Forecasting Financial Markets

VAR Analysis

Vector Autoregression

- Structural, first order VAR process
  - $y_t = b_{10} - b_{12}z_t + \gamma_{11}y_{t-1} + \gamma_{12}z_{t-1} + \varepsilon_{yt}$
  - $z_t = b_{20} - b_{21}y_t + \gamma_{21}y_{t-1} + \gamma_{22}z_{t-1} + \varepsilon_{zt}$
    - $\{\varepsilon_{yt}\}$ and $\{\varepsilon_{zt}\}$ are uncorrelated white noise processes
  - y_t and z_t have contemporaneous effect on one another

VAR in Matrix Form

- Restate in matrix form:

$$\begin{bmatrix} 1 & b_{12} \\ b_{21} & 1 \end{bmatrix}\begin{bmatrix} y_t \\ z_t \end{bmatrix} = \begin{bmatrix} b_{10} \\ b_{20} \end{bmatrix} + \begin{bmatrix} \gamma_{11} & \gamma_{12} \\ \gamma_{21} & \gamma_{22} \end{bmatrix}\begin{bmatrix} y_{t-1} \\ z_{t-1} \end{bmatrix} + \begin{bmatrix} \varepsilon_{yt} \\ \varepsilon_{zt} \end{bmatrix}$$

  - $\mathbf{Bx}_t = \Gamma_0 + \Gamma_1 \mathbf{x}_{t-1} + \boldsymbol{\varepsilon}_t$
  - $\mathbf{x}_t = \mathbf{B}^{-1}\Gamma_0 + \mathbf{B}^{-1}\Gamma_1 \mathbf{x}_{t-1} + \mathbf{B}^{-1}\boldsymbol{\varepsilon}_t$
  - $\mathbf{x}_t = \mathbf{A}_0 + \mathbf{A}_1\mathbf{x}_{t-1} + \mathbf{e}_t$

VAR in Standard Form

- Rewrite as two simultaneous equations:
  - $y_t = a_{10} + a_{11}y_{t-1} + a_{12}z_{t-1} + e_{1t}$
  - $z_t = a_{20} + a_{21}y_{t-1} + a_{22}z_{t-1} + e_{2t}$
- Error processes $\{e_{it}\}$
  - Have zero mean, constant variances and are individually serially uncorrelated
  - May be correlated with one another

VAR Stationarity

- Substitution
  - $\mathbf{x}_t = \mathbf{A}_0 + \mathbf{A}_1\mathbf{x}_{t-1} + \mathbf{e}_t$
  - $\mathbf{x}_t = \mathbf{A}_0 + \mathbf{A}_1(\mathbf{A}_0 + \mathbf{A}_1\mathbf{x}_{t-2} + \mathbf{e}_{t-1}) + \mathbf{e}_t$
  - $\mathbf{x}_t = (\mathbf{I} + \mathbf{A}_1)\mathbf{A}_0 + \mathbf{A}_1^2\mathbf{x}_{t-2} + \mathbf{A}_1\mathbf{e}_{t-1} + \mathbf{e}_t$
    - I is 2 x 2 identity matrix
- After n iterations:

$$x_t = (I + A_1 + \ldots + A_1^n)A_0 + \sum_{i=0}^{n} A_1^i e_{t-i} + A_1^{n+1} x_{t-n-1}$$

VAR Stability & Stationarity

- Stability condition
  - A_1^n ⌫ 0 as n ⌫ ∞ $x_t = \mu + \sum_{i=0}^{\infty} A_1^i e_{t-i}$

$$\mu = \begin{bmatrix} \bar{y} \\ \bar{z} \end{bmatrix}$$

$$\bar{y} = [a_{10}(1 - a_{22}) + a_{12}a_{20}] / \Delta$$

$$\bar{z} = [a_{20}(1 - a_{11}) + a_{21}a_{10}] / \Delta$$

$$\Delta = (1 - a_{11})(1 - a_{22}) - a_{12}a_{21}$$

Stationarity Conditions

- Mean is constant: $E(\mathbf{x}_t) = \mu$

- Variance is finite and time-invariant:

$$E(x_t - \mu)^2 = E\left[\sum_{i=0}^{\infty} A_1^i e_{t-i}\right]^2$$

$$= (I + A_1^2 + A_1^4 + A_1^6 + \ldots)\sum \quad = (I - A_1^2)^{-1}\sum$$

$$\sum \quad = \begin{bmatrix} \sigma_1^2 & \sigma_{12} \\ \sigma_{21} & \sigma_2^2 \end{bmatrix}$$

- Σ is variance covariance matrix of series $\{y_t\}$and $\{z_t\}$

VAR Model with Lag Operator

$$y_t = a_{10} + a_{11} L y_t + a_{12} L z_t + e_{1t}$$

$$z_t = a_{20} + a_{21} L y_t + a_{22} L z_t + e_{2t}$$

$$\Rightarrow L z_t = L(a_{20} + a_{21} L y_t + e_{2t})/(1 - a_{22} L)$$

$$\Rightarrow y_t = \frac{a_{10}(1 - a_{22}) + a_{12} a_{20} + (1 - a_{22} L) e_{1t} + a_{12} e_{2t-1}}{(1 - a_{11} L)(1 - a_{22} L) - a_{12} a_{21} L^2}$$

$$\Rightarrow z_t = \frac{a_{20}(1 - a_{11}) + a_{21} a_{10} + (1 - a_{11} L) e_{2t} + a_{21} e_{1t-1}}{(1 - a_{11} L)(1 - a_{22} L) - a_{12} a_{21} L^2}$$

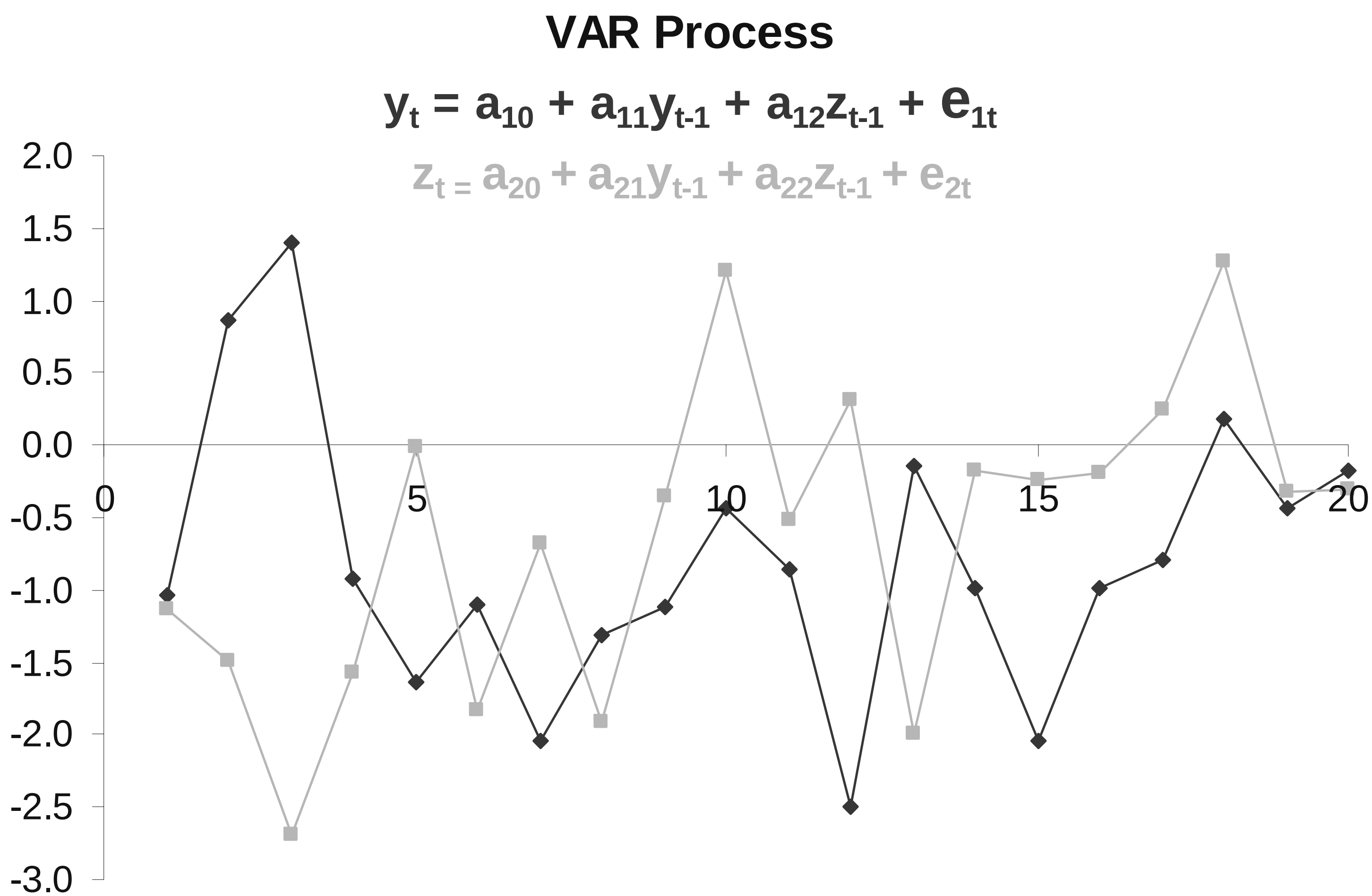
VAR Characteristic Function

- Convergence
  - Roots of $(1-a_{11}L)(1-a_{22}L) - a_{12}a_{21}L^2$ must lie outside unit circle
  - Roots can be real or complex
  - Convergent or divergent
- Solutions for $\{y_t\}$ and $\{z_t\}$ will have same roots
  - So will exhibit similar paths through time

Stationary VAR Process

- Stationary process
  - $a_{10} = a_{20} = 0$
  - $a_{11} = a_{22} = 0.7$
  - $a_{12} = a_{21} = 0.2$
- Roots of inverse characteristic fn are 1.11 and 2.0
  - Outside unit circle, hence stationary
- Series have positive cross-correlation
  - Tend to move together
  - a_{12} and a_{21} are both positive

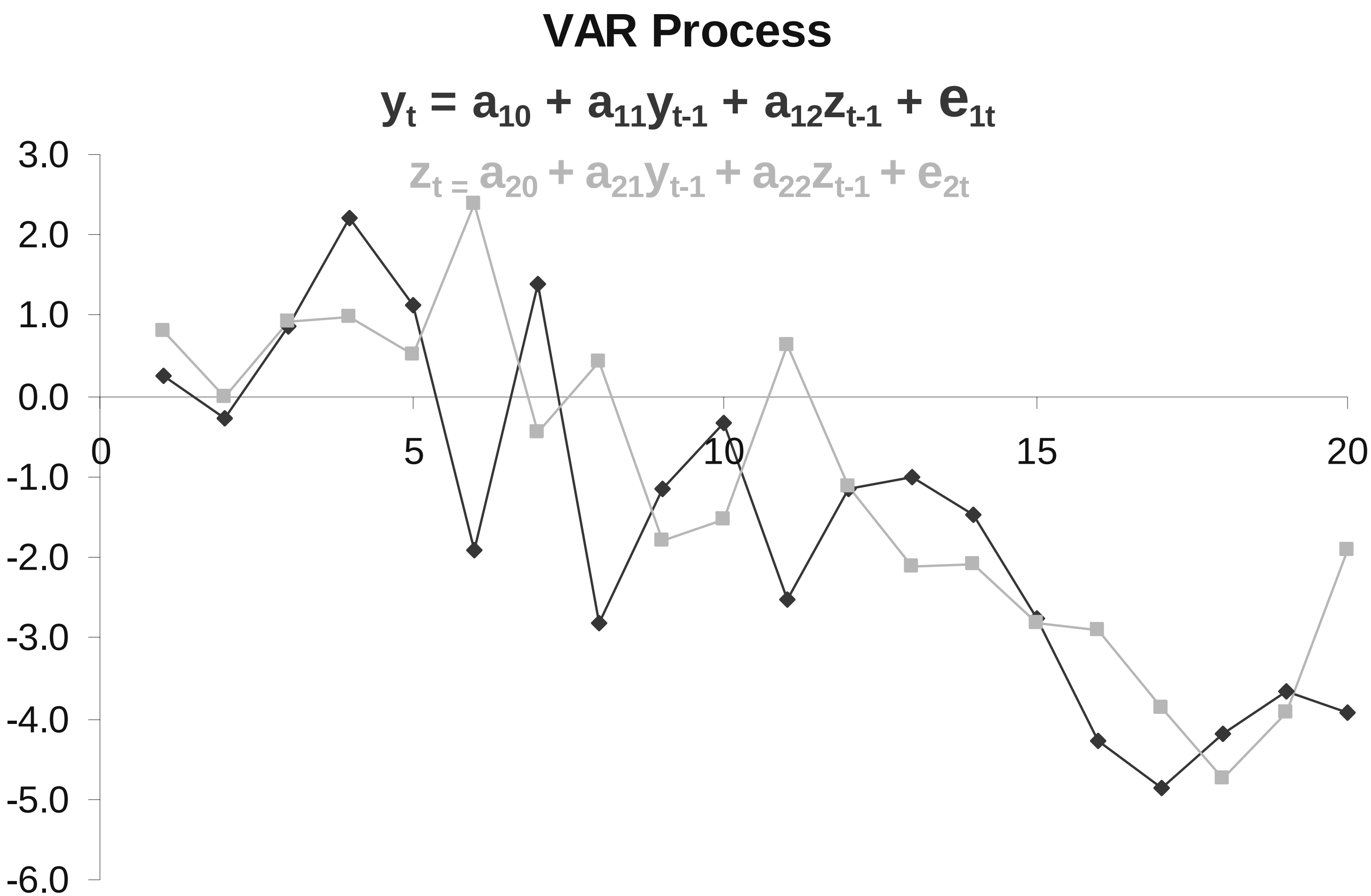
Stationary VAR Process



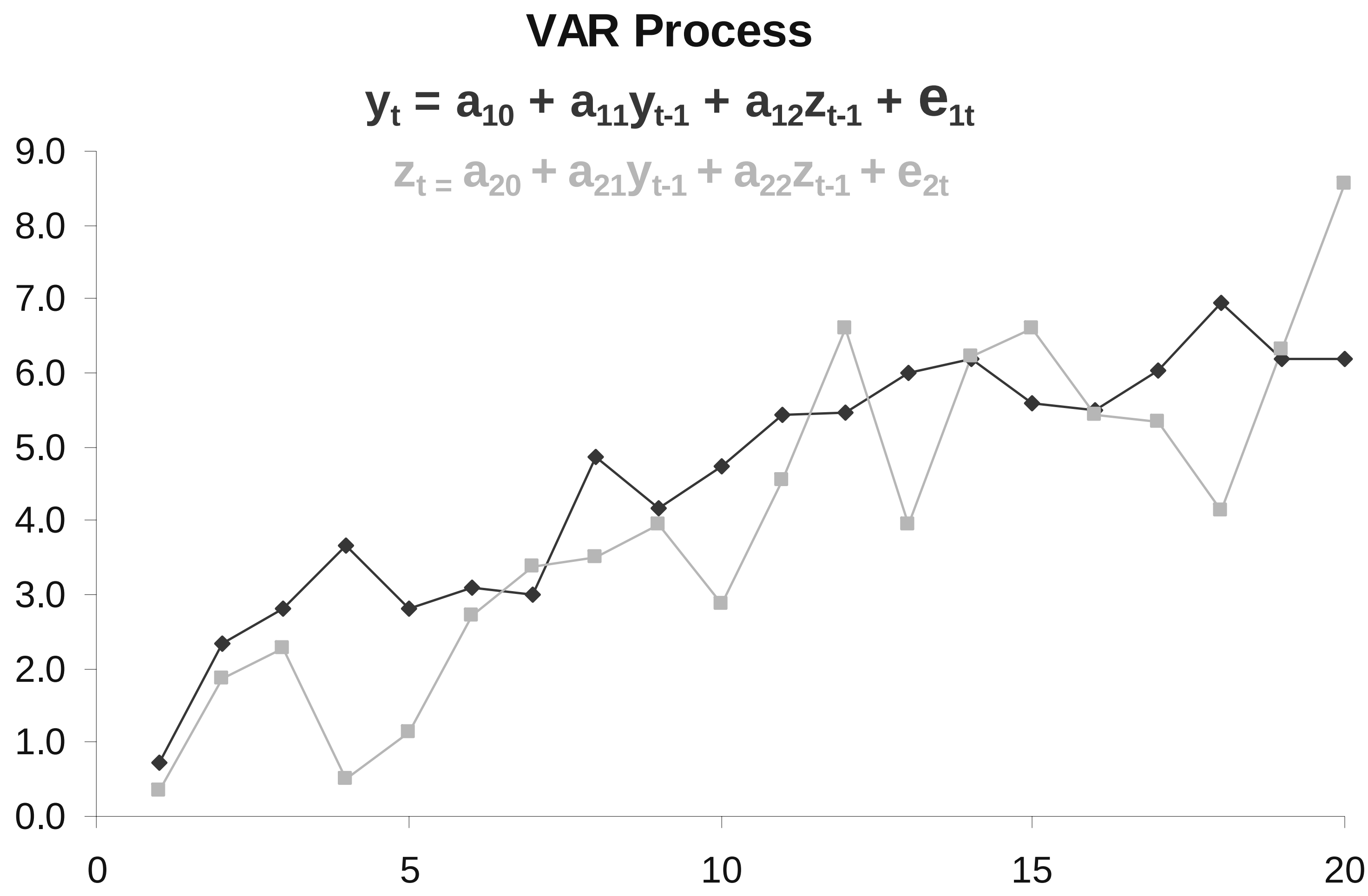
Non-Stationery VAR Processes

- Multivariate Random Walk
  - $a_{12} = a_{21} = a_{11} = a_{22} = 0.5$
  - $a_{10} = a_{20} = 0$
  - Roots are inside unit circle, hence non-stationary
- Multivariate Random Walk with Drift
  - $a_{10} = 0.5$ $a_{20} = 0$

Multivariate Random Walk



Random Walk with Drift



Generalized VAR Model

- Model form
  - $\mathbf{x}_t = \mathbf{A}_0 + \mathbf{A}_1\mathbf{x}_{t-1} + \mathbf{A}_2\mathbf{x}_{t-2} + \ldots \mathbf{A}_p\mathbf{x}_{t-p} + \mathbf{e}_t$
    - $\mathbf{x}_t$ is (n x 1) vector of n VAR process variables
    - $\mathbf{A}_0$ is (n x 1) vector of intercept terms
    - $\mathbf{A}_i$ is (n x n) matrix of coefficients
    - $\mathbf{e}_t$ is (n x 1) vector of error terms

Estimation & Identification

- Estimation requires n +pn^2 terms
  - Overparameterized
    - Some terms undoubtedly redundant
  - However, goal is to understand relationships
    - Not forecasting
  - Imposing restrictions may waste vital information
- Regression analysis
  - Regressors highly colinear
  - T-tests not reliable way of reducing model

Problems with Identification

- First Order VAR in standard form
  - Entails estimating 9 parameters by OLS
    - 6 coefficients
    - 3 variance covariance estimates for error processes
- Primitive system
  - Contains 10 parameters
- Conclusion
  - Need to restrict one parameter in order to identify primitive system

Sim's Recursive Method

- Restrict primitive system
  - Set coefficient b_{21} to zero
    - $\{z_t\}$ affects $\{y_t\}$ contemporaneously
    - $\{y_t\}$ affects $\{z_t\}$ at one-period lag
- Model form
  - $y_t = b_{10} – b_{12}z_t + \gamma_{11}y_{t-1} + \gamma_{12}z_{t-1} + \varepsilon_{yt}$
  - $z_t = b_{20} + \gamma_{21}y_{t-1} + \gamma_{22}z_{t-1} + \varepsilon_{zt}$
- Estimation
  - Estimate Standard Form coefficients using OLS
  - Solve simultaneously for primitive system coefficients

Cholesky Decomposition

- Factor error variance-covariance matrix
  - $\Sigma = B^{-1} A$
    - Where B^{-1} and A are triangular matrices

$$\Sigma = \begin{bmatrix} \sigma_1^2 & \sigma_{12} \\ \sigma_{21} & \sigma_2^2 \end{bmatrix} = \begin{bmatrix} 1 & -b_{12} \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \sigma_y^2 & 0 \\ -b_{12}\sigma_z^2 & \sigma_z^2 \end{bmatrix}$$

Vector Moving Average Systems

- VAR process

$$x_t = \mu + \sum_{i=0}^{\infty} A_1^i e_{t-i} \qquad \begin{bmatrix} y_t \\ z_t \end{bmatrix} = \begin{bmatrix} \bar{y} \\ \bar{z} \end{bmatrix} + \sum_{i=0}^{\infty} \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}^i \begin{bmatrix} e_{1t-i} \\ e_{2t-i} \end{bmatrix}$$

$$\begin{bmatrix} e_{1t} \\ e_{2t} \end{bmatrix} = \frac{1}{(1 - b_{12} b_{21})} \begin{bmatrix} 1 & -b_{12} \\ -b_{12} & 1 \end{bmatrix} \begin{bmatrix} \varepsilon_{yt} \\ \varepsilon_{zt} \end{bmatrix}$$

$$\begin{bmatrix} y_t \\ z_t \end{bmatrix} = \begin{bmatrix} \bar{y} \\ \bar{z} \end{bmatrix} + \frac{1}{(1 - b_{12} b_{21})} \sum_{i=0}^{\infty} \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}^i \begin{bmatrix} 1 & -b_{12} \\ -b_{12} & 1 \end{bmatrix} \begin{bmatrix} \varepsilon_{yt} \\ \varepsilon_{zt} \end{bmatrix}$$

Impact Multipliers

- Impact multipliers

$$\phi_i = \left[\frac{A_1^i}{(1-b_{12}b_{21})}\right]\begin{bmatrix} 1 & -b_{12} \\ -b_{12} & 1 \end{bmatrix}$$

- Restate VAR as VMA process

$$\begin{bmatrix} y_t \\ z_t \end{bmatrix} = \begin{bmatrix} \bar{y} \\ \bar{z} \end{bmatrix} + \sum_{i=0}^{\infty} \begin{bmatrix} \phi_{11}(i) & \phi_{12}(i) \\ \phi_{21}(i) & \phi_{22}(i) \end{bmatrix}^i \begin{bmatrix} \varepsilon_{yt-i} \\ \varepsilon_{zt-i} \end{bmatrix}$$

$$x_t = \mu + \sum_{i=0}^{\infty} \phi_i \varepsilon_{t-i}$$

Impulse Response Functions

- Measure cumulative effect of shocks
  - Example:
    - $\phi_{12}(0)$ measures instantaneous impact of change in ε_{zt} on y_t
- Long run multiplier
  - After n periods, cumulative effect of ε_{zt} on $\{y_t\}$ is: $\sum_{i=0}^{n} \phi_{12}(i)$
  - Limiting value is known as long run multiplier
- $\phi_{jk}(i)$ are known as impulse response functions
  - Plots of $\phi_{jk}(i)$ vs i show response of processes $\{y_t\}$and $\{z_t\}$ to shocks

Cointegration

- Engle & Granger 1987
- Non-stationary, integrated variables
  - Linear combinations may be stationary
  - Known as *cointegrated*
- System of economic variables in long-run equilibrium
  - $\beta_1 y_{1t} + \beta_2 y_{2t} + \ldots + \beta_n y_{nt} = 0$
- Equilibrium error process
  - $e_t = \boldsymbol{\beta} \mathbf{y}_t$
  - e_t are random deviations from equilibrium
  - Should be a stationary process
  - Components y_{1t}, y_{2t} , . . . are said to be cointegrated

Cointegration – Formal Definition

- Components of vector $\mathbf{y}_t$ are said to be cointegrated of order (d, b) if
- All components of are integrated of order d
  - $\Delta^d \mathbf{y}_t$ is stationary
- There exists vector $\beta = (\beta_1, \beta_2, \ldots \beta_\nu)$ s/t
  - $\beta_1 y_{1t} + \beta_2 y_{2t} + \ldots + \beta_n y_{nt}$ is integrated of order (d-b)
    - $b > 0$
- Vector β is called *cointegrating* vector

Examples of Cointegrated Processes

- Forward rates
  - Expectations theory $E_t[s_{t+1}] = f_t$
  - Error process $\varepsilon_{t+1} = s_{t+1} - f_t$
    - $\{\varepsilon_{t+1}\}$ must be a stationary process
      - Otherwise arbitrage
    - Even though {} and $\{f_t\}$ are nonstationary I(1) processes
- Currencies – PPP
  - Difference in real exchange rates must be stationary
- Econometric models in general
  - e.g. Money demand as linear function of prices, real income and interest rate

Notes on Cointegration

- Linearity
  - Possible that non-linear relationship exists
  - Cointegrating vector not unique: $\lambda\beta$
    - λ constant >0
    - Usually normalize β so coefficients sum to 1
- All variables must be integrated of same order
  - If y_{1t} is $I(d_1)$ and y_{2t} is $I(d_2)$ then any linear combination is $I(d_2)$
- Usually "cointegration" means variables are CI(1,1)
  - Residual error process is I(0)

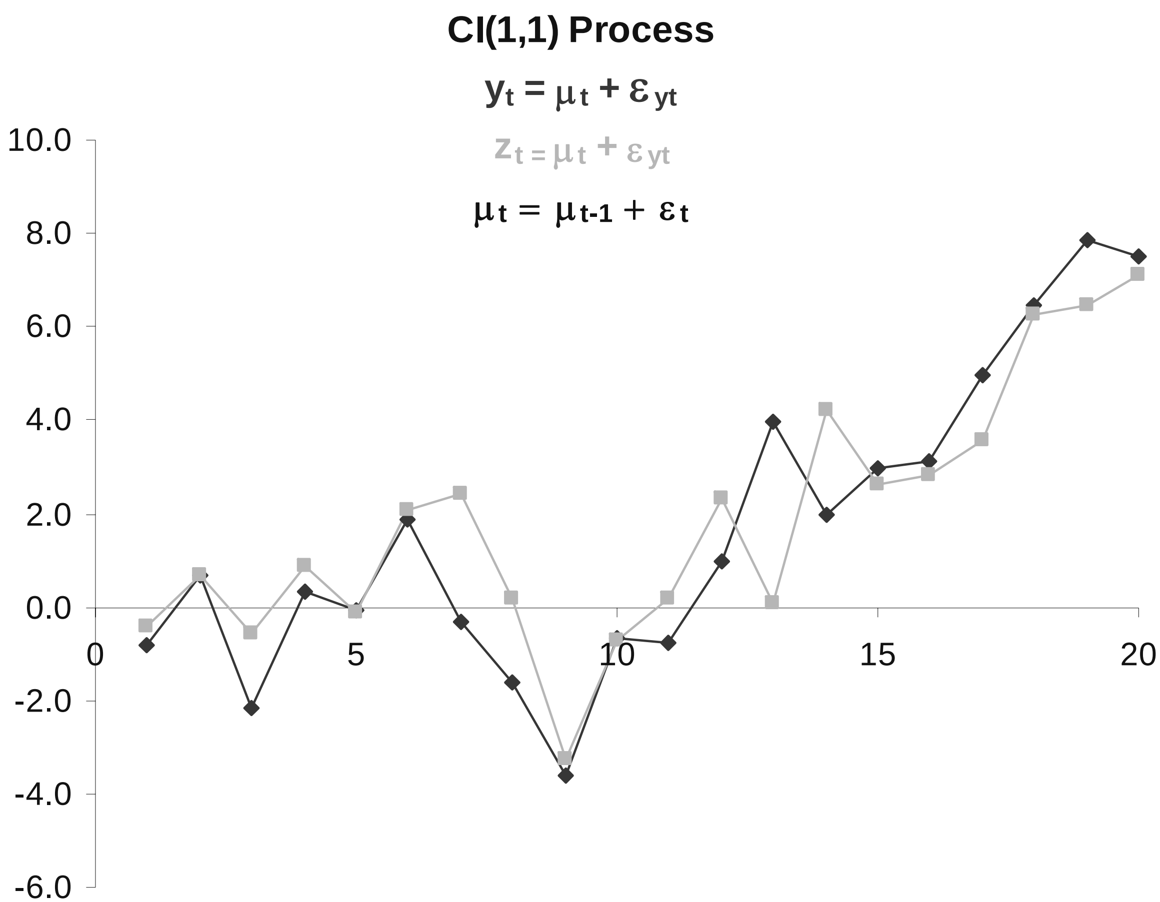
Cointegrating Rank

- Refers to # of linearly independent cointegrating vectors
- If $\mathbf{y}_t$ has n components
  - At most n-1 linearly independent cointegrating vectors

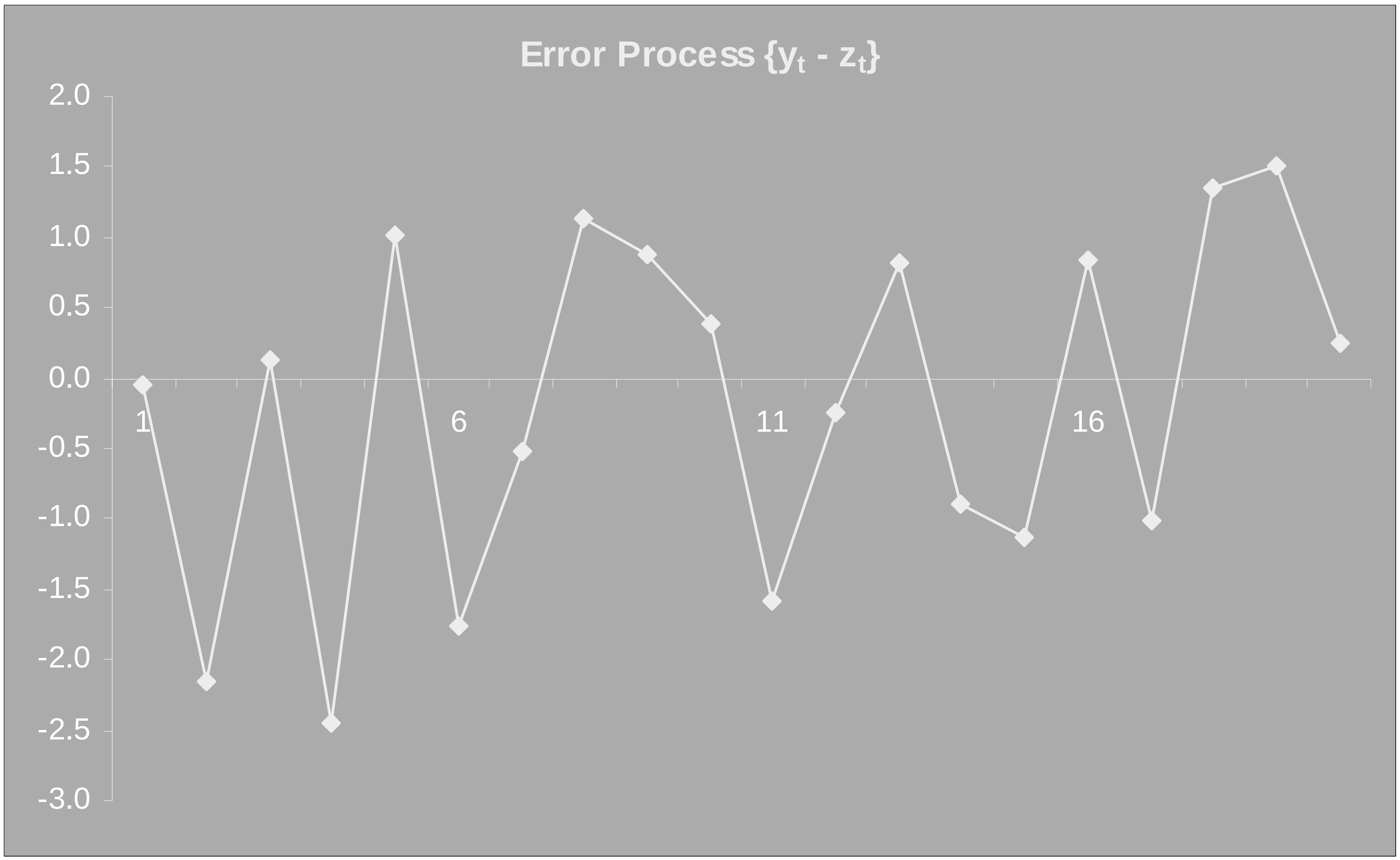
Example: CI(1,1,) System

- Two random walk processes
  - $y_t = \mu_{yt} + \varepsilon_{yt}$
  - $z_t = \mu_{zt} + \varepsilon_{zt}$
  - μ_{it} is random walk representing trend
  - ε_{it} is stationary error process
    - May not be white noise

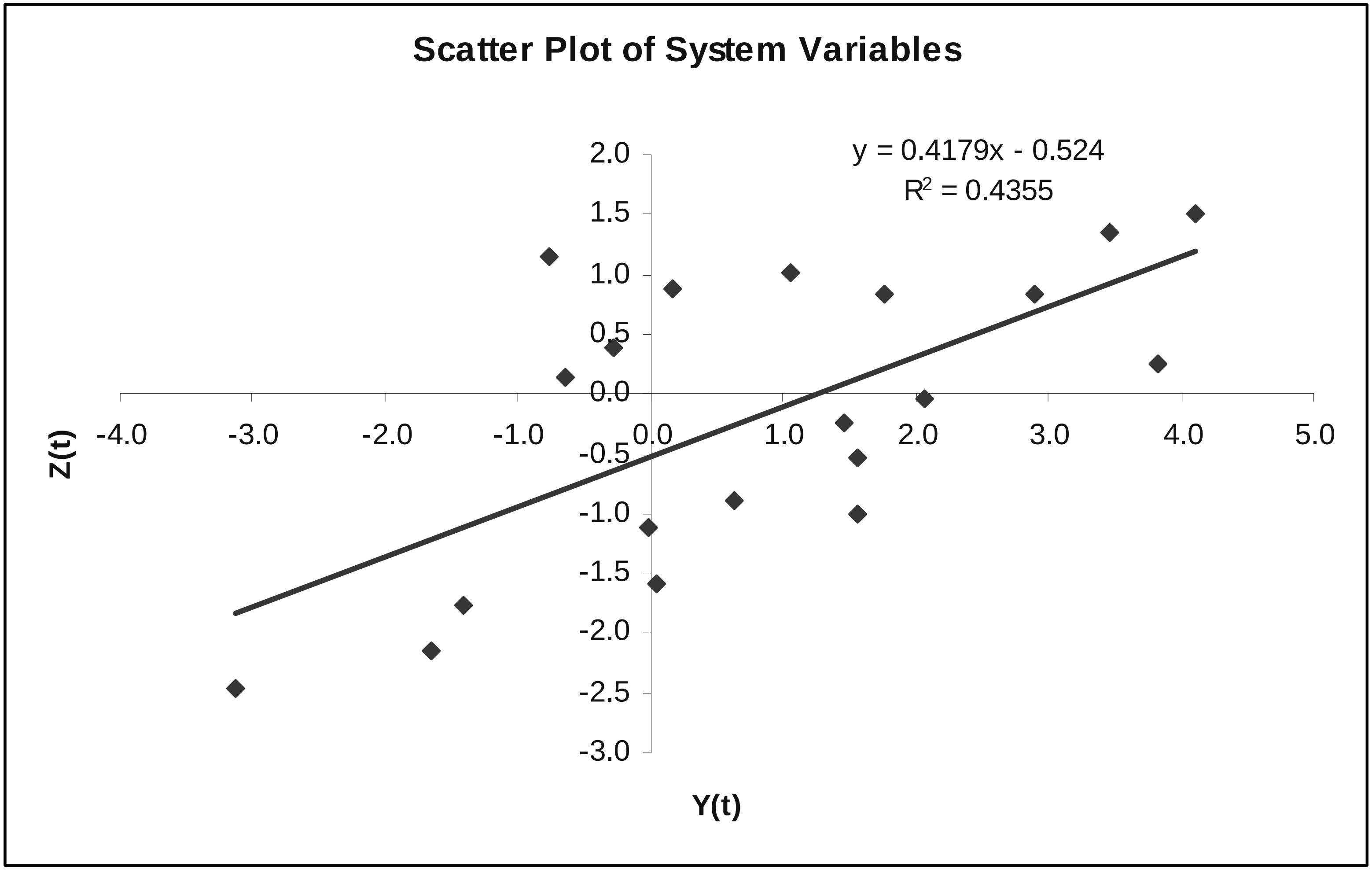
Example: CI(1,1) System



Error Process is Stationary



Scatter Plot of System Variables



Stationarity Condition for CI(1,1)

- If y_t and z_t are CI(1,1) then
  - $\beta_1 y_t + \beta_2 z_t$ is stationary
    - For some non-zero values of β_1 and β_2
  - Hence $\beta_1(\mu_{yt} + \varepsilon_{yt}) + \beta_2(\mu_{zt} + \varepsilon_{zt})$ is stationary
  - Implies $\beta_1\mu_{yt} + \beta_2\mu_{zt} = 0$
- Two processes must have same stochastic trend if they are CI(1,1) (up to scalar)
  - $\mu_{yt} = -\beta_2\mu_{zt} / \beta_1$
  - $y_t – z_t = \varepsilon_{yt} – \varepsilon_{zt}$ is stationary
  - Cointegrating vector is $\beta = (1, -1)$

Error Correction Models

- Idea:
  - Short term dynamics are influenced by deviation from long term equilibrium
- Example
  - Interest rate term structure
    - Mean reversion property
- Model implies system variables are cointegrated

Simple Term Structure Model

- Idea: short rates and long rates converge
- Model form

$$\Delta r_{st} = \alpha_s (r_{Lt-1} - \beta r_{st-1}) + \varepsilon_{st}$$

$$\Delta r_{Lt} = -\alpha_L (r_{Lt-1} - \beta r_{st-1}) + \varepsilon_{Lt}$$

  - ε_{st} and ε_{Lt} are (possibly correlated) white noise processes
  - α_s and $\alpha_L > 0$ are *speed of adjustment* (mean reversion) parameters
- Cointegration
  - Δr_s must be stationary, hence so must $(r_{Lt-1} - \beta r_{st-1})$
  - Hence short and long rates are cointegrated, vector $(1, -\beta)$

Granger Representation Theorem

- For any set of I(1) variables
  - Error correction and cointegration representations are equivalent

- Look at simple VAR model
  - $y_t = a_{11}y_{t-1} + a_{12}z_{t-1} + \varepsilon_{1t}$
  - $z_t = a_{21}y_{t-1} + a_{22}z_{t-1} + \varepsilon_{2t}$

VAR, Cointegration & Error Correction

$$\begin{bmatrix} (1-a_{11}L & -a_{12}L \\ -a_{21}L & (1-a_{22}L \end{bmatrix} \begin{bmatrix} y_t \\ z_t \end{bmatrix} = \begin{bmatrix} \varepsilon_{yt} \\ \varepsilon_{zt} \end{bmatrix}$$

$$y_t = \frac{(1-a_{22}L)\varepsilon_{yt} + a_{12}L\varepsilon_{zt}}{(1-a_{11}L)(1-a_{22}L) - a_{12}a_{21}L^2}$$

$$z_t = \frac{(1-a_{11}L)\varepsilon_{zt} + a_{21}L\varepsilon_{zt}}{(1-a_{11}L)(1-a_{22}L) - a_{12}a_{21}L^2}$$

- Characteristic function in $\lambda = 1/L$

$$\lambda^2 - (a_{11} + a_{22})\lambda + (a_{11}a_{22} - a_{12}a_{21}) = 0$$

Roots of Characteristic Function

- Both roots inside unit circle
  - $\{y_t\}$ and $\{z_t\}$ stable
  - Cannot be CI(1,1) since both stationary
- Both roots outside unit circle
  - Neither $\{y_t\}$ and $\{z_t\}$ is difference stationary
  - Hence cannot be CI(1,1)
    - If roots = 1, they are I(2), hence not CI(1,1)

Conditions for CI(1,1)

- One $\lambda_1 = 1$, $|\lambda_2| < 1$

$$y_t = \frac{(1 - a_{22}L)\varepsilon_{yt} + a_{12}L\varepsilon_{zt}}{(1-L)(1-\lambda_2 L)}$$

- Hence

$$(1-L)y_t = \Delta y_t = \frac{(1 - a_{22}L)\varepsilon_{yt} + a_{12}L\varepsilon_{zt}}{(1-\lambda_2 L)}$$

  - Stationary if $|\lambda_2| < 1$

From Cointegration to Error Correction

- Restate VAR as error correction model

$$\Delta y_t = \alpha_y (y_{t-1} - \beta z_{t-1}) + \varepsilon_{yt}$$

$$\Delta z_t = \alpha_z (y_{t-1} - \beta z_{t-1}) + \varepsilon_{zt}$$

$$\alpha_y = -a_{12} a_{21} / (1 - a_{22})$$

$$\beta = (1 - a_{22}) / a_{21}$$

$$\alpha_z = a_{21}$$

- CI(1,1) conditions ensure that this is a valid EC model
  - β <> 0
  - At least one speed of adjustment parameters <> 0

Vector Autoregression (VAR) and Granger Causality

- Let $\{X_t\}$ and $\{Y_t\}$ be stationary series such that
  - $X_t = A(L)\, X_t + B(L)\, Y_t + \varepsilon_{x,t}$
  - $Y_t = C(L)\, X_t + D(L)\, Y_t + \varepsilon_{Y,t}$
    - $\varepsilon_{Y,t}$ and $\varepsilon_{x,t}$ are separate white noise processes
    - A, B, C, D are polynomials in the lag operator
- Y *strictly Granger causes* X if
  - Some of the coefficients of B are non-zero
- X *strictly Granger causes* Y if
  - Some of the coefficients of C are non-zero

Granger Causality in Financial Markets

- Example: cash vs futures

$$R_{c,t} = \alpha + \sum_{k=-n}^{k=+n} \beta_k R_{f,t-k} + \beta_z z_{t-1} + \varepsilon_t$$

  - $R_{c,t}$ are index returns in cash
  - $R_{f,t}$ are index returns in futures
  - Z_t is difference between cash and futures index levels

- If lag coefficients (β_{-k}) are significant, futures returns lead cash index returns
- If lead coefficients (β_k) are significant, cash index returns lead futures returns

Results of Research Into Causality

- Fleming (1996)
  - S&P500 index futures lead cash by just over 5 mins
  - Chung & Ng (1990)
    - S&P 500 futures lead cash by at least 15 mins.
- Grunbichler (1994)
  - DAX index futures lead cash by 15-20 mins
- Abhyankar (1998)
  - FTSE 100 index futures lead cash by 5-15 mins
- Park & Switzer (1997)
  - 90-day t-bill futures lead forward rates

Example: Interest Rates

- Let R(k, t) be the k-period rate at time t
- $R(k, t) = (1/k)[E_t R(1,t) + E_t R(1,t+1) + \ldots + E_t R(1,t+k-1) + L(k,t)$
  - L(k,t) is the risk/liquidity premium
    - Average expected return for investing for k periods = expected return for k successive 1-period investments

Interest Rates

- Reformulate equation as interest rate spread:

$$R(k,t) - R(1,t) = \frac{1}{k}\left[\sum_{i=1}^{k-1}\sum_{j=1}^{i} E_t\{\Delta R(1,t+j)\}\right] + L(k,t)$$

- Stationarity
  - R(k,t) is I(1) and L(k,t) is stationary
  - Hence R(k,t)-R(1,t) is stationary
- Cointegration
  - From above, expect to find (n-1) cointegrating relationships between n interest rates of different maturities
  - $R(k,t)=R(1,t)+a_k \quad k = 2, \ldots, n$
    - a_k positive and increasing with maturity

UK Interest Rate Model

- Pesaran& Pesaran 1994
- VAR analysis on monthly LIBOR
  - 1m, 3m, 6m, 12m LIBOR rates
    - Endogenous I(1) variables
  - EEF = effective exchange rate
    - Percentage change lag 1
  - Dummy variables for 84(8), 85(2), 92(10)
    - Outliers (eg. exit ERM)

UK LIBOR


UK LIBOR
20.0
18.0
16.0
14.0
12.0
10.0
8.0
6.0
4.0
2.0
0.0
1M
3M
6M
12M
1980M1
1981M1
1982M1
1983M1
1984M1
1985M1
1986M1
1987M1
1988M1
1989M1
1990M1
1991M1
1992M1
1993M1
1994M1

Modeling Procedure

- Step 1 Unrestricted VAR
  - Test to find appropriate order of VAR
- Step 2 Estimate Cointegrating VAR
  - Find cointegrating vectors
- Step 3
  - Estimate Impulse-Response functions

Step 1 – Estimating VAR Order

List of variables included in the unrestricted VAR:

R1 R3 R6 R12

List of deterministic and/or exogenous variables:

INPT DLEER(-1) D84M8 D85M2 D92M10

****

Order	LL	AIC	SBC	LR test	Adjusted LR test
12	223.7027	11.7027	-317.5275	------	------
11	213.2903	17.2903	-287.0924	CHSQ(16)= 20.8248[.185]	14.1357[.589]
10	201.6576	21.6576	-257.8775	CHSQ(32)= 44.0902[.076]	29.9279[.572]
9	197.4085	33.4085	-221.2791	CHSQ(48)= 52.5885[.301]	35.6964[.905]
8	189.1103	41.1103	-188.7296	CHSQ(64)= 69.1847[.307]	46.9618[.946]
7	179.1601	47.1601	-157.8323	CHSQ(80)= 89.0853[.228]	60.4700[.949]
6	155.9621	39.9621	-140.1827	CHSQ(96)= 135.4811[.005]	91.9629[.598]
5	145.1763	45.1763	-110.1210	CHSQ(112)= 157.0529[.003]	106.6056[.626]
4	134.5754	50.5754	-79.8743	CHSQ(128)= 178.2546[.002]	120.9970[.657]
3	121.0112	53.0112	-52.5909	CHSQ(144)= 205.3830[.001]	139.4115[.592]
2	**110.4686**	**58.4686**	**-22.2859**	**CHSQ(160)= 226.4681[.000]**	**153.7238[.625]**
1	93.9668	57.9668	2.0598	CHSQ(176)= 259.4719[.000]	176.1264[.483]
0	-312.1241	-332.1241	-363.1836	CHSQ(192)= 1071.7[.000]	727.4255[.000]

AIC indicates order 2 VAR

STEP 2 – Cointegrating Vectors

Cointegration with restricted intercepts and no trends in the VAR

Cointegration LR Test Based on Maximal Eigenvalue of the Stochastic Matrix

****

175 observations from 1980M3 to 1994M9 . Order of VAR = 2.

List of variables included in the cointegrating vector:

R1 R3 R6 R12 Intercept

List of I(0) variables included in the VAR:

DLEER(-1) D84M8 D85M2 D92M10

List of eigenvalues in descending order:

.39641 .31188 .11064 .022479 .0000

****

Null	Alternative	Statistic	95% Critical Value	90% Critical Value
r = 0	r = 1	88.3497	28.2700	25.8000
r<= 1	r = 2	65.4148	22.0400	19.8600
r<= 2	r = 3	20.5200	15.8700	13.8100
r<= 3	r = 4	3.9787	9.1600	7.5300

****

Use the above table to determine r (the number of cointegrating vectors).

Conclude there are 3 cointegrating vectors

Step 3 – Cointegrating Vectors

ML estimates subject to exactly identifying restriction(s)

Estimates of Restricted Cointegrating Relations (SE's in Brackets)

Converged after 2 iterations

Cointegration with restricted intercepts and no trends in the VAR

****

	Vector 1	Vector 2	Vector 3
R1	.98321	.93871	.86499
	(.011437)	(.028626)	(.050086)
R3	-1.0000	0.00	0.00
	(*NONE*)	(*NONE*)	(*NONE*)
R6	0.00	-1.0000	0.00
	(*NONE*)	(*NONE*)	(*NONE*)
R12	0.00	0.00	-1.0000
	(*NONE*)	(*NONE*)	(*NONE*)
Intercept	.29809	.85018	1.7407
	(.13362)	(.33287)	(.58088)

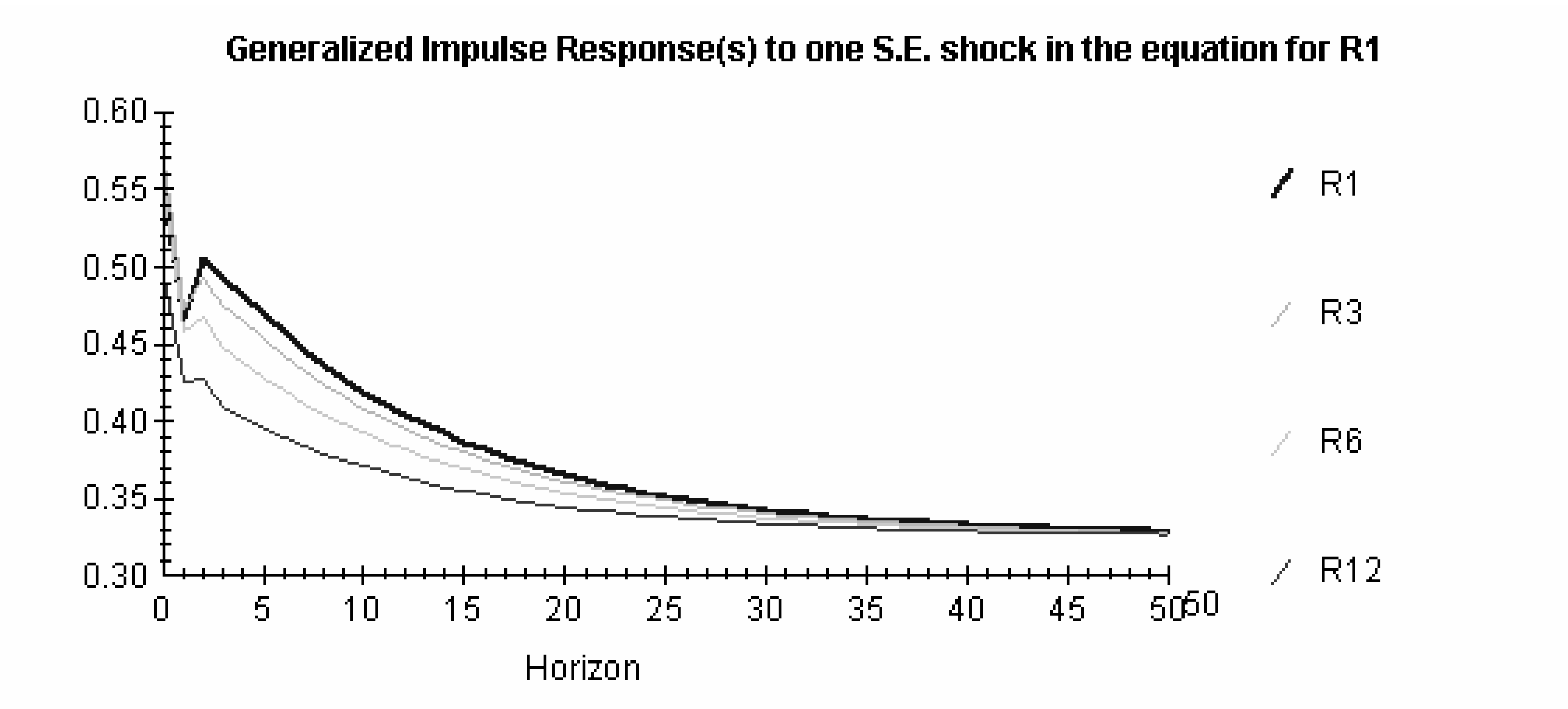
****

LL subject to exactly identifying restrictions= 67.8656

****

Impulse Response Function

- How shocks in R1 affect Term Structure



Summary

- VAR models
  - Extensions of simple univariate analysis
  - Emphasis on understanding relationships
- Cointegration
  - Important idea in finance: linear combinations of non-stationary processes may be stationary
- Error Correction models
  - Model return to long term equilibrium
  - Equivalence to cointegrated systems