2L} + w_3 D_{3L} = D_L$

When One Asset is Cash

- Sensitivity of cash to all interest rates is zero
  - $w_1 D_{1S} + w_2 D_{2S} = D_S$
  - $w_1 D_{1L} + w_2 D_{2L} = D_L$
- Cash holding is residual
  - $w_3 = 1 - w_1 - w_2$

Lab: Bond Hedging Exercise

- Worksheet: Bond Hedging
- Scenario:
  - You have a short position in 8-year bonds
  - Have to hedge using 3 and 15 year bonds
- Hedging
  - Create conventional duration hedge
  - Test under 4 scenarios
  - Create 2-factor duration hedge
  - Repeat test & compare
- See Notes & Solution

Solution: Bond Hedging Exercise

- Hedge Structure

Method	Holdings			
	Cash	3yr	8yr	15yr
Conventional	0.00	0.3538	-1.000	0.6462
Two-Factor	-.0089	0.4599	-1.000	0.5490

- Hedge Performance (Profit/Loss)

Scenario	Conventional	2-Factor
I	-27bp	3bp
II	-29bp	3bp
III	28bp	2bp
IV	25bp	2bp

Advanced Interest Risk Modeling

- Index rate contingent cash flows
  - Key Treasury Rate Duration
- Interest rate options
  - Option-adjusted duration
- Analytical methods
  - Deterministic
  - Monte Carlo simulation

Duration Risk Measurement

- Recall: (dP/P) = - D* x dr
- Modified Duration D* = -(dP/dr) x 1/P
  - For swaps & derivatives concept of duration is ambiguous
- Need to measure sensitivity to changes in:
- Index Rate
  - $DUR_{INDEX} = -(dP/dr_{index}) \times 1/P$
- Discount Rate
  - $DUR_{DISC} = -(dP/dr_{disc}) \times 1/P$

Calculating Duration - Perturbation Method

- IRD
  - Add small increment dr (1bp) to index rate
  - Recompute PV
- DRD
  - Add small increment dr (1bp) to discount rate
  - Recompute PV
- DURATION = $[PV_{Orig} - PV_{New}]/PV_{Orig} \times 1/dr$

Discount Rate

- Can be found by assuming cash flows are non-contingent
- YTM of comparable fixed coupon note of same maturity
- Hence DUR_{DISC} = Duration of vanilla note
  - E.g. for 3-yr note DRD = 2.8 yrs
- Exception: Note which has indeterminate maturity

Index rate

- E.g. 3-yr FRN
- Coupon = 3-month LIBOR, paid quarterly
- What is appropriate index rate?
  - NOT 3-month LIBOR
  - Aggregate of all floating rate components
    - 12 different IR's in this example
- Solution: swap rate
  - Summarizes entire LIBOR cash flow stream
  - Expressed as a spread over 3-year treasury rate
  - Hence $DUR_{INDEX} = -2.8$ approx.

Net Duration

- 3-year FRN, coupon 3-m LIBOR

DRD	=	2.8
IRD	=	-2.8
NET DURATION	=	0

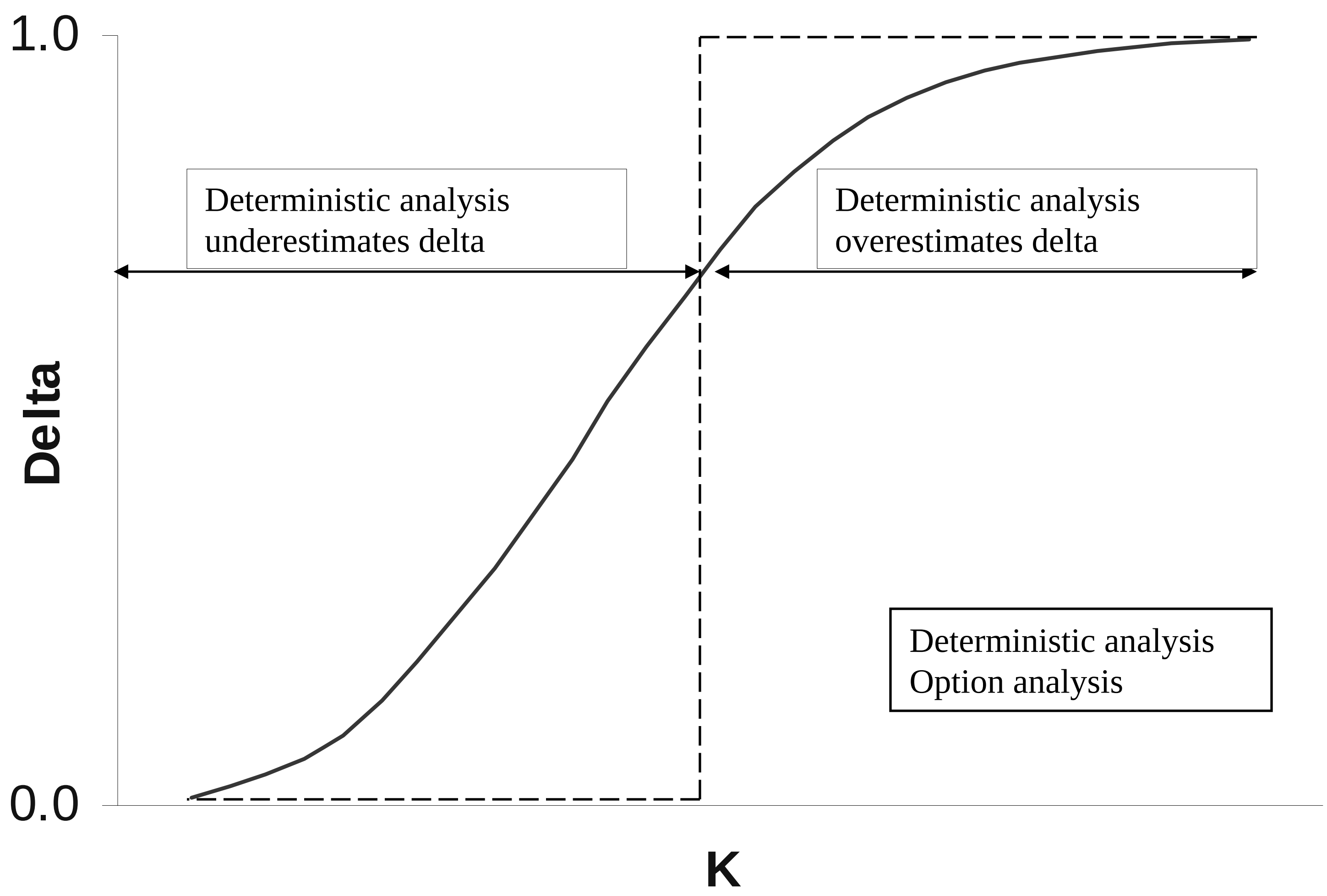
Key Treasury Rate Duration (KTRD)

- Calculates change in price wrt change in one segment of the Treasury curve.
- Used when Index rate and Discount rate are not equal

Duration for Derivative Structures

- E.g. Capped FRN
  - Like capped floating leg of swap
- Option Adjusted Duration
  - $OAD = DUR \times P / P_C \times (1 - \Delta)$
    - DUR = Duration of uncapped FRN
    - P = price of uncapped FRN
    - P_C = price of capped FRN
    - Δ = cap delta

Deterministic Analysis & Option Delta

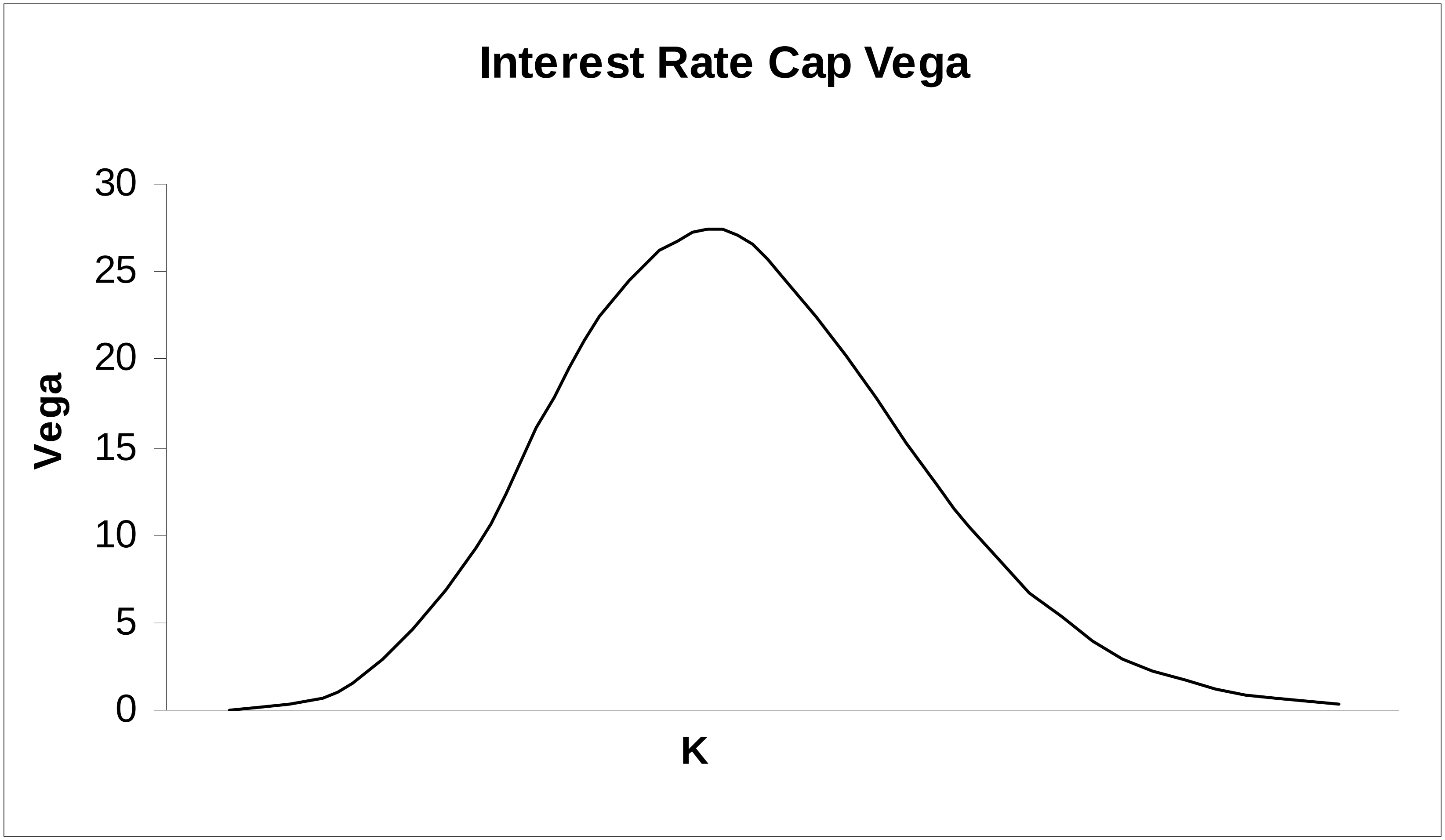


Volatility Duration

- Applies to securities with embedded optionality
- $DUR_{VOL} = -(1/P) \times (dP/d\sigma)$

  $= -(1/P) \times \text{Vega}$
- Vega greatest for ATM options

Cap Vega



Evaluating Risk

- Deterministic Analysis
  - Assume know rates in advance
  - Determines cash flows, yield
    - Duration estimated using perturbation method
- Simulation Analysis
  - Monte Carlo simulation model of interest rates
  - Statistical analysis of:
    - Cash flows
    - Yield
    - Duration

Deterministic Analysis

- Forward Analysis
  - Assumes index spot rates move to forwards
  - Problem of bias
    - Forward rates typically exceed future spot rates
- Expectation analysis
  - Projects 'expected' spot rates

Linear Smooth Expectation (LSE) Analysis

- Set final index spot rate
  - E.G. from forward rate
- Estimate intermediate index rates using linear interpolation
- Compute cash flows, yield, duration in normal way
- Repeat for range of final index rates

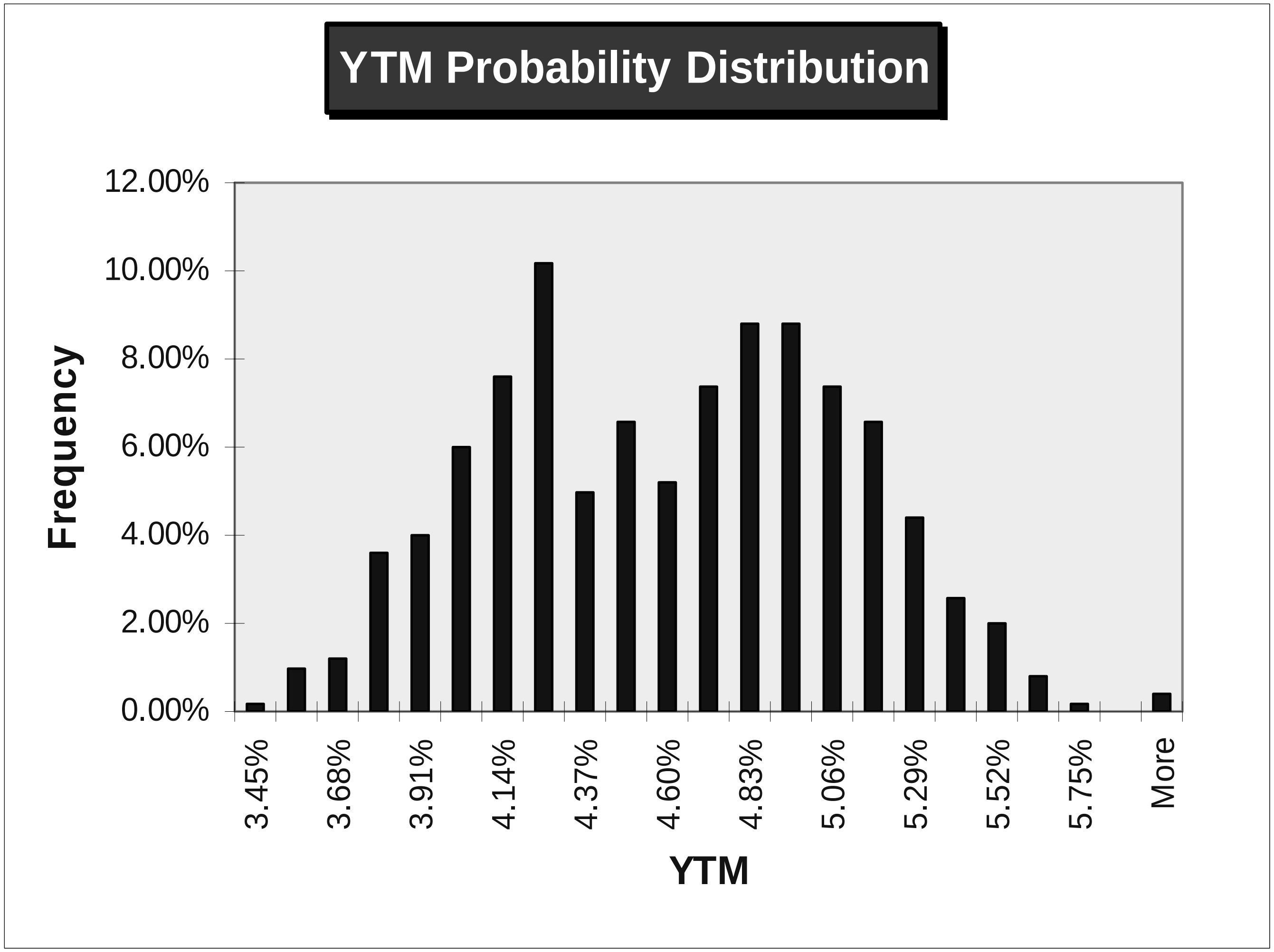
Monte-Carlo Methodology

- Simulate movement in index rates
- Calculate cash flows, PV’s, yield, duration
- Repeat large no of times
- Create histogram of yield, duration values
  - Calculate average yield, duration

Generating Simulated Index Rates

- $R + \Delta R = R \times \text{Exp}[(\mu - \sigma^2/2)\Delta t + \sigma \Delta z]$
  - ΔR is change in index rate
  - μ is drift factor
  - σ is volatility
  - $\Delta Z = \varepsilon(\Delta t)^{1/2}$
  - ε is normal random variable, No(0,1)
- Procedure:
  - Generate ε (random)
  - Compute new index rates, cash flows, etc
  - Estimate duration using perturbation method
  - Repeat many times (10,000+)

Example

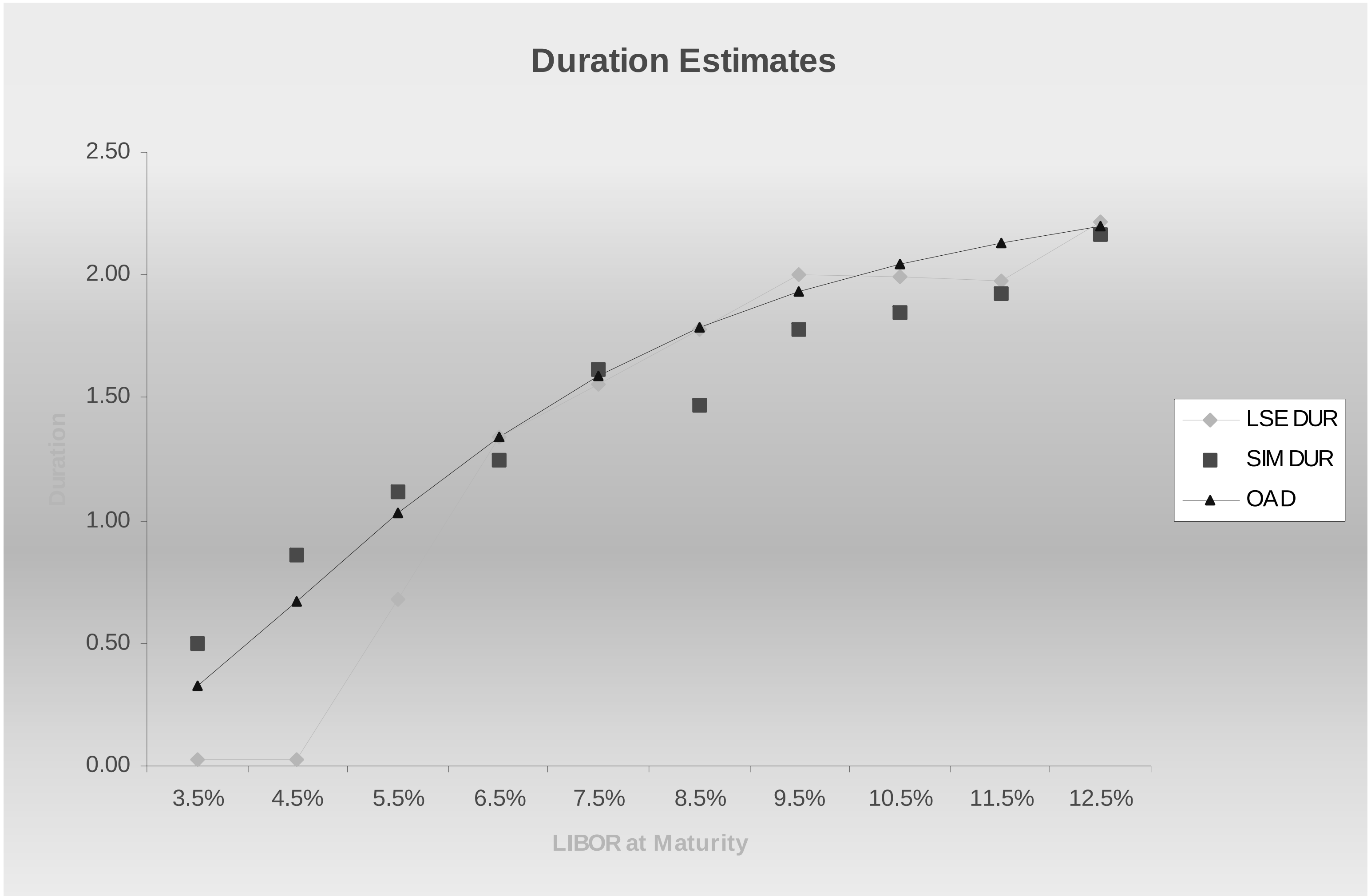


YTM Probability Distribution
12.00%
10.00%
8.00%
6.00%
4.00%
2.00%
0.00%
Frequency
3.45%
3.68%
3.91%
4.14%
4.37%
4.60%
4.83%
5.06%
5.29%
5.52%
5.75%
More
YTM

Lab: Capped FRN

- Start with simple 3-year FRN, quarterly LIBOR
  - Confirm IRD = - DRD
- FRN Coupon LIBOR + 0.5%, 5.5% Cap
  - Calculate IRD, DRD, Net Duration
  - Use simulation analysis to estimate yield, duration
  - Use LSE analysis to compute yield, duration
  - Compare LSE & simulation analysis
  - Compare OAD with deterministic & simulation analysis

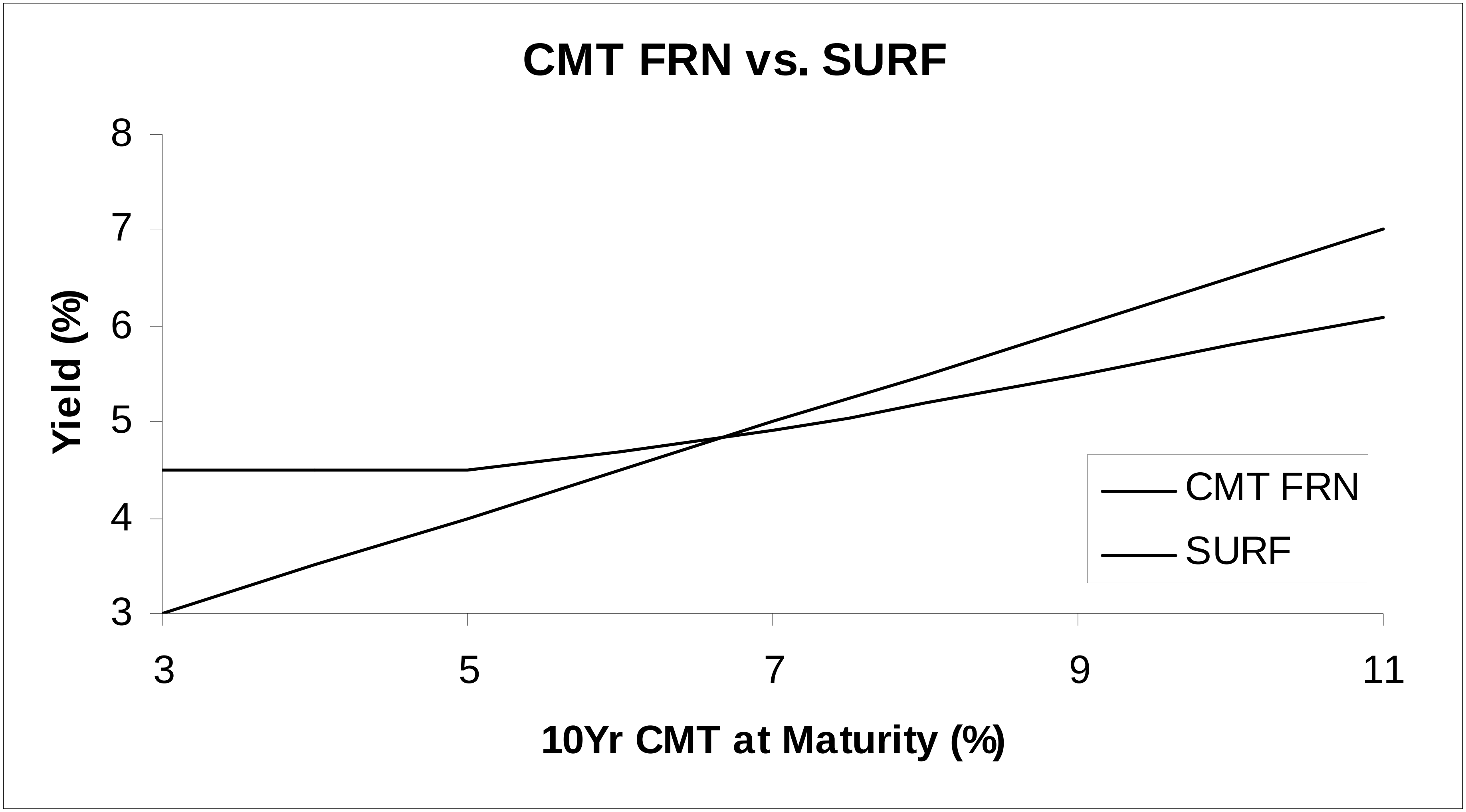
Solution: Capped FRN



Step-up Recovery Floaters (SURFs)

- Objective
  - Provide higher floating yield than CMT or LIBOR FRNs
- Structure
  - Above-market floor, some upside participation
  - Example: 5-year note
    - Coupon = 0.5*(10-year CMT) + 1.5%
    - Floor 4.5%
- Equivalent Position
  - Short T-Bonds
  - Long ITM Bond Call Options

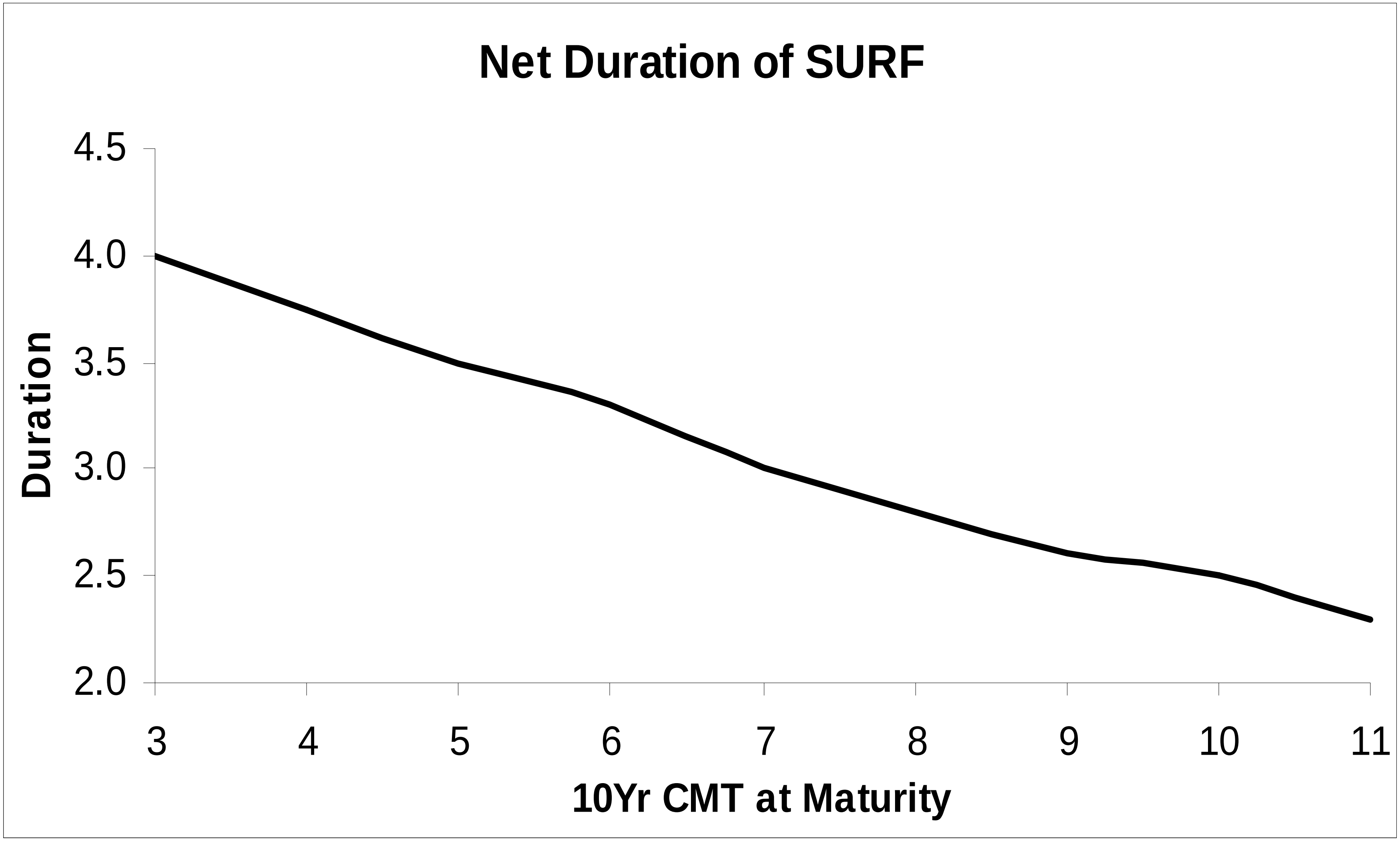
SURF vs CMT FRN



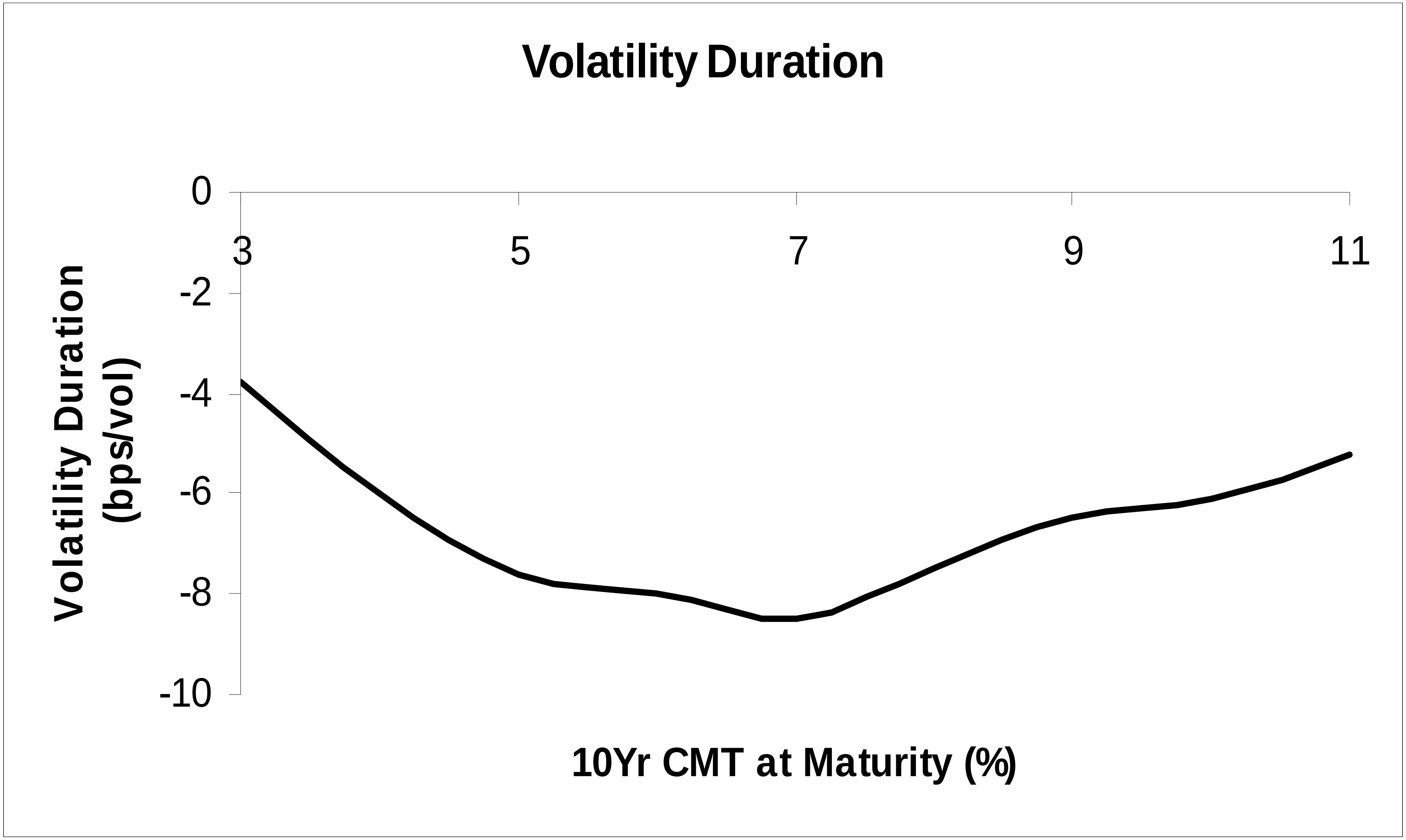
SURF Risk Factors

- Net Duration
  - A lower rates, behaves like a fixed income security
    - Due to coupon floor
    - Hence higher duration a low rates
  - At higher rates, behaves more like an FRN
    - Hence lower duration at high rates
- Volatility Duration
  - Long a floor option, positive Vega
  - Hence negative Vol. Duration
  - Value of floor (and note) increases with volatility

SURF - Net Duration



SURF - Volatility Duration



Range Floaters / LIBOR Enhanced Accrual Notes (LEANs)

- Typical Structure
  - 4 Year FRN
  - Coupon LIBOR + 50bp
    - Only paid if LIBOR in range
  - Year 1-2 range 5% - 6%
  - Year 3-4 range 6% - 7%
    - Ranges increase due to upward sloping forward curve
- Investor has written series of binary calls and puts
    - Compensated by higher spread
    - Taking advantage of high implieds
    - Betting that volatility will be lower than anticipated

LEANs - Risk Factors

- Net Duration
  - Close to zero within range
  - Changes dramatically outside range
    - Negative below range
      - note value rises with rates
    - Positive above range (>> maturity)
      - -note value falls as rates rise
- Volatility Duration
  - Positive in range
    - Note loses value if volatility increases
  - Negative outside range
    - Note gains in value if volatility rises

Multi-Index Notes

- Coupon based on sum or difference between multiple indices
- Most common structures:
  - CMT-LIBOR Differential Notes
  - Prime-LIBOR Differential Notes

Example: CMT-LIBOR Diff. Note

- Note features:
  - Issuer: US Agency
  - Maturity: 3 years
  - Annual Coupon: (10-year CMT - 12m LIBOR) +2.00%
- Discount Rate Duration
  - DR is to-maturity Treasury rate
  - Hence DRD = 2.8 years approx.
- Index Component
  - 10-year CMT
  - LIBOR

CMT-LIBOR Diff. Note - Overview

- Investor Outlook
  - Achieve higher coupon than either CMT or LIBOR
- Risk
  - Yield curve flattening will rapidly erode the note’s yield advantage
- Equivalent Position:
  - Long CMT FRN
  - Long Eurodollar Futures

CMT-LIBOR Diff. Note

- 10-Year CMT:

$$CouponPV = \frac{T_{1,10}}{(1+r_1)^1} + \frac{T_{2,10}}{(1+r_2)^2} + \frac{T_{3,10}}{(1+r_3)^3}$$

- 1bp change in T_{10} produces approx. 1bp change in 10-year forward rate $T_{1,10}$
- Hence value of note will increase by PV01 in each year

CMT-LIBOR Diff. Note: Key Treasury Rate Durations

Key Rate	PV01	Duration
T_{10}	$-1/(1+T_1)^1$	-0.95
T_{11}	$-1/(1+T_2)^2$	-0.91
T_{12}	$-1/(1+T_3)^3$	-0.86

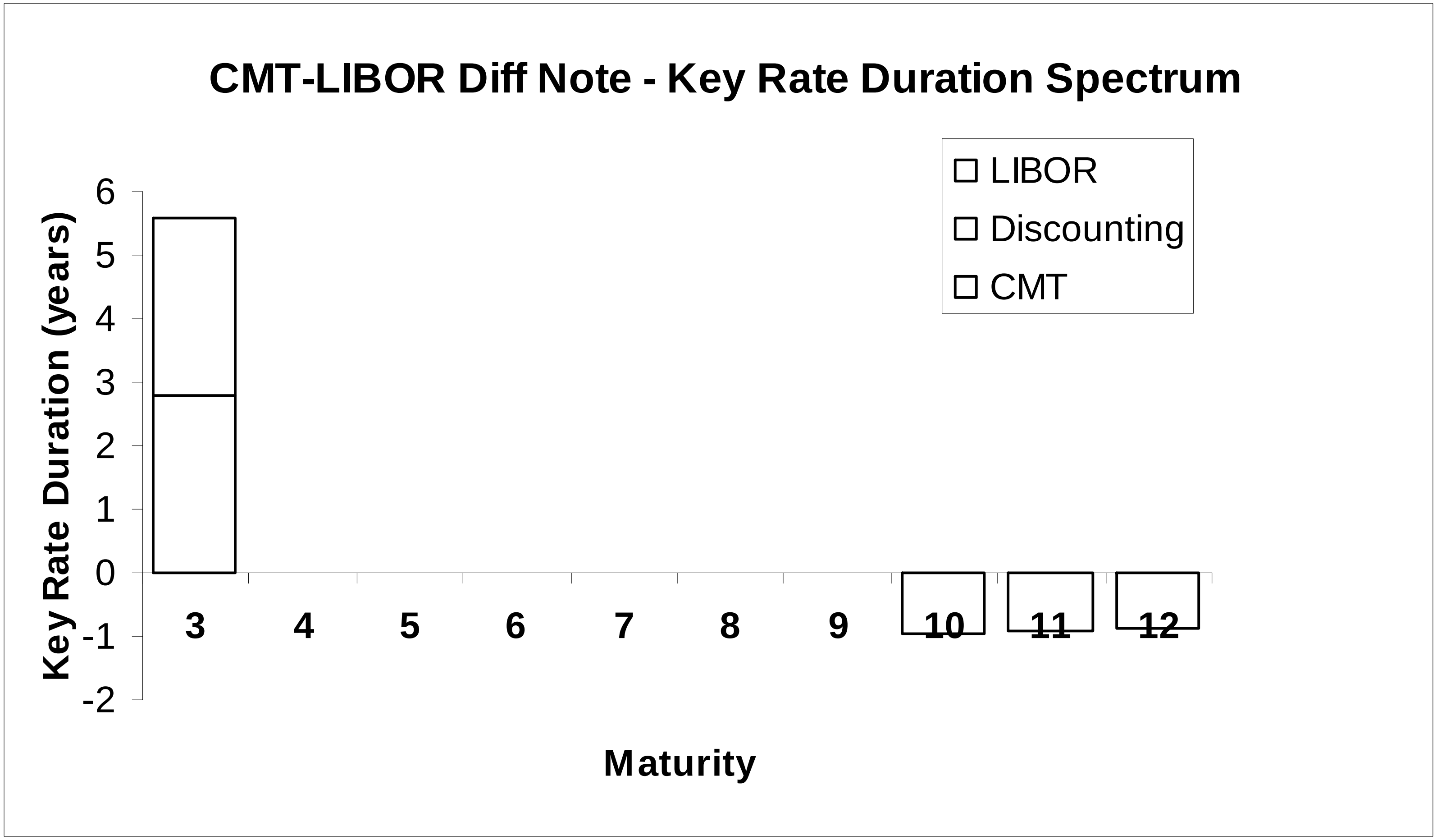
CMT-LIBOR Diff. Note: LIBOR Component

- Equivalent to 3-year swap
- Corresponds to to-maturity Treasury rate
- Hence duration is equiv. to fixed coupon 3-year note
- KTRD for LIBor component is 2.8 years

CMT-LIBOR Diff Note: KTRD's

Component	Index	KTR	KTRD
Index rate	10-yr CMT	T_{10}	-0.95
		T_{11}	-0.91
		T_{12}	-0.86
	12-m LIBOR	T_3	2.8
Discounting Rate	To-maturity Treasury	T_3	2.8

CMT-LIBOR Diff Note: KTRD Spectrum



Summary: Risk Management

- Risk Measurement
  - Duration Concepts
  - Index Rate Duration
  - Key Treasury Rate Duration
- Risk Analysis
  - Deterministic
  - Simulation