Forecasting Financial Markets

Testing Strategies

Overview

- Measuring Estimator Performance
- Measuring Profitability
- Equity Curve Measures
- Portfolio Performance Measures

Measuring Estimator Performance

- Correlation coefficient
- Theil's information coefficient
- Akaike information criteria
- Schwartz Bayesian information criterion
- Average relative variance
- Directional change predictor
- Bull-bear statistic

Correlation Coefficient

- Most Common Measure of Prediction Accuracy

$$R^2 = \frac{\left\{ \sum_{i=1}^{n} (y_i - \bar{y})(f_i - \bar{f}) \right\}^2}{\sum_{i=1}^{n} (y_i - \bar{y})^2 \sum_{i=1}^{n} (f_i - \bar{f})^2}$$

- Adjusted $R^2 = R^2 (n+k) / (n-k)$
  - Penalizes for model complexity
    - k is the number of model parameters

Theil's Information Coefficient

- Advantages vs. standard measures
  - Comparison with naïve forecast ($f_{t+1} = y_t$)
  - Considers disproportionate cost of large errors (in MSE)

$$U = \sqrt{\frac{\sum_{t=1}^{n-1}(FPE_{t+1} - APE_{t+1})^2}{\sum_{t=1}^{n-1} APE_{t+1}^2}} = \sqrt{\frac{\sum_{t=1}^{n-1}\left(\frac{f_{t+1} - y_{t+1}}{y_t}\right)^2}{\sum_{t=1}^{n-1}\left(\frac{y_{t+1} - y_t}{y_t}\right)^2}}$$

  - $FPE_{t+1} = (f_{t+1} - y_t) / y_t$ (forecast relative change)
  - $APE_{t+1} = (y_{t+1} - y_t) / y_t$ (actual relative change)

Interpretation of Theil's U-Statistic

- $U = 1$
  - Naïve method is as good as technique being evaluated
    - $FPE_{t+1} = 0$; so $f_{t+1} = y_t$, as for naïve method
- $U < 1$
  - Technique is better than naïve method
    - Since $FPE_{t+1} < APE_{t+1}$
    - Smaller U the better
- $U > 1$
  - Naïve method will produce better results
    - Since $FPE_{t+1} > APE_{t+1}$

Average Relative Variance (Mean Reversion)

- Trivial Predictor: Historical Mean

$$T_{\mu} = \frac{\sqrt{\sum_{t=1}^{n}(y_t - f_t)^2}}{\sqrt{\sum_{t=1}^{n}(y_{t+1} - \bar{y})^2}}$$

- With $T_{\mu} < 1$ the forecasting method is making better predictions than simply predicting the mean

Akaike Information Criteria

- Adjusts MSE to take account of complexity of estimator

$$A = \frac{1}{n}\sum_{i=1}^{n}(y_i - f_i)^2\left[\frac{n+k}{n-k}\right]$$

  - K is the # free parameters in estimator

Bayesian Information Criterion

- Adjusts MSE for model complexity

$$B = Ln\left[\frac{1}{n}\sum_{i=1}^{n}(y_i - f_i)^2\right] + \frac{k}{n}Ln(n)$$

  - K is # free parameters in model (weights)

Directional Change Predictor

- Directional change predictor
  - Correctness of *sign* predictions

$$d = \frac{1}{n}\sum_{i=1}^{n} z_i$$

    - $z_i = 1$ if $(y_{t+1} - y_t)(f_{t+1} - y_t) > 0$; 0 otherwise
- Interpretation
  - D = 1 means perfect prediction of directional changes
  - D > 0.5 is better than tossing a coin
  - D = 0 implies 0% predictive ability
    - Note: easy to obtain large d in trending market

The Bear-Bull Statistic

- Measures forecasting ability
  - P_1 is percentage of correct bull market forecasts
  - P_2 is percentage of correct bear market forecasts
  - $B = P_1 + P_2 - 100\%$
- Example:
  - Predictor which is always right on bull and bear calls:
    - $P_1 = P_2 = 100\%$; $B = 100\%$
  - Predictor which calls all the bulls (but no bears)
    - $P_1 = 100\%$; $P_2 = 0\%$; $B = 0\%$

Measuring Profitability

- Net returns
- Buy and hold test
- Distance from the ideal

Net Returns

- Test of investment strategy
  - Long positions when expected returns are positive
  - Short positions when expected returns are negative

$$r = \sum_{t=1}^{n} p_t (y_{t+1} - y_t)$$

$$p_t = \begin{cases} 1 & if\ (f_{t+1} - y_t) > 0 \\ -1 & if\ (f_{t+1} - y_t) < 0 \\ 0 & if\ (f_{t+1} - y_t) = 0 \end{cases}$$

Buy and Hold Test

- Benchmark to quantify excess returns
  - Tests whether profitability is due to predictive ability or just general market conditions

$$r = \frac{c + (y_{t+n} - y_t)}{y_t}$$

- C is stock dividend or bond coupon

Distance From the Ideal

- Measures returns from trading system against perfect predictor d

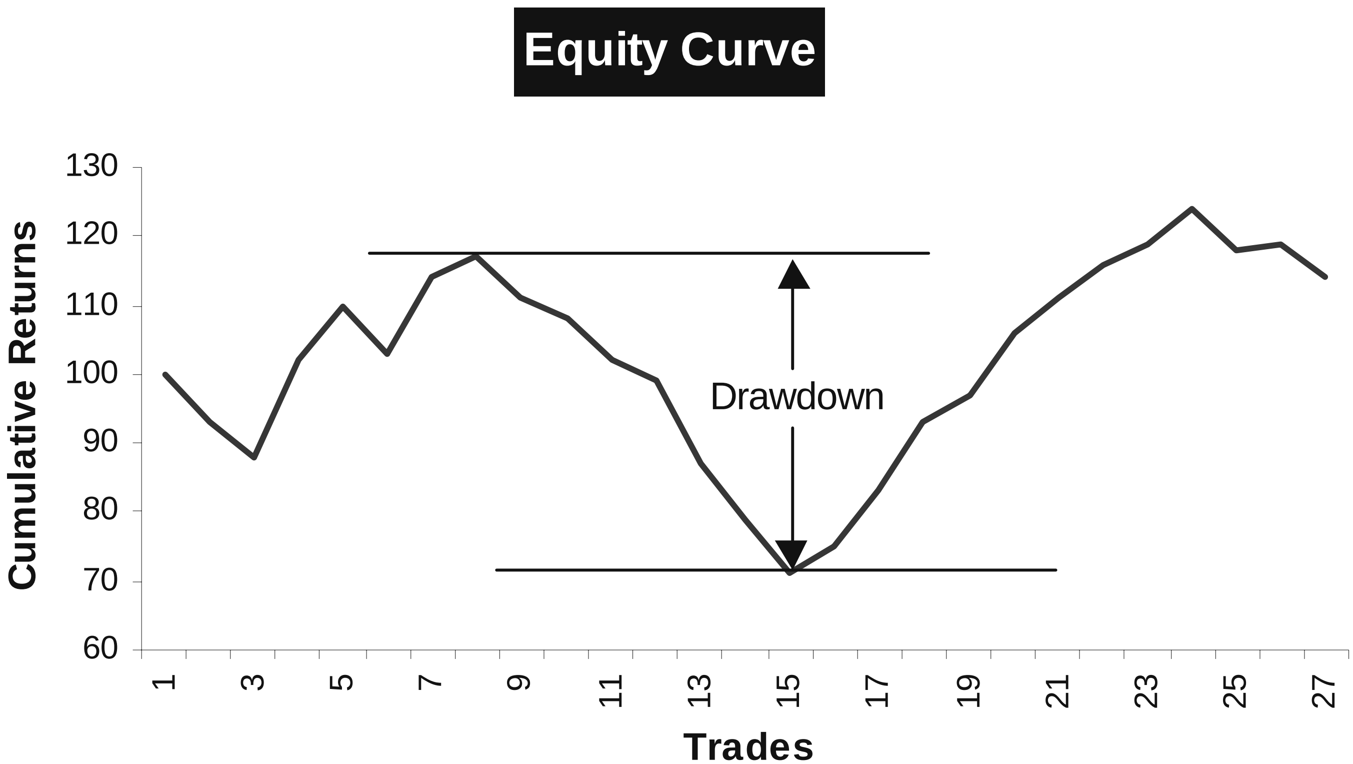
$$r_d = \frac{\sum_{t=1}^{n} p_t (y_{t+1} - y_t)}{\sum_{t=1}^{n} | y_{t+1} - y_t |}$$

  - P_t as previously defined

Equity Curve Measures

- Drawdown
- Luck coefficient
- Stirling ratio
- Risk of ruin

Drawdown



Equity Curve
130
120
110
100
90
80
70
60
Cumulative Returns
Drawdown
1
3
5
7
9
11
13
15
17
19
21
23
25
27
Trades

Drawdown

- Systems with large drawdowns hard to trade
  - Requires lots of capital & confidence!
- Smooth equity curve is desirable
- Usually harder to obtain than high net return

Luck Coefficient

- How much of total profit was dependent on most profitable (k) trades(s)?

$$l(k) = \frac{Max_k\{r_0, r_1, ..., r_n\}}{\sum_{i=1}^{n} r_i}$$

- Large L indicates system success unlikely to be repeatable

Stirling Ratio

- Penalizes average returns for drawdown

$$s = \frac{\frac{1}{n}\sum_{i=1}^{n} r_i}{10 - d_i}$$

  - d_i is the i-period maximum drawdown.
- Can be too slow to change
  - Recalculate frequently

Risk of Ruin

- Probability that capital will be depleted
  - Depends on
    - Probability of successful trade p
    - Payoff ratio (av. Win / av. Loss)
    - Fraction of capital exposed to trading
  - Assume:
    - Payoff ratio is 1
    - We risk all capital
    - K sequential trades
  - $R \sim [(1-p)/p]^{k}$

Portfolio Performance Measures

- Sharpe ratio: $(r_p - r_f) / \sigma_p$
  - Measures reward to total risk trade-off
- Treynor's measure: $(r_p - r_f) / \beta_p$
  - Excess return per unit of systematic risk
- Jensen's measure: $\alpha_P = r_p - [r_f + \beta_p(r_M - r_f)]$
  - The portfolio's alpha - abnormal return above that predicted by CAPM
- Appraisal ratio: $\alpha_P / \sigma(e_p)$
  - Abnormal return per unit of specific risk that could be diversified away using a market index portfolio

Which Measure to Use

- Suppose you have invested in a portfolio P
- Case 1: P is your entire investment fund
- Case 2: P is your active portfolio and:
  - You are also investing in the passive market index portfolio
- Case 3: P is one of many portfolios
  - Combined in a large investment fund
  - E.g. You are one of a number of portfolio managers

Case 1: P Is Your Entire Investment Fund

- Compare P’s Sharpe ratio with other fund:
  - Passive index fund
  - Professionally managed active funds

Case 2: P Is Your Active Portfolio

- Recall: $S^2_C = S^2_M + [\alpha_P / \sigma(e_P)]^2$
  - S_C is the Sharpe ratio of the combined portfolio (M and P)
- "How much does your active portfolio P add to the Sharpe ratio S_M of your passive market index portfolio?"
- Use appraisal ratio: $[\alpha_P / \sigma(e_P)]$

Case 3: P Is One of Many Portfolios

- P's contribution to the entire diversified fund is α_P
- So could use Jensen's measure (portfolio alpha)
  - But this takes no account of risk
- Better to use Treynor's measure: $(r_p - r_f) / \beta_p$
  - Measure P's excess return against the systematic risk (beta) rather than the total diversifiable risk (s.d.)

Lab: Portfolio Performance Measurement

- Advise a client in choice of funds
  - Use different performance measures
- Excel lab: portfolio performance measurement
  - Complete worksheet
  - See solution worksheet
- See written notes and solution

Portfolio Performance Measurement - Solution

	Fund P	Fund Q	Benchmark M
Sharpe	0.43	0.49	0.19
Alpha (Jensen)	1.63%	5.26%	0.00%
Beta	0.70	1.40	1.00
Treynor	3.97	5.38	1.64
$\sigma(e)$	1.92%	9.35%	0
Appraisal ratio	0.85	0.56	0.00
R^2	91.12%	63.82%	100.00%

Portfolio Performance Measurement - Solution

- Both P & Q outperform M:
  - Higher Sharpe ratios, positive alphas
- Fund Q is preferred:
  - If this fund is the client's only investment
    - Higher Sharpe ratio than P
  - As one of a mix of portfolios
    - Higher Treynor measure than P
- P is preferred if used as an active fund
  - In conjunction with a passive index fund
    - Higher appraisal measure than Q

Summary: Testing Strategies

- Appropriate testing metric depends on application
  - Forecasting
  - Trading system development
  - Portfolio management
- Models unlikely to perform equally on every basis
  - E.G. Models with low R^2 may generate significant profits
  - Models with good statistical fit may trade badly
- Moral
  - Decide objective and testing strategy *before* modeling!