Forecasting Financial Markets

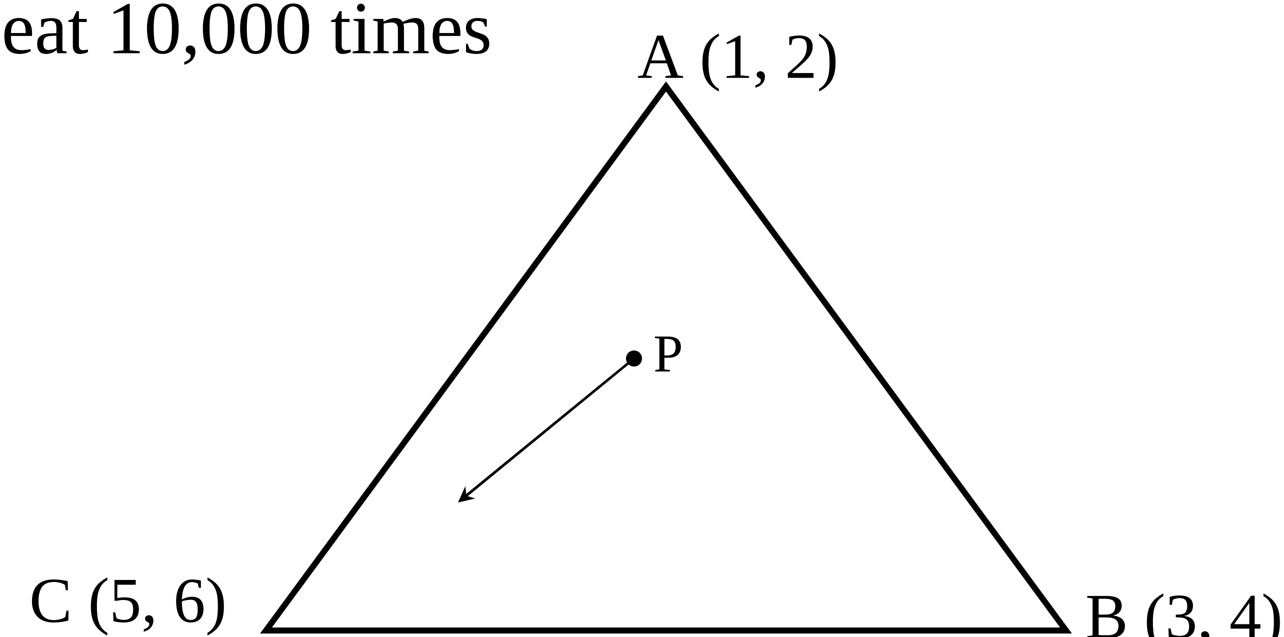
Chaos Theory

Overview

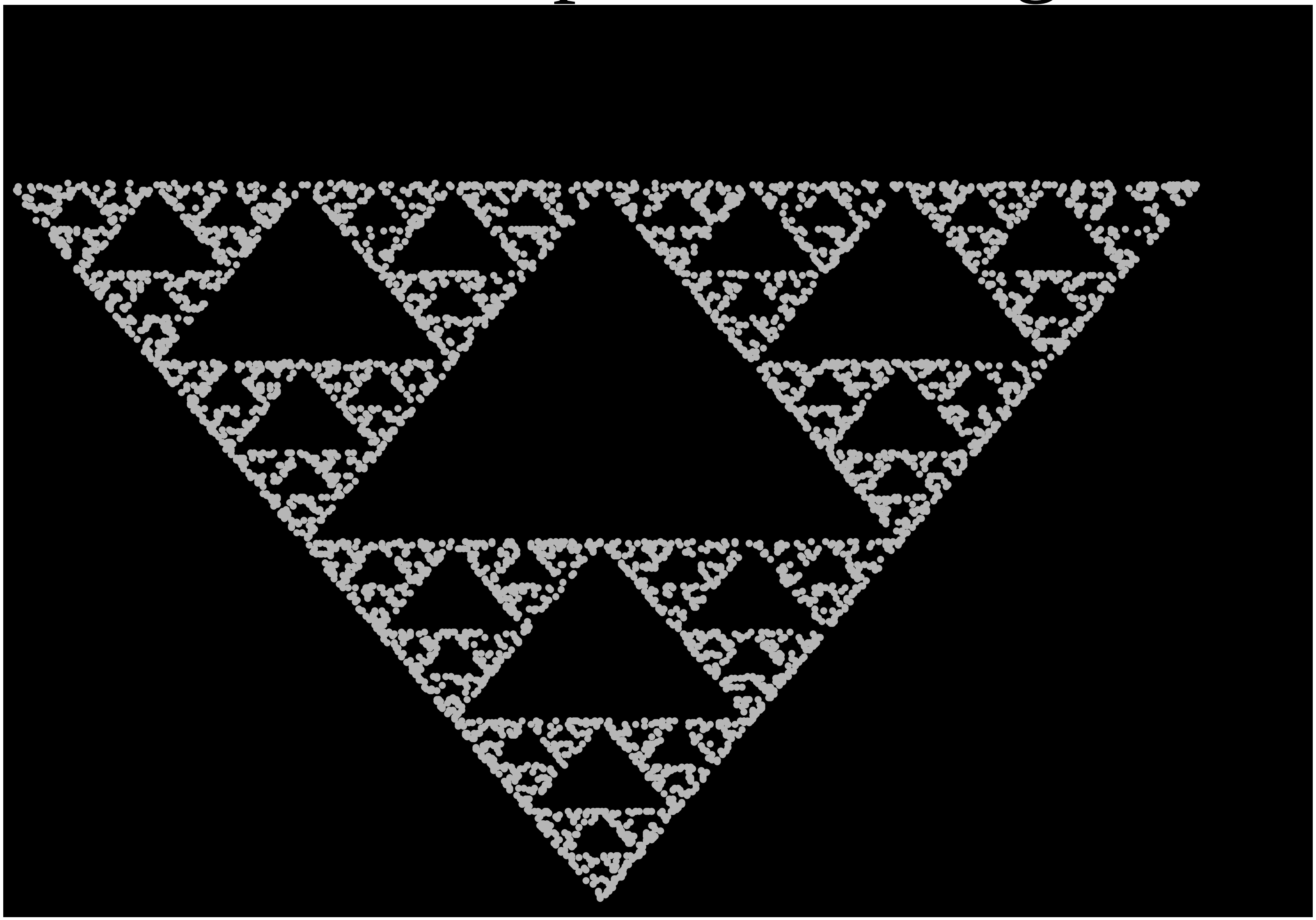
- Fractals
  - Self-similarity
- Fractal Market Hypothesis
- Long Term Memory Processes
  - Rescale Range Analysis
  - Biased Random walk
  - Hurst Exponent
- Cycles
- Phase Space
- Chaos & the Capital Markets

The Chaos Game

- Plot point P at random
- Roll dice
- Proceed halfway from P to point labeled with rolled number & plot new point
- Repeat 10,000 times



The Sierpinksi Triangle



Chaos

- Local randomness, global determinism
  - Apparently random process may contain deterministic pattern
- Stable, self-similar structure
- Sierpinksi Triangle
  - Plot order impossible to predict
  - But odds of plotting each point are not equal
    - Empty spaces in each triangle have zero probability
    - Local randomness does not equate to independence

Characteristics of Fractals

- Self-similarity
  - The part is similar to the whole
    - Precise similarity in case of Sierpinski triangle
- Scale Invariance
  - Sub-parts not to same scale as parent
- Dimension
  - Euclidean space features integer dimensions
  - Fractals occupy fractional dimension
    - E.g dimension of Sierpinski triangle is more than a line but less than a plane ($1 < d < 2$)

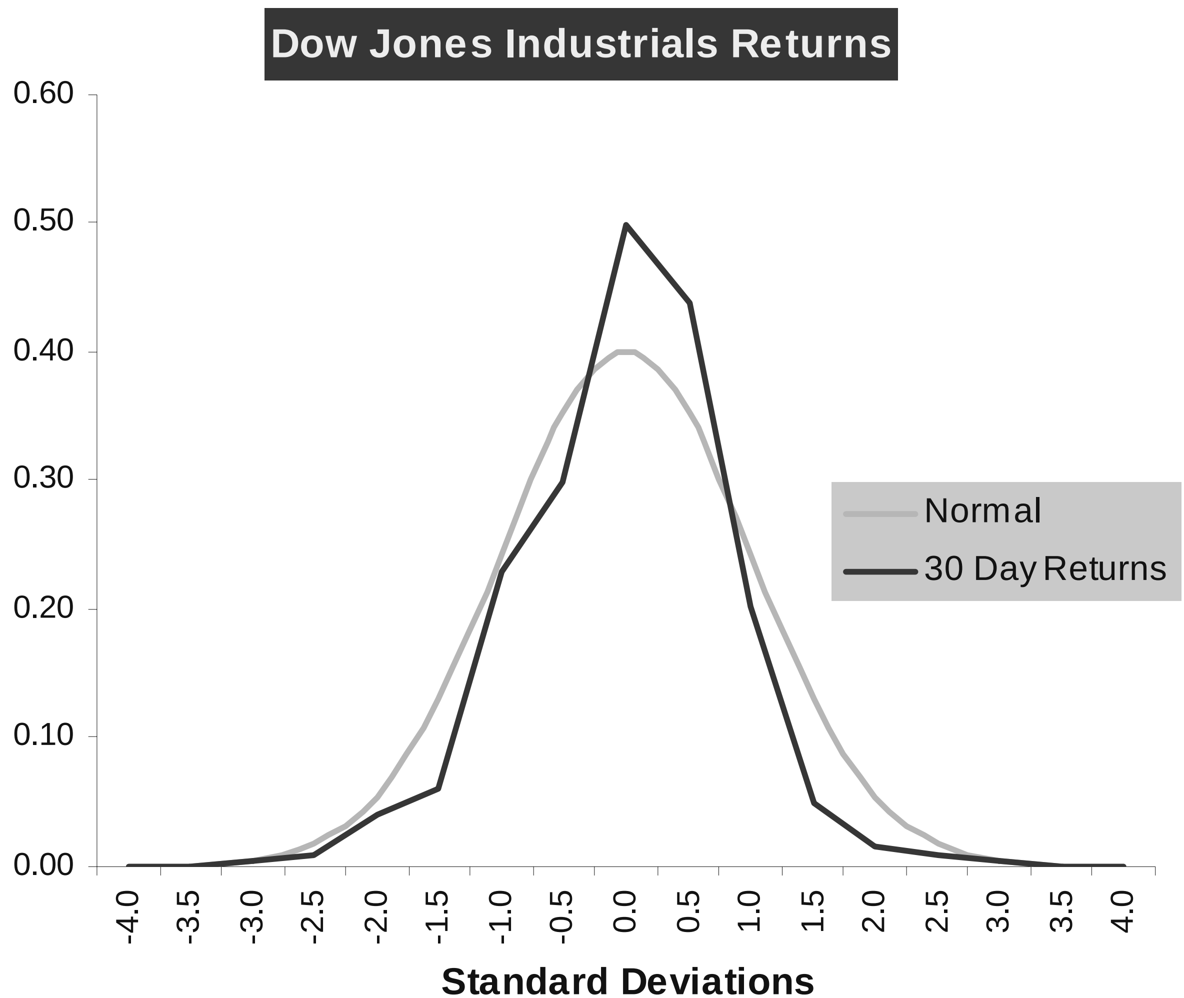
Fractal Time Series

- Dimension measures how “jagged” series is
  - Straight line has fractal dimension of 1
  - Random time series has fractal dimension of 1.5
    - 50% chance of rising or falling
  - A line can have fractal dimension between 1 and 2
  - At values $\neq 1.5$ series is less or more jagged than a random series
    - Non-Gaussian

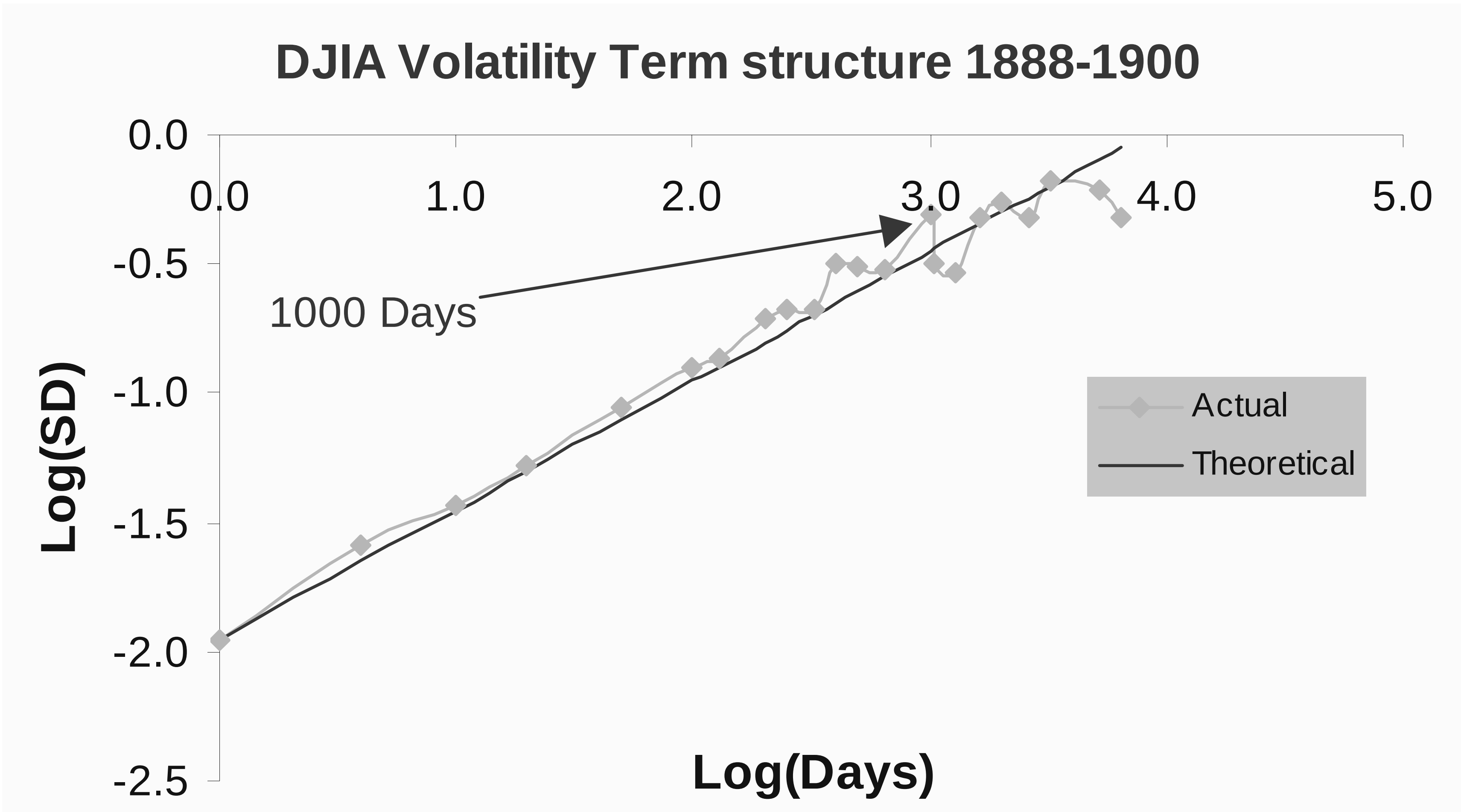
Non-Gaussian Properties of Financial Markets

- Distribution of Returns
  - Higher peak at mean than Normal
  - Fatter tails
    - Uniformly fatter
      - As many observations at 4σ away from mean as at 2σ
  - Markets tend to stay still or make major moves more often than theory predicts
    - Reflected in option volatility smiles
- Term Structure of Volatility
  - Scales at faster rate than $T^{1/2}$

Example: Returns on the DJIA



Volatility Term Structure



Regression Analysis of Term Structure

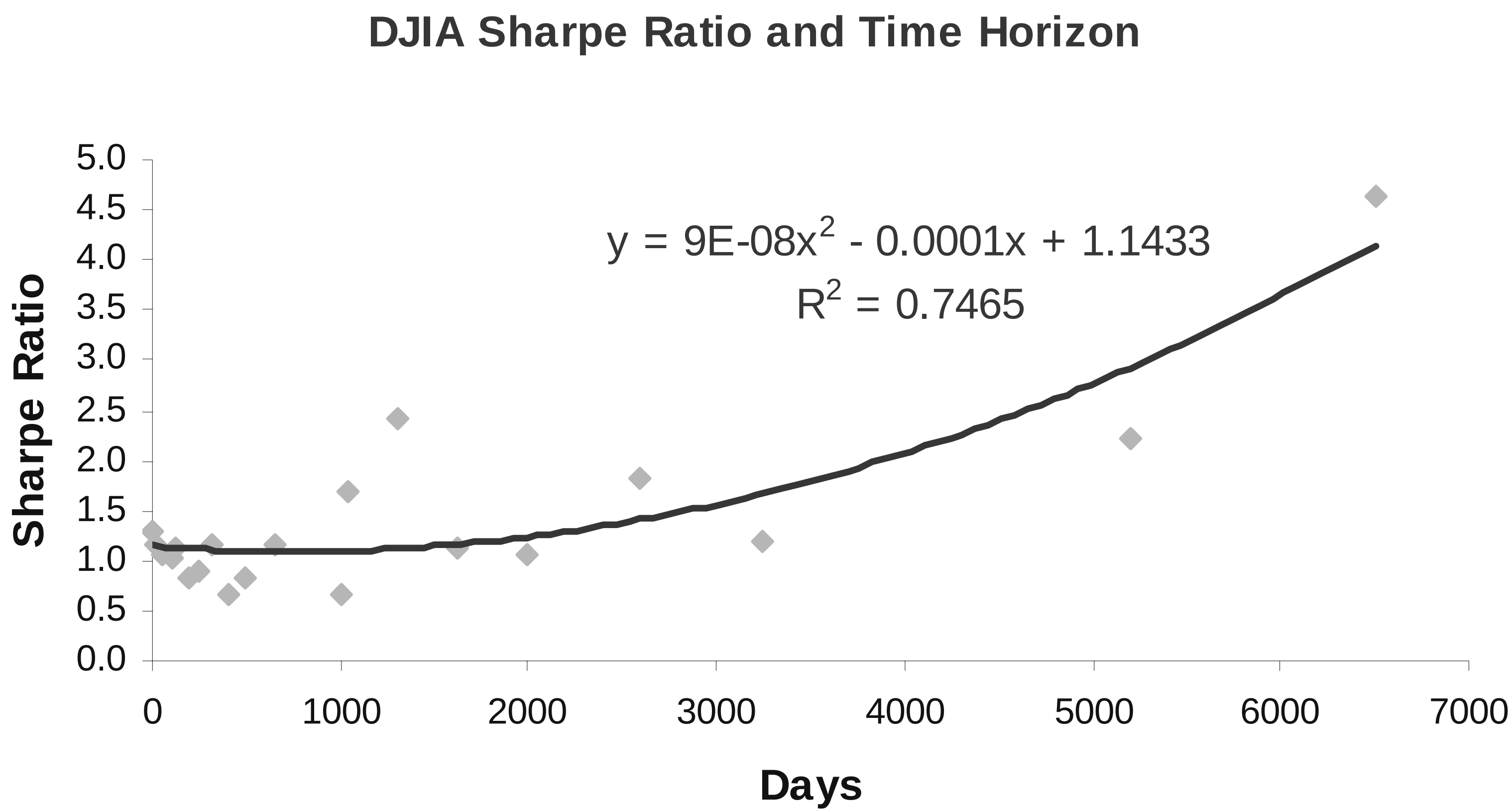
- Chart indicates clear breakdown in volatility term structure after n = 1,000 days
- Regression analysis confirms this:

	Days < 1,000	**Days > 1,000**
Intercept	-1.95	-1.38
Slope	0.53	0.31
R^2	99.3%	47.2%
F	1823.0	5.4
p	0.00%	5.99%

Conclusion Re term Structure

- Risk-Return
  - Riskier to invest for period < 4 years
  - Increasingly less risk incurred beyond 4 years
  - Tied to business cycle?
- Sharpe Ratio
  - Gets larger for longer time horizons

Sharpe Ratio & Time Horizon



Fractal Theory of Markets

- Stable markets
  - Investors with many investment horizons
    - Ensures liquidity
- Information set depends on investment horizon
  - Short term: market sentiment & technical factors
  - Long term: fundamental analysis
- Unstable markets
  - Occur when LT traders exit market or trade ST
- Prices set by combination of ST & LT valuation
  - ST trends are noise. LT trends tied to economic cycles

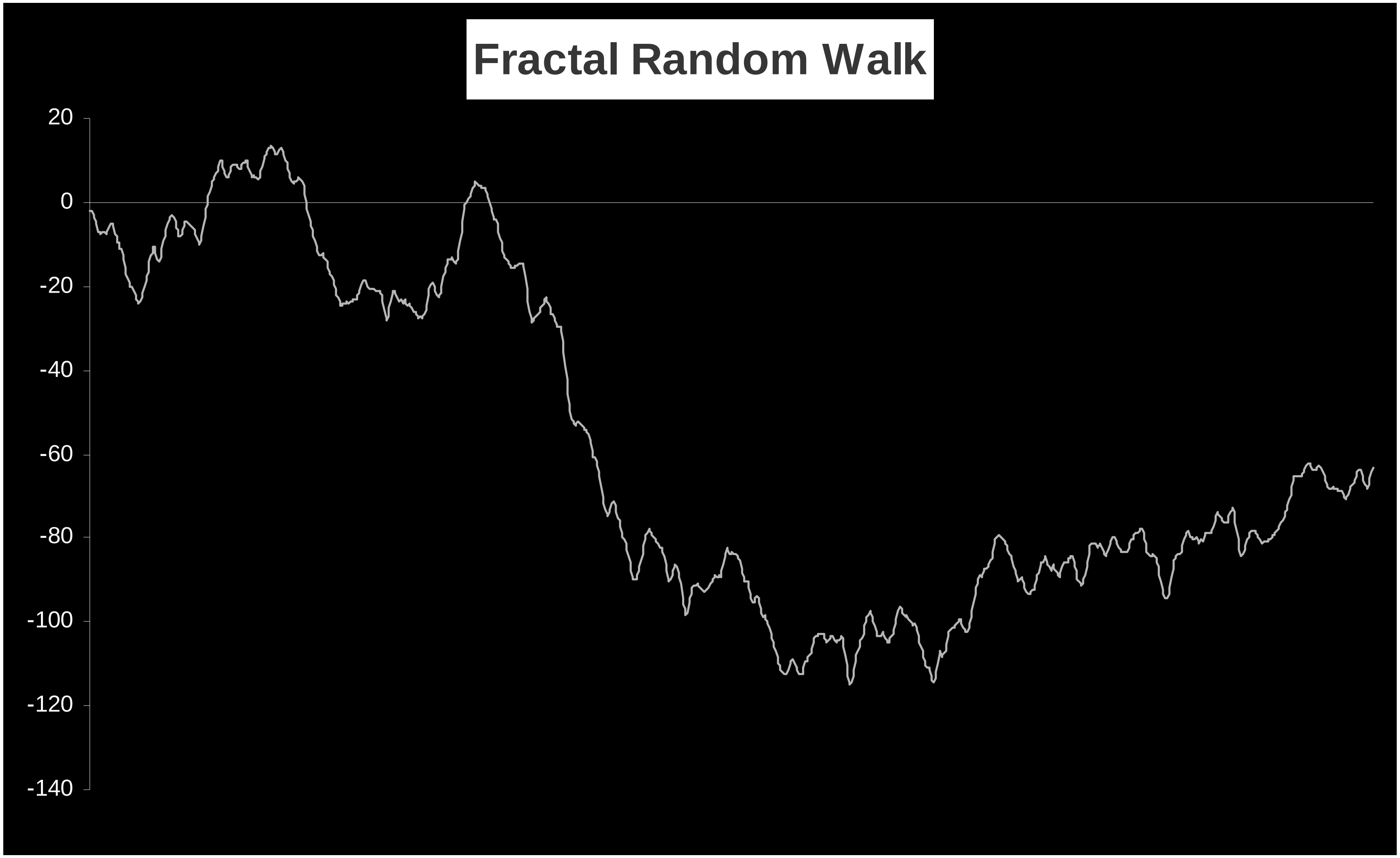
Rescaled Range Analysis

- Developed by H.E. Hurst 1950's
- Brownian Motion
  - Distance traveled $R \propto T^{0.5}$
- Hurst Exponent
  - $(R/S)_T = cT^H$
    - H is the Hurst Exponent
    - c is a constant
    - T is # observations
    - $(R/S)_T$ is the rescaled range, a standardized measure of distance traveled
    - Note for random time series H = 0.5

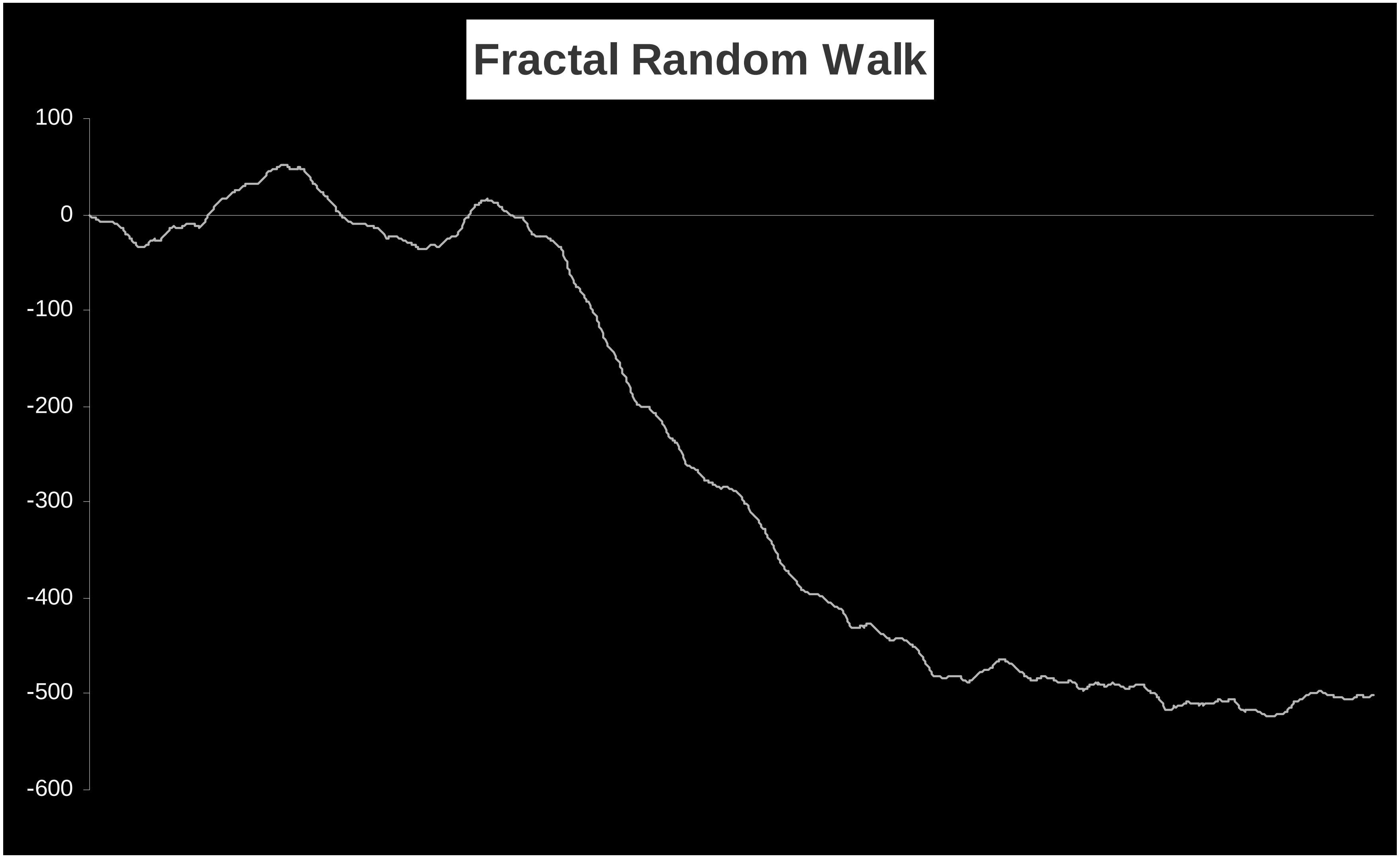
Hurst Exponent & Market Behavior

- H measures *persistence*
- Correlation $C = 2^{(2H-1)} - 1$
- White Noise: $H = 0.5$, $C = 0$
- Black Noise: $0.5 < H < 1$, $0 < C < 1$
  - Persistent, trend reinforcing series
  - “Long memory”
- Pink Noise: $0 < H < 0.5$, $C < 0$
  - Antipersistent, mean-reverting
  - Choppier, more volatile than random series

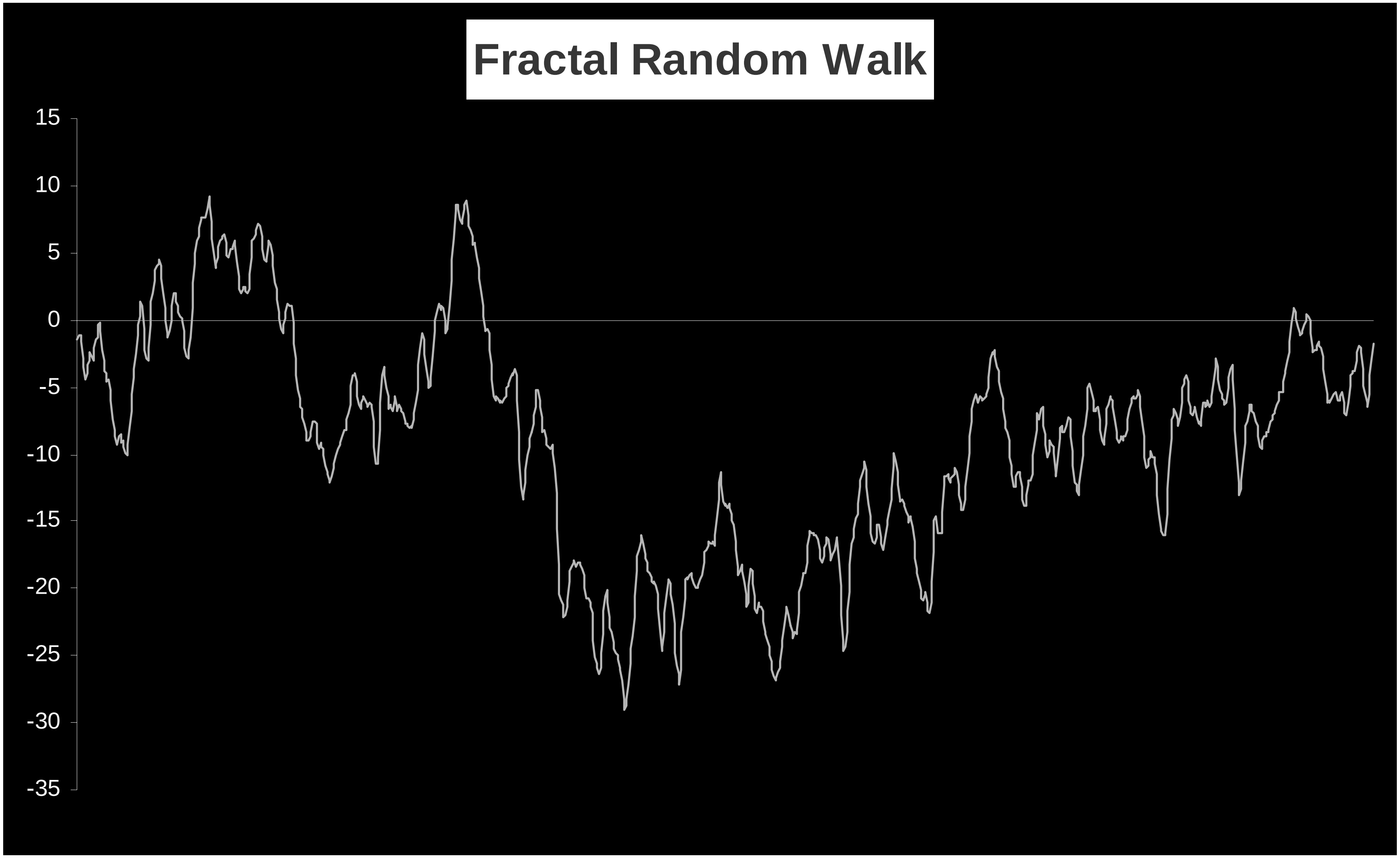
White Noise Process



Black Noise Process



Pink Noise Process



Simulating a Fractal Random Walk

- Feder (1988):

$$\Delta y_H(t) = \left(\frac{n^{-H}}{\Gamma(H+0.5)} \right) \times \left\{ \sum_{i=1}^{nt} i^{(H-0.5)} E_{(1+n(M+1)-i)} + \sum_{i=1}^{n(M-1)} \left[(n+i)^{(H-0.5)} - i^{(H-0.5)} \right] E_{(1+n(M-1+t)-i)} \right\}$$

- E_i is a strict white noise process, No(0, 1)
- M is the number of periods for which long memory is generated
- n is set to 5
- t is set to 1
- H is Hurst exponent

Calculating (R/S)

- Form series of returns
  - $r_t = \mathrm{Ln}(P_t / P_{t-1})$ for t = 1, 2, . . . , T
- Divide into A contiguous sub-periods
  - Length n, such that An = T
- Compute average for each sub-period $\bar{r}_a = \sum_{k=1}^{n} r_{ak}$
- Form cumulative series

$$X_{ka} = \sum_{i=1}^{k} \left(r_{ia} - \bar{r}_a \right)$$

- Define range $R_a = \mathrm{Max}(X_{k,a}) - \mathrm{Min}(X_{k,a})$

Calculating (R/S)

- Compute standard deviation for each subperiod

$$S_a = \left[\frac{1}{n} \sum_{k=1}^{n} \left(r_{ka} - \bar{r}_a \right)^2 \right]^{1/2}$$

- Calculate average R/S for each n

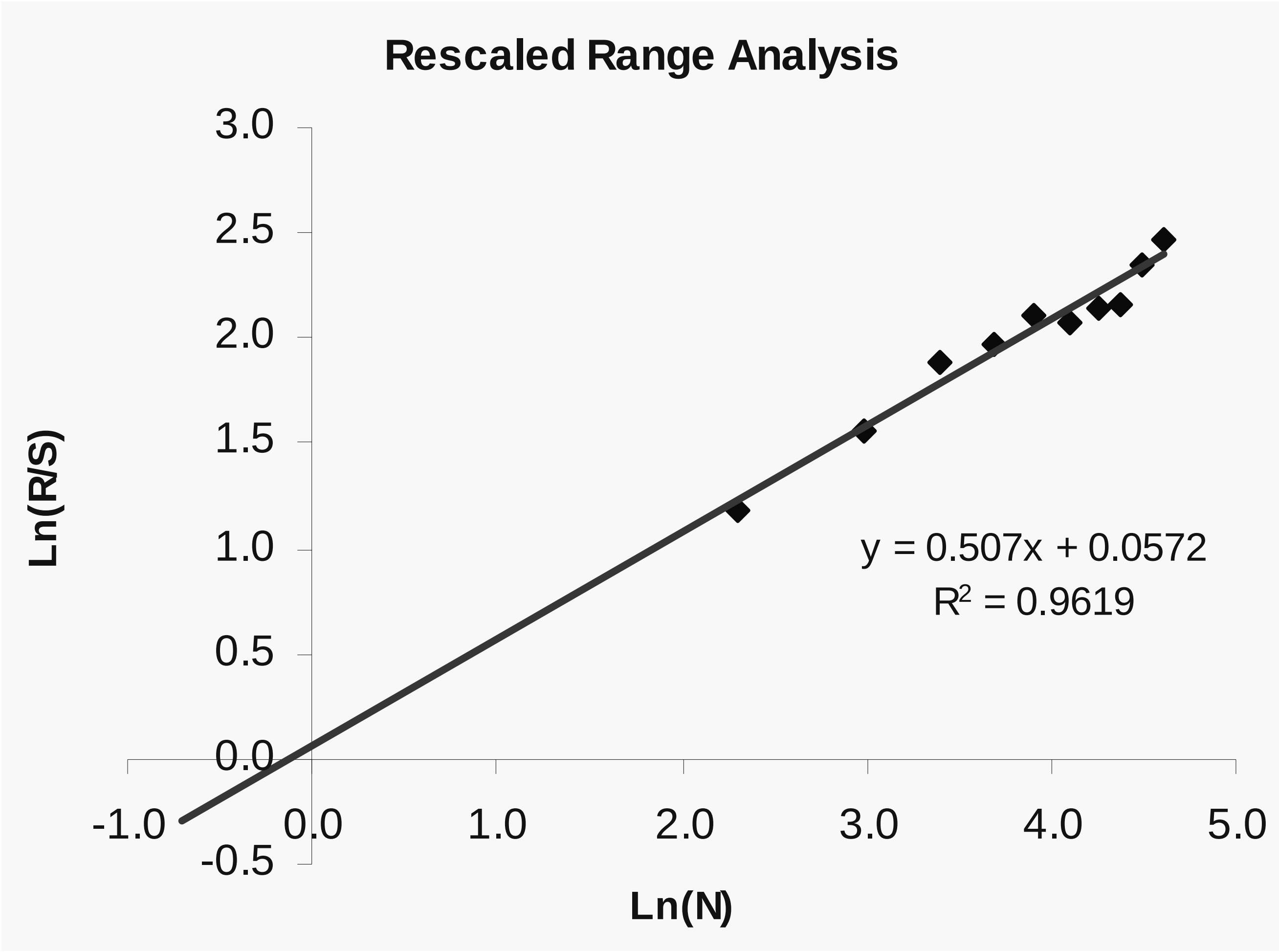
$$(R/S)_n = \frac{1}{A} \sum_{a=1}^{A} (R_a / S_a)$$

- Use OLS Regression to Estimate H
  - $\text{Ln}(R/S)_n = \text{Ln}(c) + H\,\text{Ln}(n)$

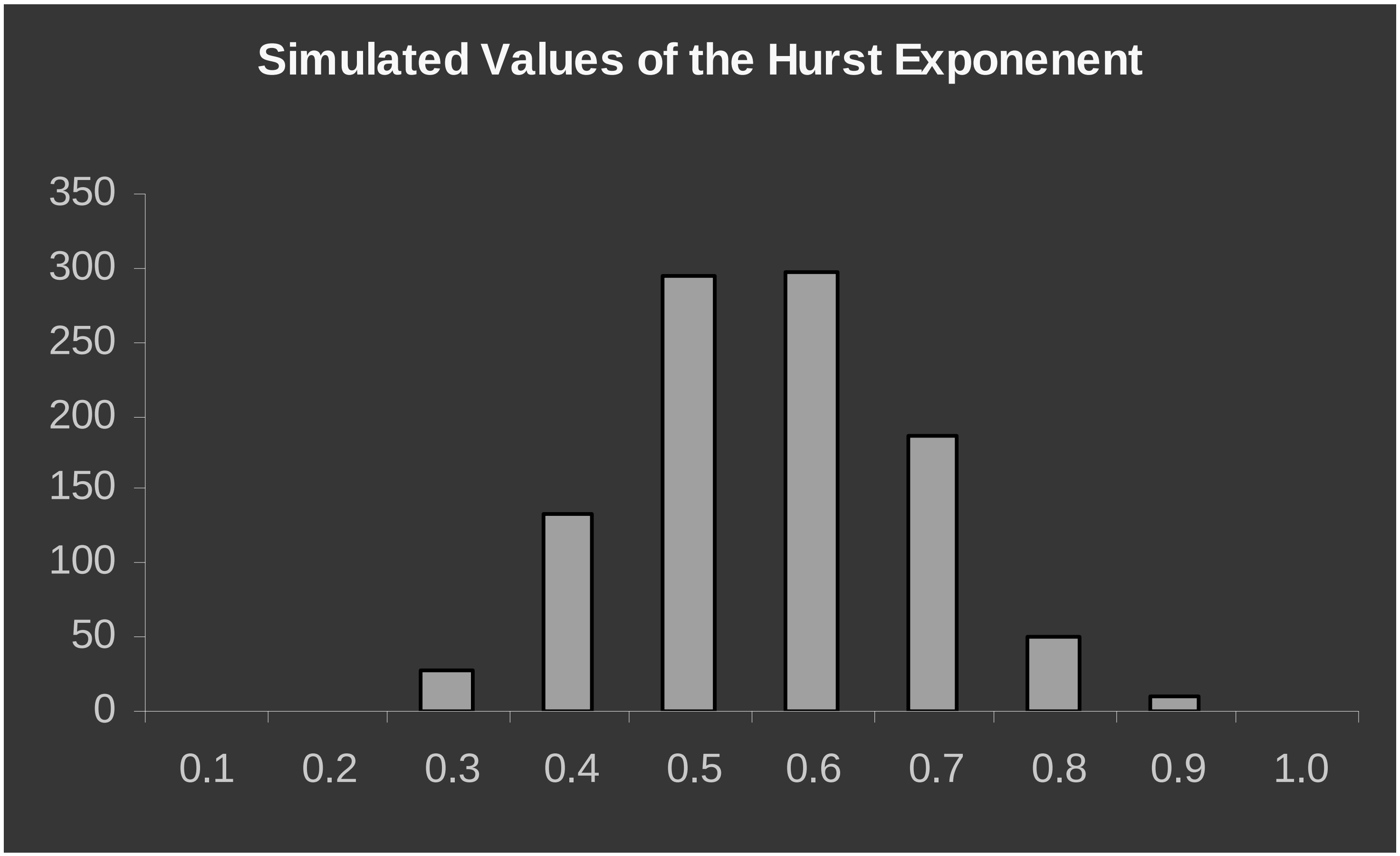
Example: RS Worksheet

- Generate random sequence using RAND() fn.
  - Periods of length n = 10, 20, . . ., 100
- Calculate mean and SD for each sub-period
- Form cumulative series
- Calculate R, R/S and Ln(R/S) for each sub-period
- Repeat 10 times
- Plot Ln(n) against average Ln(R/S)
- Fit linear trend
  - OLS slope estimate = H

Example: RS Analysis



Hurst Exponent for Random Series 1,000 Simulations



Testing (R/S): $E(R/S_n)$

- Anis & Lloyd (1976)

$$E(R/S_n) = \frac{\Gamma[0.5(n-1)]}{\sqrt{\pi}\,\Gamma(0.5n)} \sum_{i=1}^{n-1} \sqrt{\frac{n-i}{i}}$$

- For large n (>350) Use Stirling Function

$$E(R/S_n) = \left(\frac{n-0.5}{n}\right)\left(\frac{n\pi}{2}\right)^{-0.5} \sum_{i=1}^{n-1} \sqrt{\frac{n-i}{i}}$$

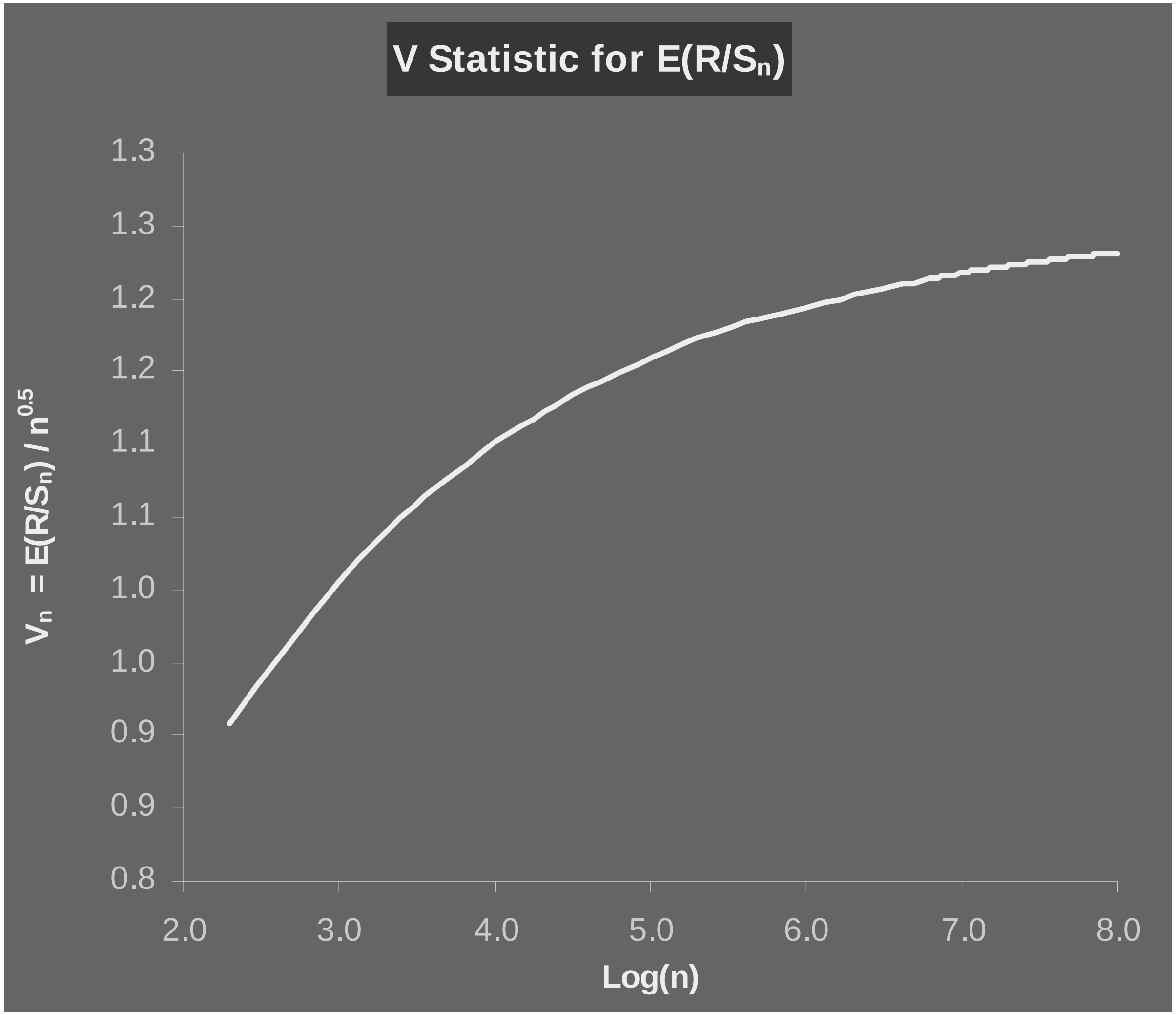
Testing (R/S)

- $E(H) = Ln(E[R/S_n]) / Ln(n)$
- $Var(H) = 1 / T$
  - If underlying process is random Gaussian, H will be Normally distributed
- V-Statistic
  - Recall $(R / S_n) = cn^H$
  - V-Statistic
    - Divide by $\sqrt{n}$
    - $V(n) = (R / S_n) / \sqrt{n} = cn^{(H-0.5)}$

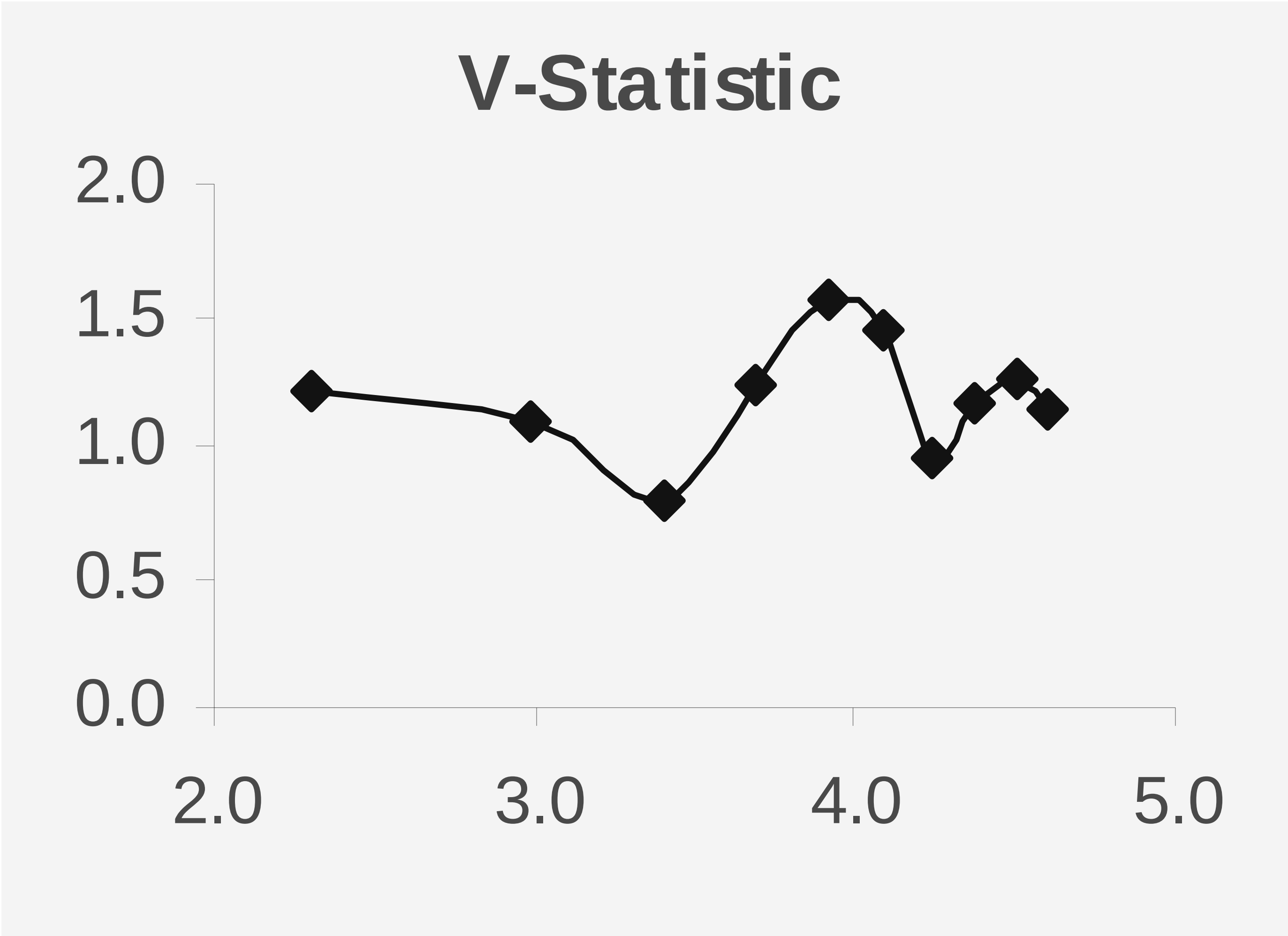
V-Statistic

- V-Statistic $V(n) = (R / S_n) / \sqrt{n} = cn^{(H-0.5)}$
- For *Persistent* Series H > 0.5
  - V(n) is increasing fn. of n
- For *Random* Process H = 0.5
  - V(n) is constant
- For *Antipersistent* process H < 0.5
  - V(n) is declining fn. of n

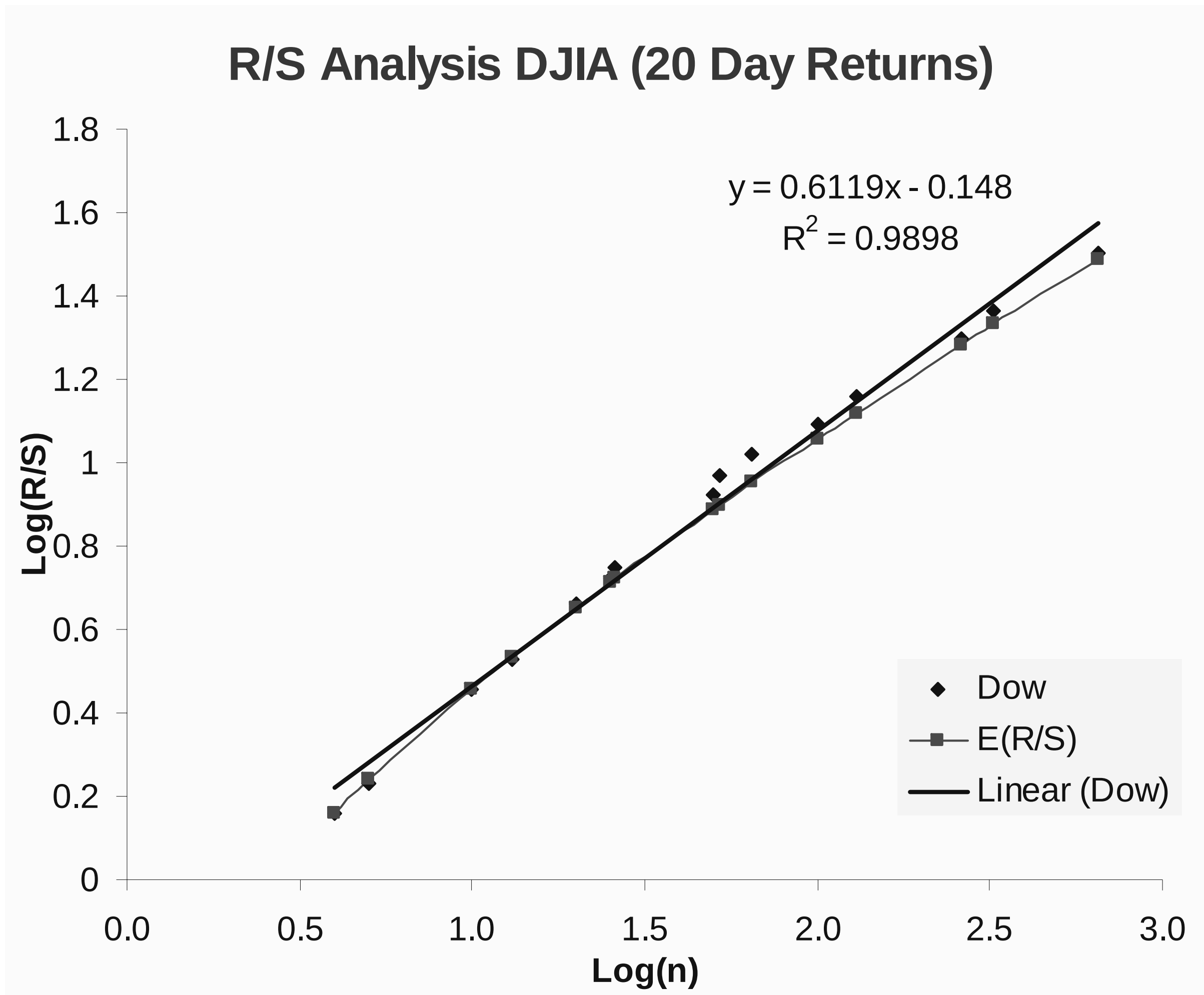
V Statistic for $E(R/S_n)$



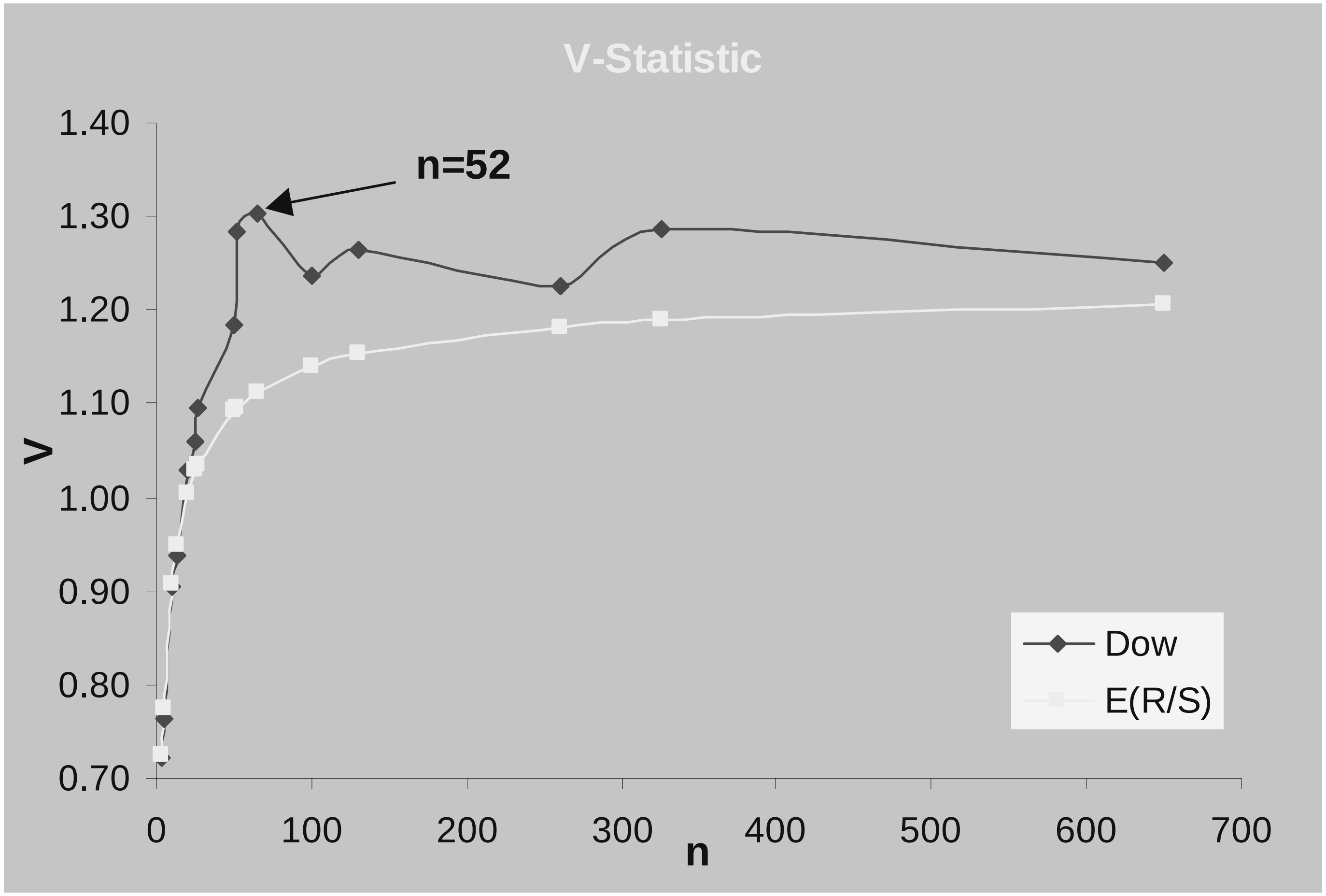
V-Statistic for Random Series



Example: Long memory Process in DJIA (20-day Returns)



Example: Long memory Process in DJIA (20-day Returns)



Dow R/S Regression Analysis

- Estimated Hurst Exponent
  - H = 0.62
    - OLS estimate of regression slop coefficient
  - Indicates fractal persistent memory process
- 10 < n < 52
  - H = 0.71, R^2 = 99.9%
    - Highly persistent Hurst process
- 52 < n < 650
  - H = 0.49, R^2 = 99.8%
    - White noise process

Dow: Conclusions

- R/S analysis indicates long memory process
- Average cycle length approx 4 years
  - Tied to economic cycle
  - Events occurring today affected by events up to 4 years ago
- Long memory effects dissipated after 4 years

R/S Analysis of Stocks

	Hurst Exponent	Cycle (months)
IBM	0.72	18
Xerox	0.73	18
Apple	0.75	18
Coca-Cola	0.70	42
McDonald’s	0.65	42
Con Edison	0.68	90

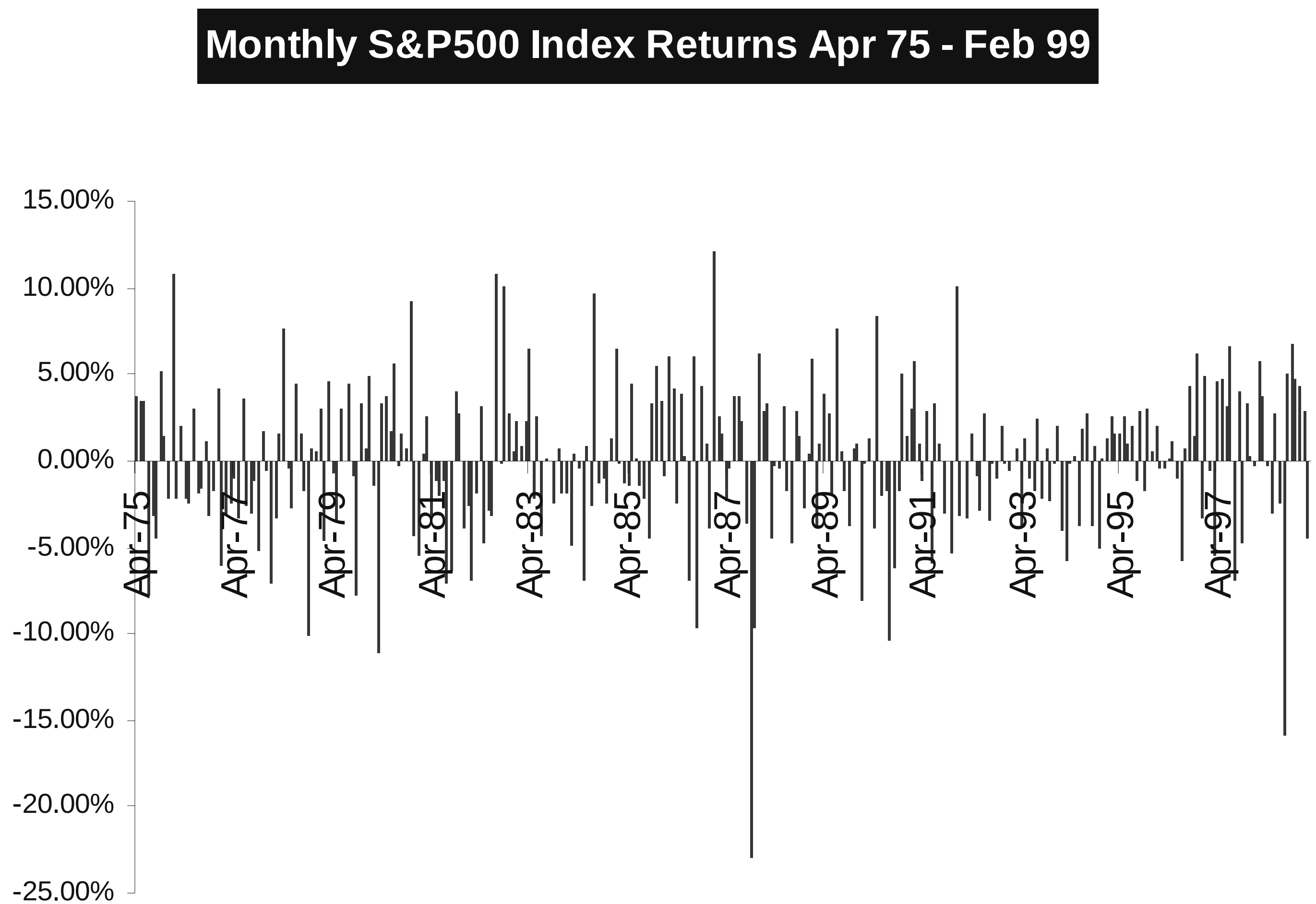
R/S Analysis: Conclusions About Stock

- Innovative, high growth firms
  - Have high H and short cycles
- Stable, low growth firms
  - Have low H and long cycles
- Implications for risk
  - High H firms are less risky
    - Less noise in series
    - Contradicts standard theory
  - What about diversification?
    - Dow index has one of the highest H exponents

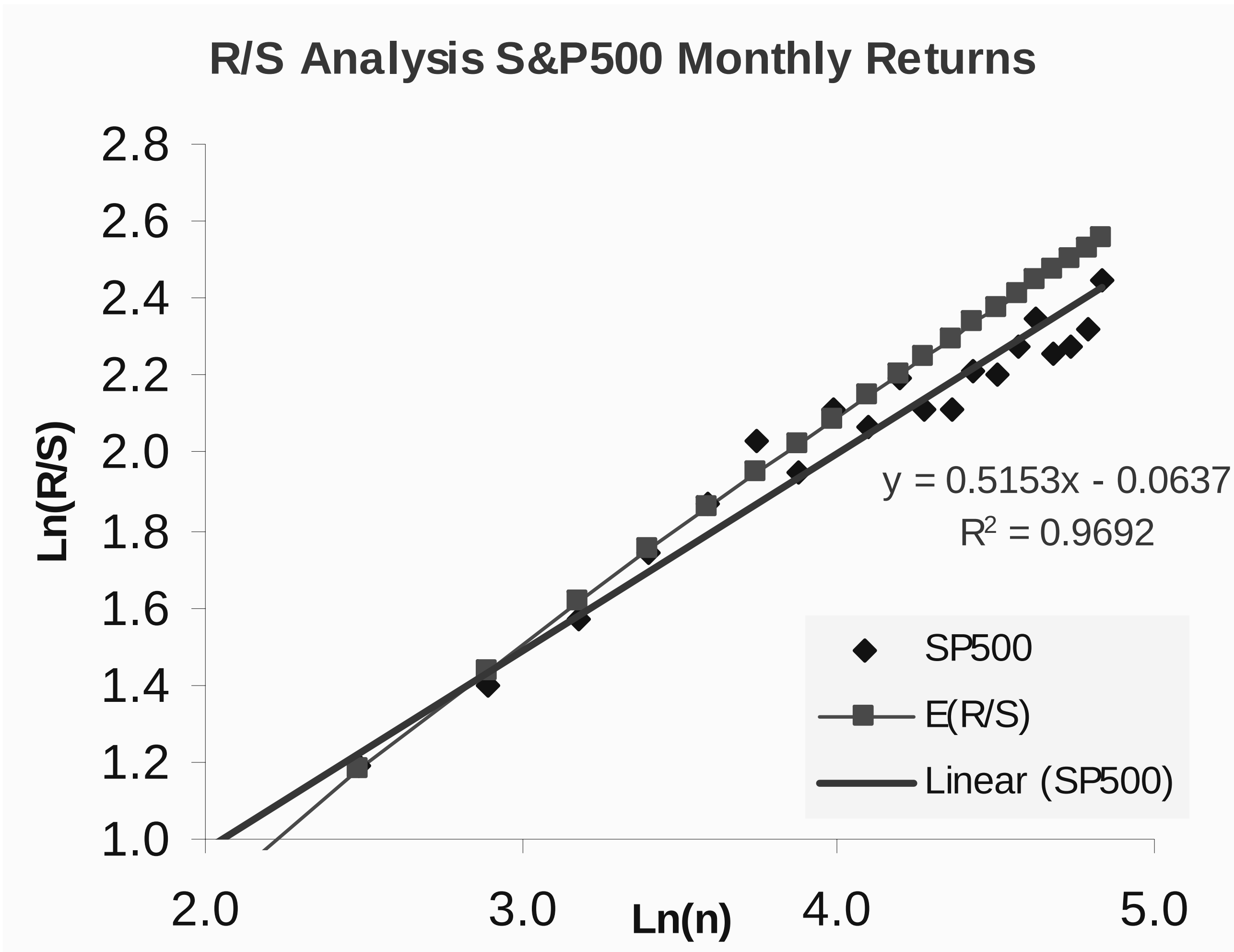
Lab: R/S Analysis of S&P 500 Index

- Monthly returns April 75 to Feb 99
  - Detrended to remove short term memory effects
- Calculate
  - RS, E(R/S), v-statistic (actual and expected)
- Plot
  - Ln(n) vs. Ln(R/S)
  - N vs. V-statistic
- Estimate
  - Cycle length
  - Hurst exponents pre and post cycle

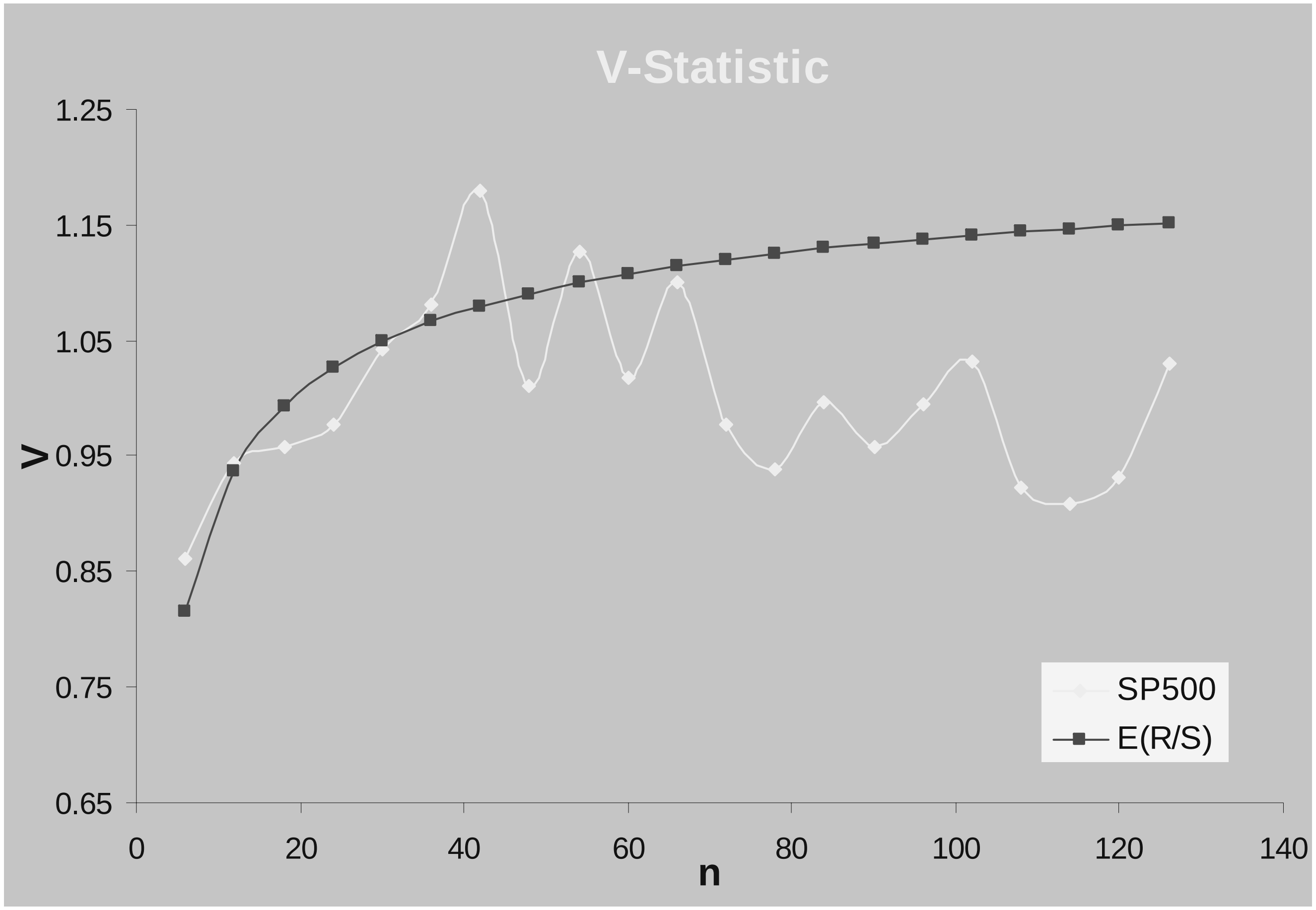
Lab: R/S Analysis of S&P 500 Index



Solution: R/S Analysis of S&P 500 Index - Ln(R/S) Plot



Solution: R/S Analysis of S&P 500 Index - V-Statistic



SP500 Regression Analysis

Regression	6-126	6 - 42	42-126
Slope	0.515	0.641	0.349
Intercept	-0.064	-0.421	0.666
SE	0.021	0.023	0.044
t	0.73	6.01	3.42
p(slope = 0.5)	47.70%	0.18%	1.89%
R^2	96.9%	99.3%	82.8%

Economic Indicators

- Industrial Production Index
  - H = 0.79
    - Based on monthly data, Jan 1946 - Jan 1999
    - Strongly persistent
  - Cycle 42 months
    - Shorter than 4-year cycle accepted by economists
  - Ties in with S&P 500 Index

R/S Analysis: Currencies

- True Hurst process: no cycle length
  - R/S continues scaling at rate H indefinitely with n
  - Infinite memory process
  - Not tied to economic cycles
  - No "fundamental" valuation of currency
  - Less persistent, more volatile than stocks
- H Exponents (based on daily data)
  - Yen: H = 0.64
  - GBP: H = 0.63
  - DM: H = 0.62

R/S Analysis: Other Financial Markets

- Treasury Bonds
  - H = 0.68
    - Based on daily yields, Jan 1950-Dec 1989
  - Cycle length 5 years
- Gold
  - Some evidence of 4-year cycle
  - H = 0.58, but not significant
- Volatility
  - A true pink noise antipersistent process
  - H = 0.31 for S&P 500 Index vol. (realized)

Chaos Theory & Financial Markets - Summary

- R/S Analysis confirms aperiodic cycles in many series
  - Stocks, stock indices, bonds, economic indicators
  - Cycle length related to economic cycle
- Currencies are true Hurst process
  - Scale indefinitely
- Volatility is only known antipersistent financial time series (apart from Wheat futures!)
  - Mean-reverting?