# Fixed Income Securities

# Fixed Income Securities

- Treasury Securities
  - Bonds, Notes & Bills
- The Yield Curve

# Time Value of Money

$100    T = 1 year    $110

R = 10%

- *Future Value* of $100:
  - $100 * (1 + 10%) = $110
- *Present Value* of $110:
  - $110/ (1 + 10%) = $100

# Compounding & Discounting

- Compounding
  - Computing *future value* from current value is called compounding
  - $100 ⟶ $110
- Discounting
  - Computing *present value* from future value is called discounting
  - $100 ⟵ $110

# Compounding Over Multiple Periods

- Suppose interest rate = 10% and I have $100 to invest
- What will I get in 1 year time?
  - $\$100 \times (1 + 0.1) = \$110$
- What will I get after 2 years?
  - $\$100 \times (1 + 0.1)^2 = \$121$
- After N years?
  - $\$100 \times (1 + 0.1)^N$

# Time Value of Money

- $ today is worth more than $ tomorrow
- If I invest $X today, I will expect more than $X tomorrow i.e. $P_N > P_0$
- $P_0 \quad \times \quad (1 + r)^N \quad = \quad P_N$

•Current Price
•Price at time 0
•Net Present Value

•Discount rate
•Internal rate of return
•Yield to maturity
•Compound Factor

•Ending Price
•Price at time N
•Future Value

# Compounding Frequency

- Interest rates quoted on an annual basis
- Compounding Frequency:
  - Annual: $(1+r)^n$, applied every year
  - Semi-annual: $(1+r/2)^{2n}$, applied every 6m
    - typically used for treasuries
  - Quarterly: $(1+r/4)^{4n}$, applied every qtr.
  - Daily: $(1+r/365)^{365n}$, applied every day.
  - n times a year: $(1+r/n)^{nt}$
  - Continuous: $e^{rt}$, limit as n increases infinitely

# Discounting

- Discounting is just the reverse of compounding:
- $P_n$ in n years time is worth $P_0 = P_n / (1+r)^n$ today

$$P_0 \quad = \quad P_n \quad \times \quad 1 / (1 + r)^n$$

- Current Price
- Price at time 0
- Net Present Value

- Ending Price
- Price at time n
- Future Value

Discount Factor

# Simple Interest

- An old convention: pre-calculator
- Invest \$100 for 90 days at 10%, simple interest
- Many markets: 360 day year
- After 90 days you have:
- \$100 (1 + 10% x 90 / 360) = \$102.50

# Daycounts

- How many days in a month and year
  - 30/360 (Money Market)
    - in one month, get 1+(30/360)r
  - Actual/360 (LIBOR)
    - in one month get 1 + (31/360)r if 31 days
  - Actual/365 (Treasury)
    - (or actual/actual: adjust for leap year)

# Discount Factors and Compounding

- Notation:
  - R = % Interest rate, T = Time (days), D = Discount Factor
- R is simple:
  - $D = 1 / (1 + R \times T / 360)$
  - $R = (-1 + 1/D) * 360 / T$
- R is annually compounded:
  - $D = 1 / (1 + R)^{T/360}$
  - $R = -1 + (1 / D)^{360/T}$
- R is continuously compounded:
  - $D = e^{-RT/360}$
  - $R = -Ln(D) \times 360 / T$

# Zero Coupon Bond

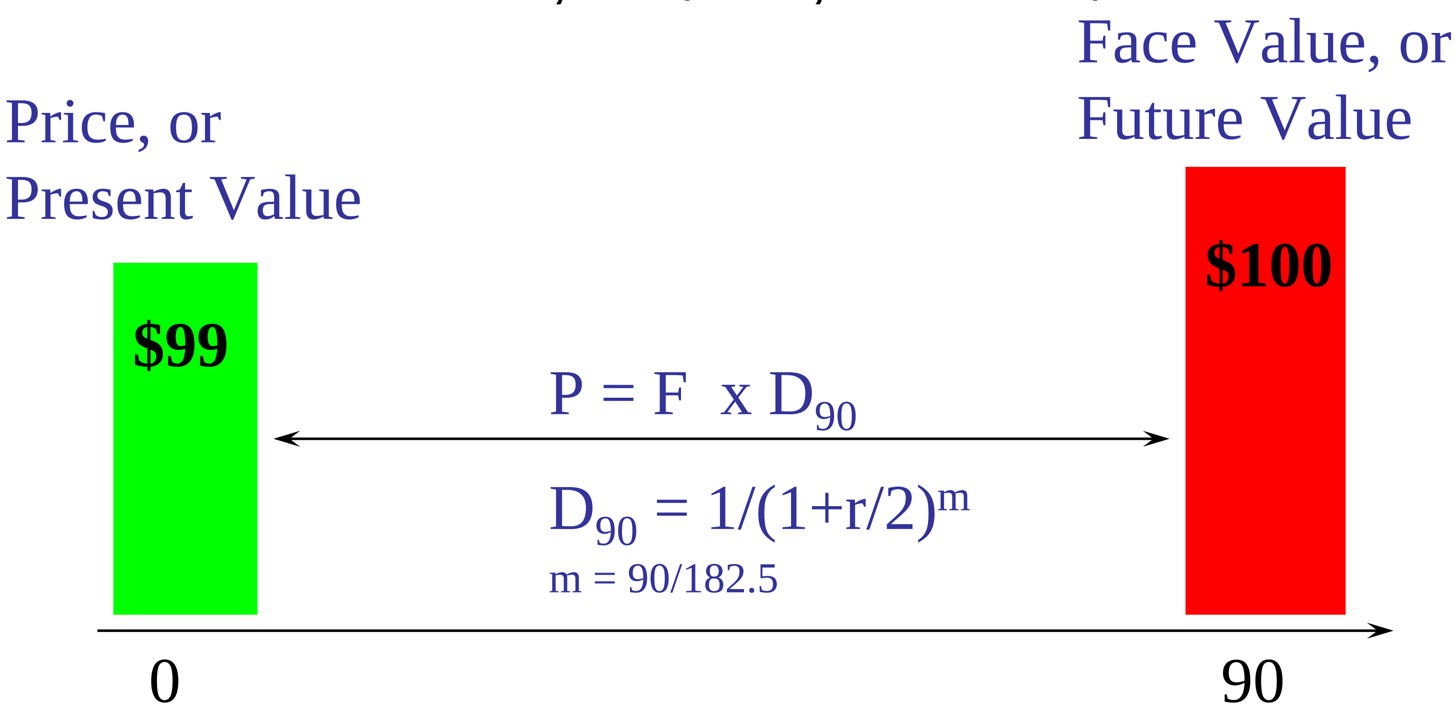
- Pays a fixed sum (face value) at some future date (maturity)
- No interest paid in between (zero coupon)
- Sells today for a discounted price
- E.g.. $100 paid in 90 days
- Price today is $99
- What is the interest rate?

# Zero Coupon Bond Spot Rate

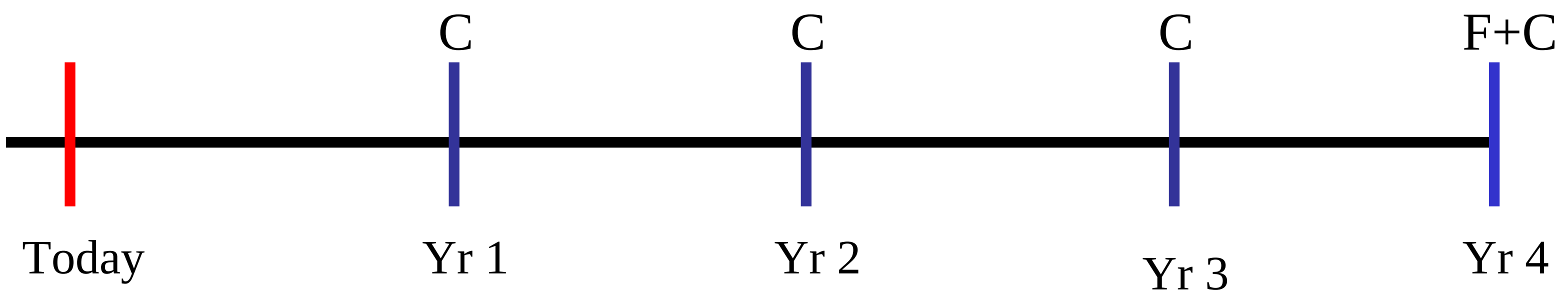
- $99 today, $100 in 90 days
- Semi-annual: (bond equivalent basis)
  - $100 = 99(1+r/2)^m$
  - m is number of semi-annual periods
  - here, m is 90/182.5
  - $r = 4.12\%$
- R is called the (zero coupon) *Spot Rate*

# Pricing a Zero

- The price of a bond is the *present value* of its future cash flows
- Given r = 4.12%, F=$100, then P=$99

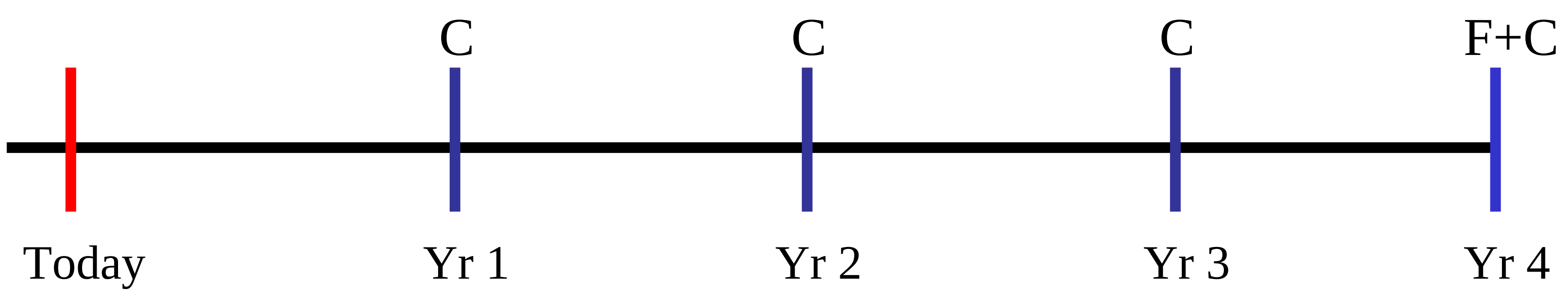


# Simple Coupon Bond Pricing




- Bond Price = Present Value of all Cash Flows
- Price = $CD_1 + CD_2 + CD_3 + (C+F)D_4$
  - $D_n$ = Discount factor period n
  - $D_n = 1 / (1 + y_n)^n$
- $y_n$ is the period-n spot rate

# Yield to Maturity




- Price = $\dfrac{C}{(1+Y)^1} + \dfrac{C}{(1+Y)^2} + \dfrac{C}{(1+Y)^3} + \dfrac{C}{(1+Y)^4} + F$
- Yield to Maturity (YTM):
  - at what ‘average’ interest rate Y can we discount all future cash flows so that the present value of the cash flows equals the price?
- Y is a complex average of spot rate

# Treasury Securities

- Apply concepts to markets
  - Discount bonds
    - Treasury bills, strips
  - Coupon Bonds
    - Definitions
    - Yield to maturity
    - Gilts, Treasury notes and bonds
    - Examples, calculations

# Treasury Markets

- US
  - Approx. $2.5 trillion in govt. debt
    - other debt as well
  - Types
    - Cash-management bills
    - Treasury Bills
    - Treasury bonds and notes
- UK
  - Approx. £300b
  - Types
    - Treasury Bills
    - Cash management notes
    - Gilts (about 75%)

# Treasury Market

- US
  - T-Bills
    - 3 mo, 6mo, 1yr
  - Notes
    - 2 yr, 3 yr, 5yr, 10 yr
  - Bonds
    - 30 yr
- UK
  - T-Bills
    - 3 mo, 6 mo, 1yr
  - Short Gilts
    - less than 5 year
  - Medium Gilts
    - 5-15 years
  - Long Gilts
    - greater than 15 year

# Treasury Market

- Auction
  - determines price/yield
    - US: experimenting with auction design
    - UK: discriminatory price auction
- “When issued” market
  - trading on yield in auction
- Secondary market
  - subsequent trading

# Treasury Bill Quotations

- Bills quoted as a “bank discount yield”, y
  - annualized yield on a bank discount basis
  - n = number of days to maturity
- $y = (1-P/F)(360/n)$
  - return on basis of *face value* rather than price
    - F-P = total gain
    - (F-P)/F = (1-P/F) = return relative to face value
    - (1-P/F)/n = return “per day”
    - (1-P/F)(360/n) = return on 360 day (‘bankers’) year

# Treasury Bill Pricing

- From quoted yield, calculate the price:
  - $P=F[1-(n/360)y]$
    - Example:  Bill that matures in 360 days and sold at a discount of 7% will be priced at 93
    - note: higher y implies lower price
    - bid/ask reversed here
- What is the return on the investment?

# Bond Equivalent Yield (BEY)

- What is the return on the investment?
  - Depends on how we quote interest rates
  - Need to convert the discount to a “bond type” yield for comparison purposes:
- BEY = (F/P-1)(365/n)
  - Return on basis of *price*, in a 365 day year
  - This is how yield is reported
  - Discount = F - P
  - Discount % = (F-P)/P = (F/P -1)
  - Discount % per annum = (F/P -1) * (365/n)

# T-Bill Example

- Treasury Calculator
  - Today: Jan 4, 2001
  - Maturity: May 11, 2001
    - 127 days
  - Discount: 4.93%
  - Reported yield = 5.09%
  - Purchase Price = $982,608

# T-Bill Example

| T-Bill | |
|---|---|
| **Face Value** | $1,000,000 |
| **Settlement** | 4-Jan-01 |
| **Maturity** | 11-May-01 |
| Remaining Term | 127 |
| **Discount yield** | 4.93% |
| **Price** | **$982,608.06** |
| **BEY** | **5.09%** |

# Gilts, US Treasury Notes and Bonds

- Long term government debt, 2 year-30 year when auctioned
  - US Notes: 2-10 year; bond: 30 year
  - Short, medium, long Gilts
- Are coupon bonds
  - coupon paid semi-annually
    - unlike discount bonds (pay a zero coupon)
    - leads to "accrued interest" adjustment

# Treasury Coupon Bonds

- Specifications
  - Maturity date
  - Face Value (paid at maturity)
  - Coupon interest rate
    - coupon dates: usually semi-annual working back from maturity
      - e.g. matures on Dec 15, 1998, so last coupon on that date; previous to last is June 15, 1998, 6 months prior
    - if c = coupon interest rate, semi-annual, then get Fc/2 on every coupon date

# Coupon Bonds


- Time line
  - Shows “cash-flow dates”
    - times when money changes hands
  - Accrued interest
    - if I sell you the bond today, I get a part of the next coupon payment, since I owned the bond for part of this coupon period (m days out of m+n days)

# Coupon Bonds: Accrued Interest

- Suppose there are n days to the next coupon and m days from the previous coupon
  - so n+m days between coupons
  - cF/2 = semi-annual coupon that will be paid at the next coupon date
  - Then, the seller gets the following part of the next coupon payment:

$$Accrued\,Interest = \frac{m}{(n+m)}(cF/2)$$

# Clean and Dirty Prices

- Quoted price called the “clean” or “flat” price
  - Does not include accrued interest
- Price paid called the “dirty” price
  - Pay price plus accrued interest
    - portion of interest that has accrued since the last coupon
    - calculated as a proportion:
      - AI = (coupon per period)*(days elapsed)/(days between coupons)

# Coupon Bonds

- Quotations
- in 32'nds
- so a quote of 100:23 or 100.23 or 100'23 means 100 and 23/32.
  - sometimes in 64'ths and sometimes in 16'ths
    - even 128'ths

# Treasury Bond Example

- US Treasury bond on Jan 4, 2001
- Matures 15 Feb 2025
  - 8,808 days to maturity
  - Coupon = 7.25%
  - Ask: 126 7/32  (clean price)
- Accrued Interest 2.7976
- Price = 129.0163 (dirty price)
- Reported yield = 5.31 %

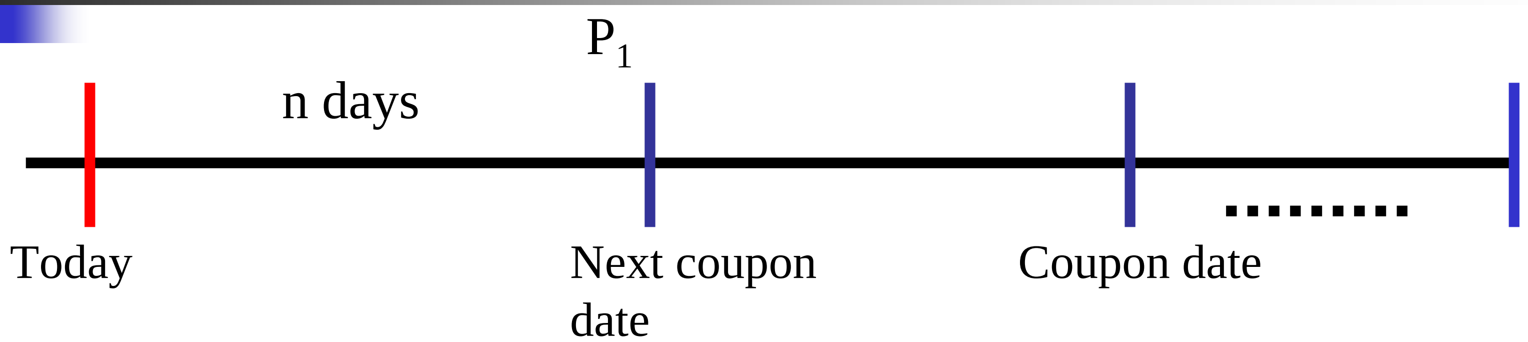
# Treasury Bond Example

| Bond | |
|---|---|
| Face Value | 100 |
| Quoted Price | 126 7/32 |
| Settlement | 4-Jan-01 |
| Maturity | 15-Feb-25 |
| Remaining Term | 8,808 |
| Coupon | 7.25% |
| Accrued Interest | 2.7976 |
| Dirty Price | 129.0163 |
| Yield to Maturity | 5.31% |
| Modified Duration | 12.42 |

# Coupon Bond Yields

- Quotations have a reported “yield.”
- This yield is the answer to following question:
  - at what interest rate can we discount all future cash flows so that present value of cash flows equals the (dirty) price?

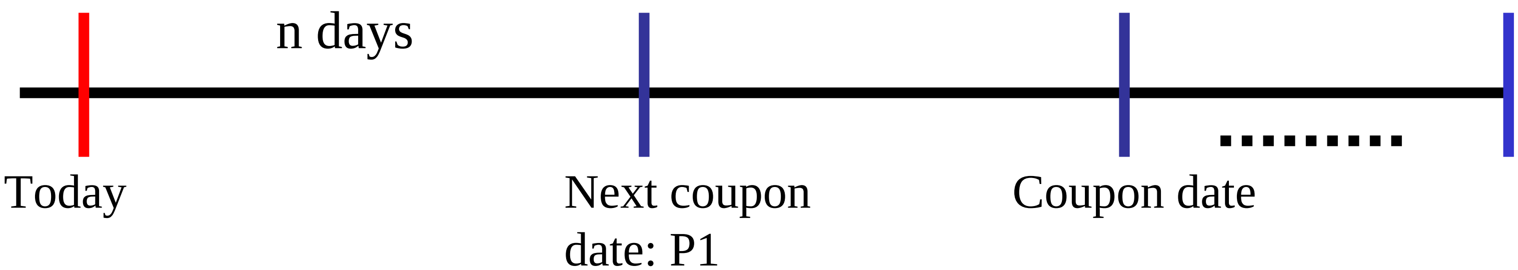
# Yield to Maturity




- Fix a yield, say y.
- Calculate the value of the bond at the next coupon date; call this value $P_1(y)$
  - m = number of coupon periods left after the next one

$$P_1(y) = cF/2 + \frac{cF/2}{(1+\frac{y}{2})} + \frac{cF/2}{(1+\frac{y}{2})^2} + \ldots + \frac{F + cF/2}{(1+\frac{y}{2})^m}$$

# Yield to Maturity




- Now, discount $P_1$ back to today
  - based on proportion of 1/2 a year left

$$PV(y) = \frac{P_1(y)}{(1+\frac{y}{2})^{\frac{n}{n+m}}}$$

# Coupon Bond Yield

- P = price (= quote + accrued interest)
- The YTM is the y such that PV(y) = P
  - A single interest rate such that if all future cash flows are discounted using it, then the present value of the cash flows equals the bond price.
- Also called the ***Bond Equivalent Yield***
  - Note: re-investment assumption. This would be the yield we would get if we could re-invest the coupons at the same yield.

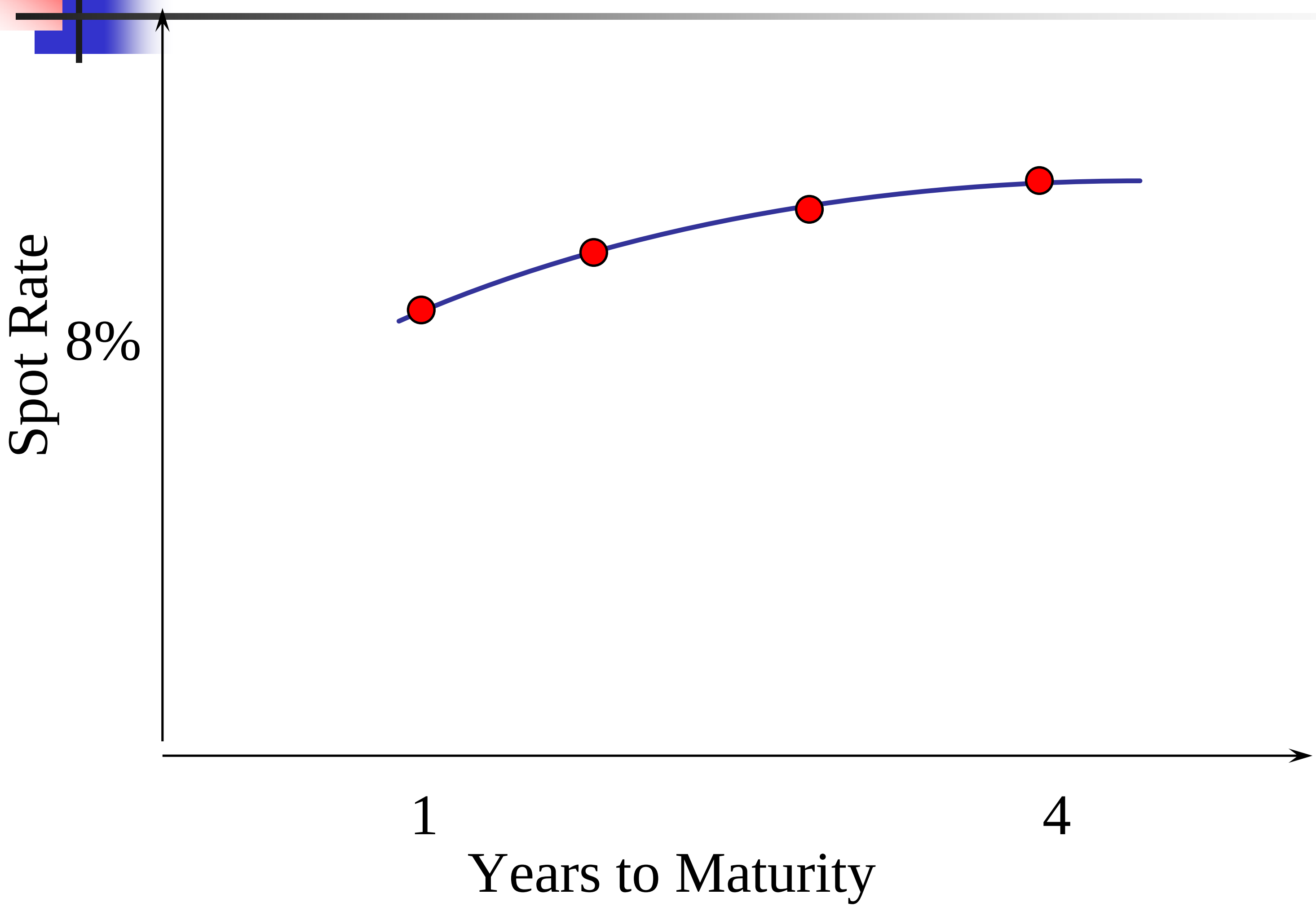
# The Yield Curve

- Zero-Coupon Bonds, Face Value $1,000:
- Term / Price / Discount / YTM

| Term | Price | Discount | YTM |
|---|---|---|---|
| 1 | 925.93 | $1/(1+y_1)$ | 8.000% |
| 2 | 841.75 | $1/(1+y_2)^2$ | 8.995% |
| 3 | 758.33 | $1/(1+y_3)^3$ | 9.660% |
| 4 | 683.18 | $1/(1+y_4)^4$ | 9.993% |

- Spot Yield (Zero Coupon Yield)
  - $y_1$ is called the one year spot rate
  - $y_2$ is called the two year spot rate

# Yield Curve Example



# Building a Yield Curve

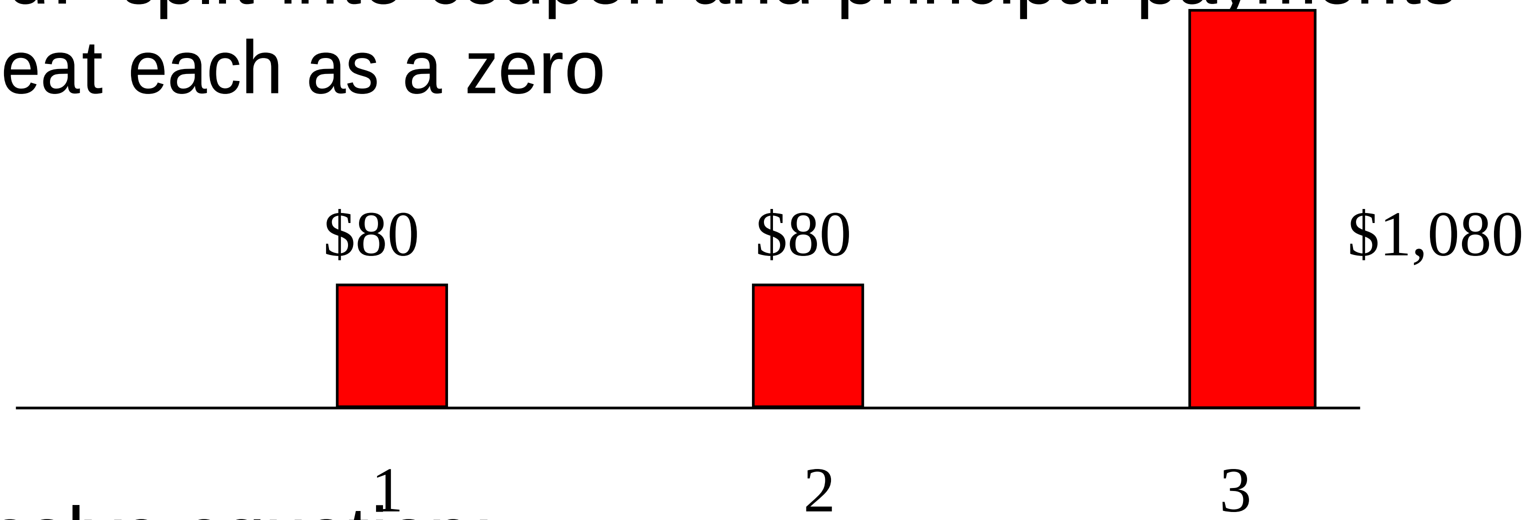
- In practice we have coupon bonds, not just zeros
- Term / Price / Discount / YTM

| Term | Price | Discount | YTM |
|---|---|---|---|
| 1 | 925.93 Z | $1/(1+y_1)$ | 8.000% |
| 2 | 841.75 Z | $1/(1+y_2)^2$ | 8.995% |
| 3 | 952.40 C | | |

- Bond in year 3 is a coupon bond
  - Pays 8% coupon ($80 per year)
  - How do we proceed?

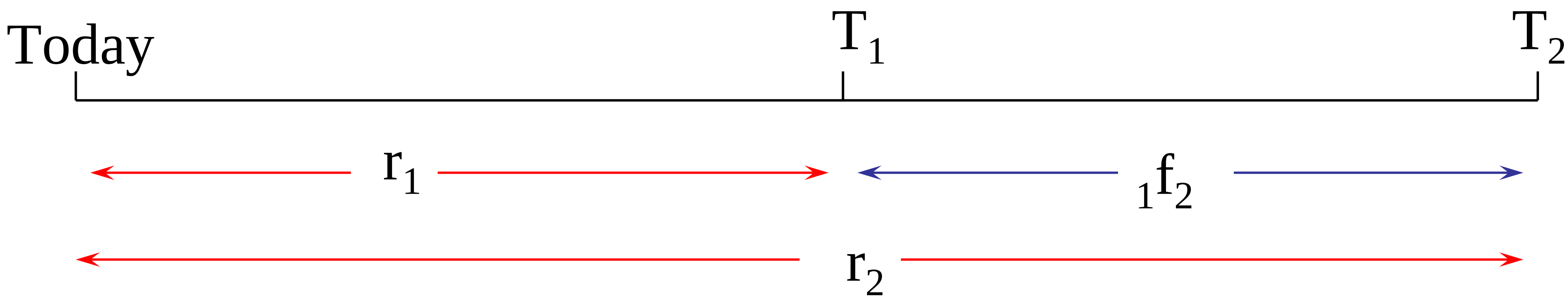
# Bootstrapping

- Method: split into coupon and principal payments and treat each as a zero




- Then solve equation:
  - $952.40 = \$80/(1+y_1) + \$80/(1+y_2)^2 + \$1080/(1+y_3)^3$
  - $y_1$ & $y_2$ are known
  - $y_3 = 10.020\%$

# Forward Rates

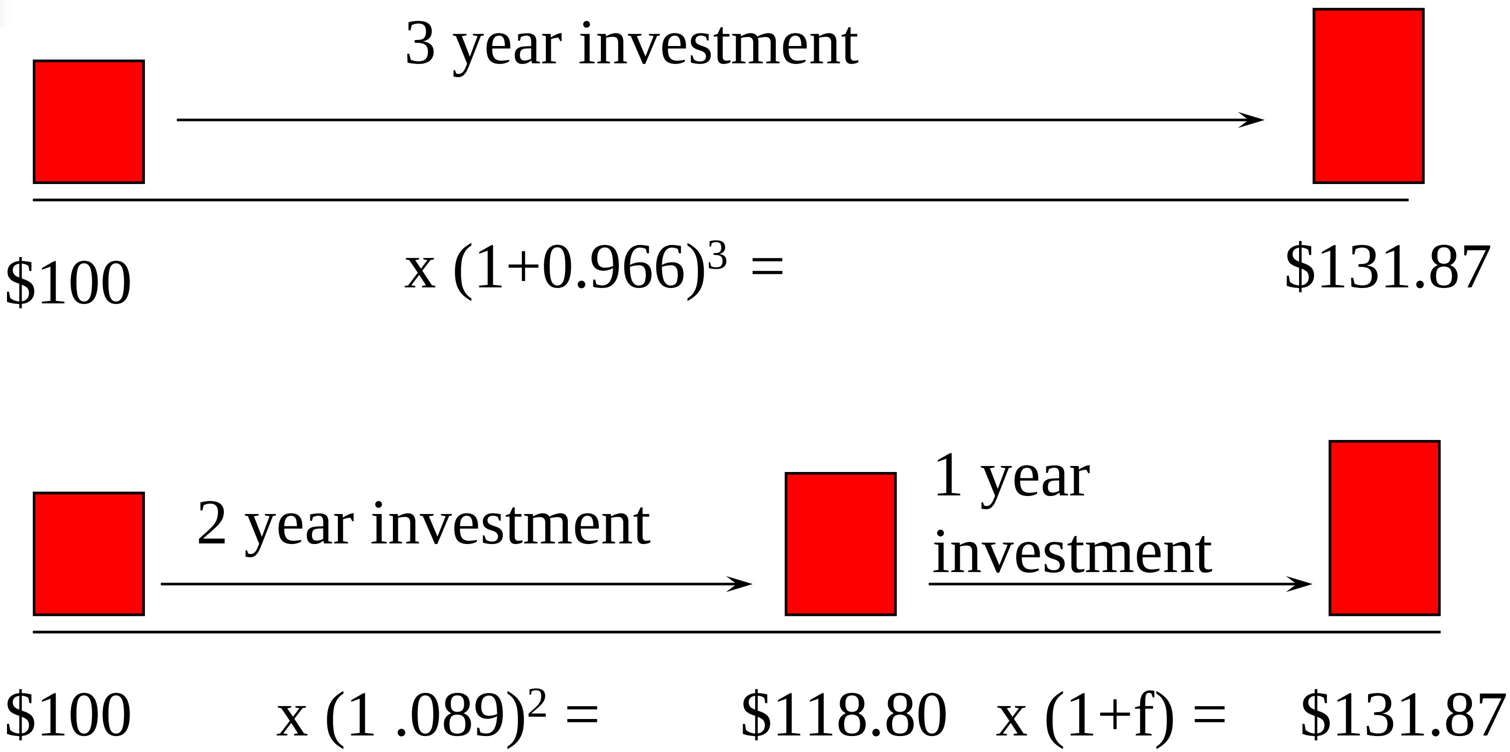



- Interest rates at which you can borrow in future
  - “locking in” interest rates in future
- Define forward rate ${}_1f_2$
  - $(1+r_1)^{t1}(1+ {}_1f_2)^{t2-t1} = (1+r_2)^{t2}$

# Forward Rate Example

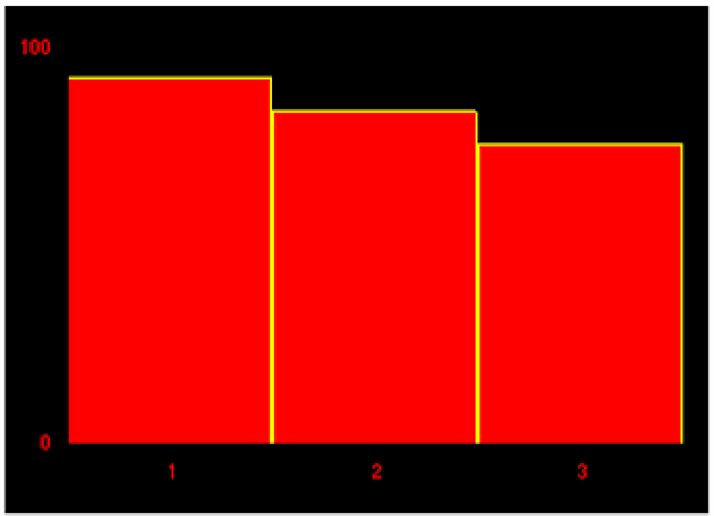
- Example:
  - Treasurer has $100mm to invest for 3 years
  - Alternative 1:
    - Invest for 3 years (at 3-yr spot rate)
  - Alternative 2:
    - Invest for two years (at 2-yr spot),
    - then reinvest at the end of year 2 for one more year
  - Question:
    - What rate will he get in that third year?
  - Spot Rates: Yr 2 = 8.9%, Yr 3 = 9.66%

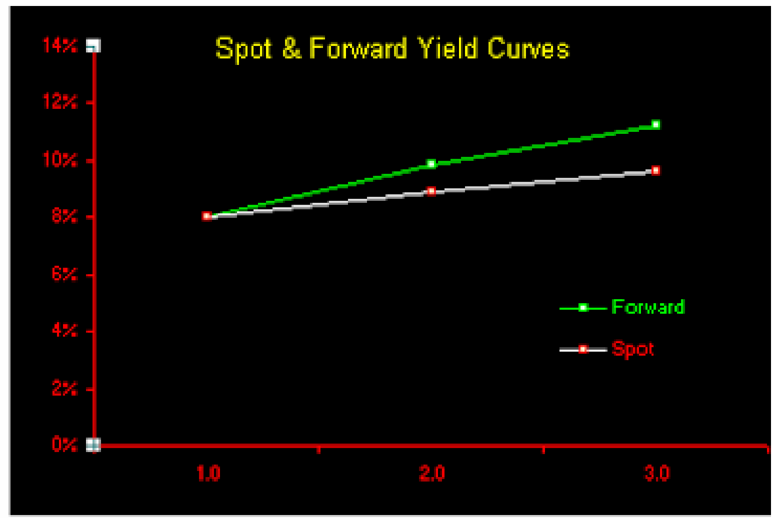
# Forward Rate Example




$$\$131.87 = \$100(1+0.966)^3 = \$118.80(1+{}_2f_3) \Rightarrow {}_2f_3 = 11.2\%$$

# Forward Rates




**Zero Coupon Bonds**

| Maturity | Spot | Forward | Zero | Units |
|---|---|---|---|---|
| 1.00 | 8.000% | 8.000% | 92.593 | |
| 2.00 | 8.900% | 9.807% | 84.323 | |
| 3.00 | 9.660% | 11.196% | 75.832 | |

Default | Portfolio Value

**Forward Contract Delivery Dates**

| BOND | SPOT | 1.00 | 2.00 | 3.00 |
|---|---|---|---|---|
| 1 | 92.59 | | | |
| 2 | 84.32 | 91.07 | | |
| 3 | 75.83 | 81.90 | 89.93 | |

# Forward Contracts

- How do you “lock-in” forward rates?
- Use *forward contract*
- An agreement to exchange a bond:
  - At an agreed future date
  - At a price, F, agreed today

# Forward Contracts

Today — $T_1$ — $T_2$

Agree contract, price F

Delivery

Bond matures

- Forward contract to deliver at $T_1$:
  - Zero coupon bond maturing $T_2$
  - Price F
- $F = 100 / (1 + {}_1f_2)^{T2-T1}$

# Forward Contract Example

- E.g. I arrange to sell you a zero coupon bond:
  - for delivery in two years time
  - maturing at the end of year 3
  - for face value $100
  - at price F
- What is price F?

# Forward Contract Example

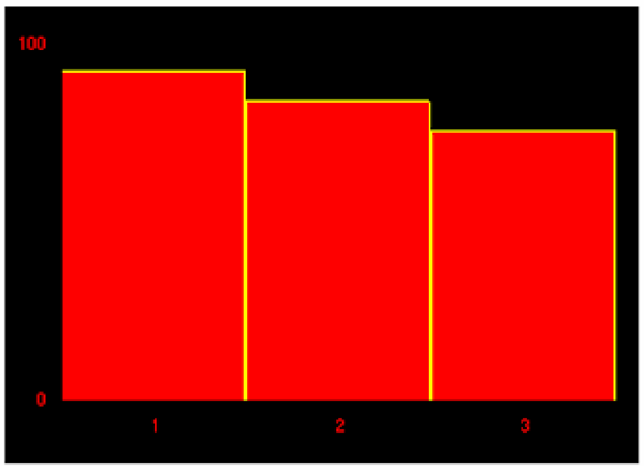

Now:
agree
price F

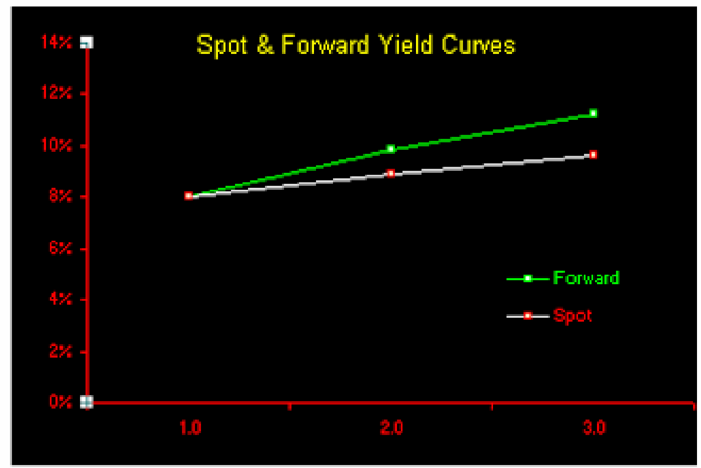
YEAR 2:
Delivery of
zero, pay
price F

YEAR 3:
Receive $100
= $F(1+$f_3$)

- This is just a variant on forward rate example
- F is determined by the forward rate (11.2%)
- $F = 100 / (1+{}_2f_3) = 100/(1.112) = \$89.93$

# Forward Contracts

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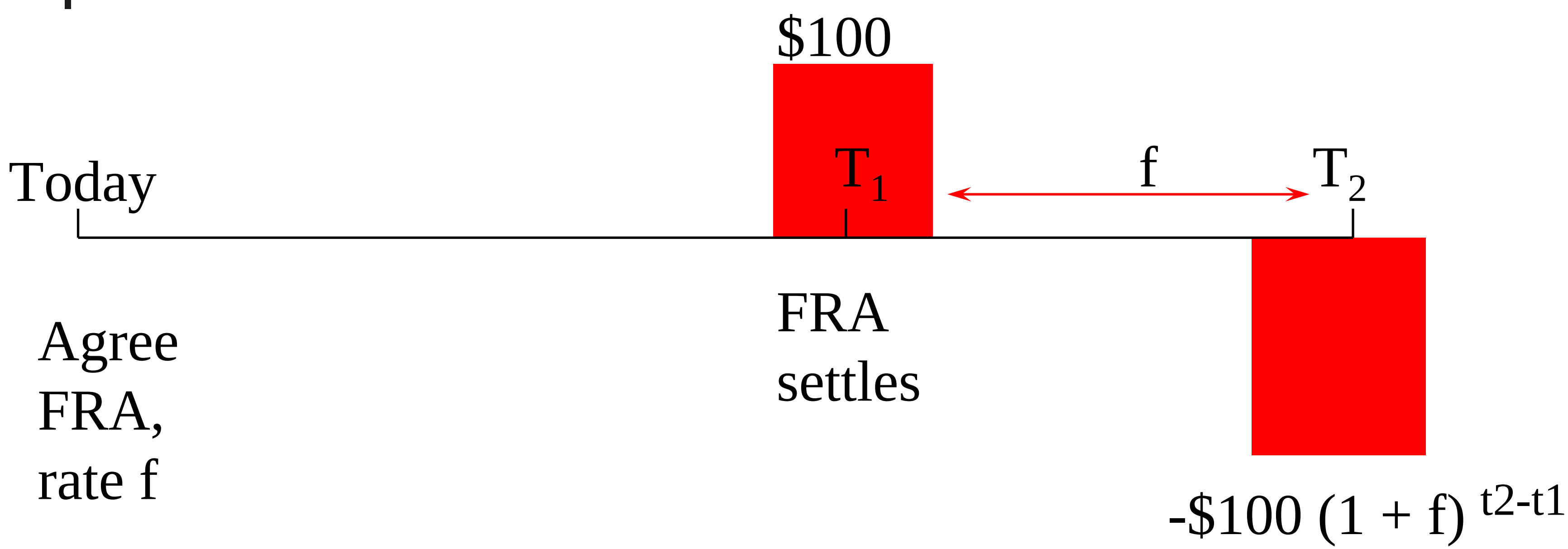
# Forward Rate Agreements

- Like each cashflow of floating side of a swap
  - Agreed forward rate
  - Agreed period
  - Quoted e.g. 9x12
    - Starts in 9 months, applies for 3 month period
  - Buyer pays fixed
    - Protects against rising rates
    - Seller protects against falling rates

# Forward Rate Agreements

- Strips of FRAs
  - Like floating side of swap
  - Used to hedge swaps
  - Used to hedge interest rate risk
- Advantages (vs. futures)
  - No margins
  - Customized dates, amounts
  - Limited credit risk (only net amount exchanged)

# FRA Contract




- The FRA contract rate f is just the forward rate

# FRA Settlement

- Settlement calculated on money market basis:
  - $C = P \times (f - s) \times (T_2 - T_1)/360$
    - P = Notional principal
    - f = FRA contract rate
    - s = spot LIBOR rate at fixing date (usually $T_1$ - 2)
    - $(T_2 - T_1)$ is contract period in days
- Buyer: typically a borrower
- Seller: typically a bank
- Buyer hedges against rising interest rates
    - Receives C if the LIBOR rate s exceeds the FRA rate f

# Money Market Instruments

- Euro-markets
- Certificates of Deposit (CD’s)
- Banker’s Acceptances (BA’s)
- Commercial paper

# Euro-Markets

- Currencies deposited outside country of origin
  - Euro dollar, Euro Yen, Euro Sterling, Euro DM
- LIBOR (London Interbank Offered Rate)
  - Term structure of LIBOR rates

# LIBOR Spot Rates

- Spots quoted as Add-on interest
- Actual/360 daycount
- Example: 3 month deposit
  - Today is Jan 12 1999
  - Deposit matures April 12, 1999
  - Number of days:  91
  - Rate is r, P is principal
- Value at maturity:  P x (1 + r x 91 / 360)

# Forward LIBOR Rates

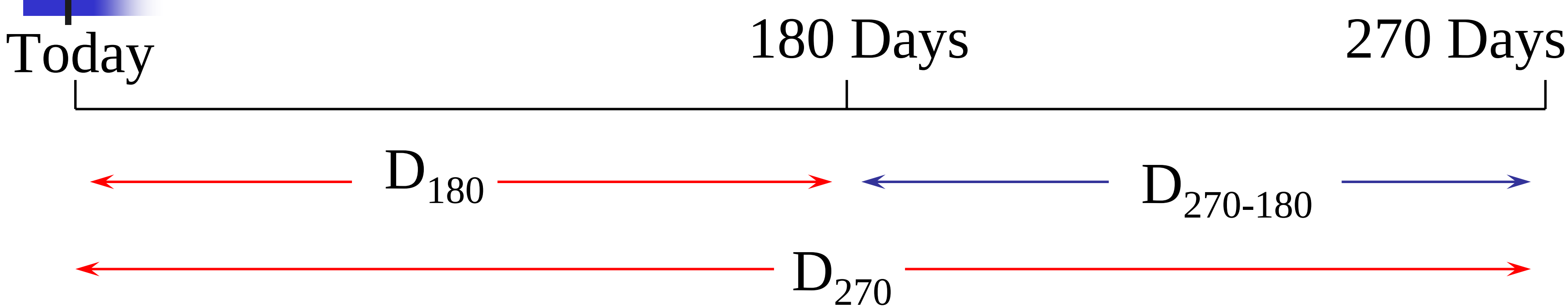
- Principal of equivalent return
  - Deposit @ LIBOR 6-month spot vs.
  - Roll over Two 3-month LIBOR deposits

$$(1 + \text{LIBOR}_{6m} \times \text{Actual Days} / 360) =$$

$$(1 + \text{LIBOR}_{3m} \times \text{Actual Days}/360) \times$$

$$(1 + \text{LIBOR}_{forward} \times \text{Actual Days} / 360)$$

# Forward rates Using Discount Factors

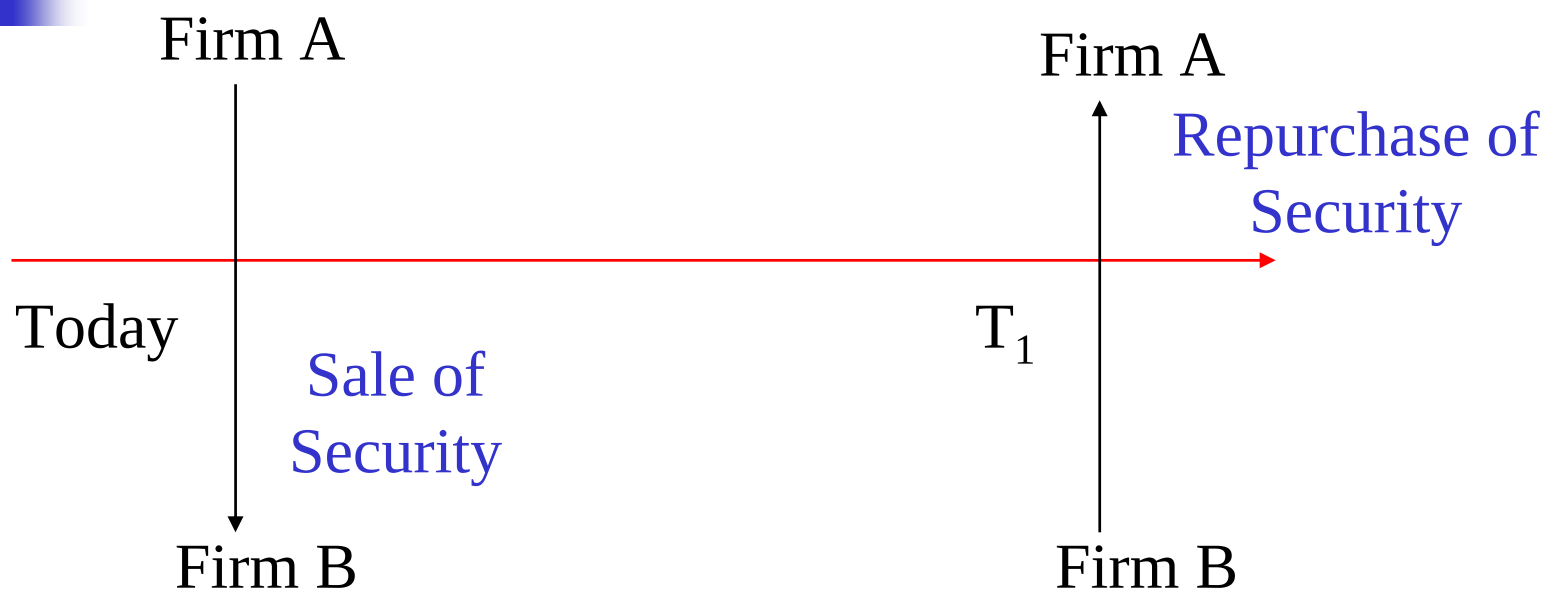



- Discount Factor:
  - $D_{270} = D_{180} \times D_{270\text{-}180}$
- Forward Rate:
  - ${}_{180}f_{270} = (-1 + 1 / D_{270\text{-}180}) \times 360/(270\text{-}180)$

# Other Money Market Instruments

- Certificates of Deposit
  - Negotiable fixed rate interest bearing term instrument
- Bankers Acceptances
  - Discounted time draft drawn on bank
  - Bank “accepts” draft, i.e. assumes responsibility for payment
- Commercial Paper
  - Discount bearer securities issued by corporations

# Repos




- Firm A funds itself by doing a *repo*
  - Pays interest to the buyer at the *repo rate*
- Firm B lends money by doing a *reverse repo*
  - *Regarded as a collateralized loan*

# Repo Trades

- Repo Master Agreement
- Term
  - Mainly short term:  overnight (70%) to 1 week (20%)
  - Long term up to one year (‘Term repos’)
- Repo Rate
  - Can be paid as interest or by setting repurchase price above sale price
  - Simple add-on interest, 360 day year: $(1 + r \times n/360)$
  - Overnight repo rate typically spread below Fed Funds

# Repo Trades

- Securities ("Collateral")
  - Mainly Treasuries & Agency securities, but also CD's BA's, CP, MBS
- Credit risk:  applies to both parties
- Margin  ("Haircut")
  - Good faith deposit paid by borrower to lender
  - Sells securities worth $100, borrows $98
- Right of substitution
  - Borrow may pay extra 2-3 bp for right to offer lender other collateral

# Repo Markets

- Borrowers of collateral (reverses)
  - Mainly dealers wanting to short specific issues
  - The “specials” market
- Lenders of collateral (repos)
  - Banks, S&L’s Munis
- Brokers
  - Garvin, Prebon, Tullet

# Trading Applications

- Customer Arbs
  - Reverses to maturity
- Tails

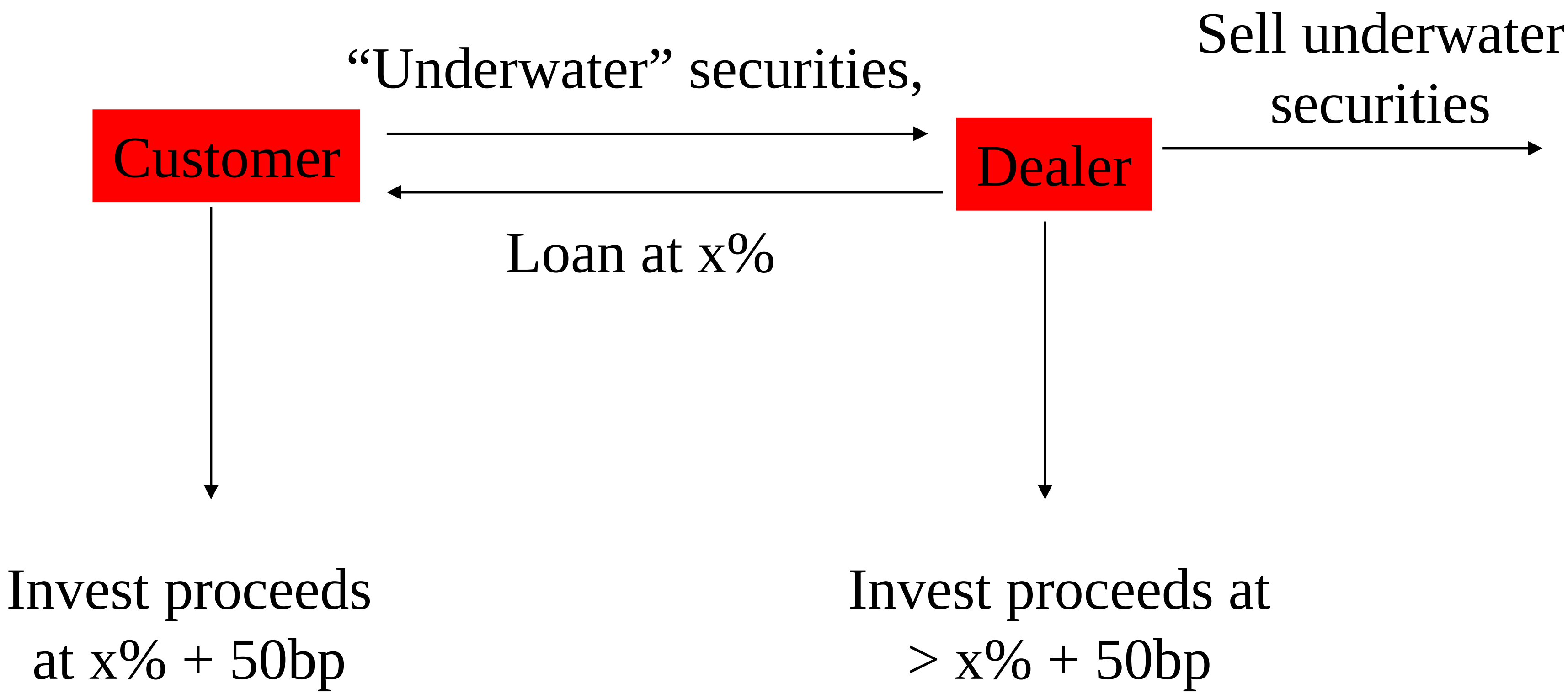
# Customer Arbs

- Reverses to maturity
  - Yields have risen, customer portfolios are underwater
  - Portfolio managers can't take a loss
  - Carrying securities at book value, rather than current lower market value
- Choice:
  - Sell securities, book loss, & reinvest proceeds at higher yields
  - Hang onto underwater securities, avoid booking a loss, earn a lower yield

# Reverses to Maturity

- Dealer offers to reverse in underwater securities for remaining term
  - Sells securities in market
  - Invests proceeds in securities of equal maturity at yield spread above break-even reverse rate
- Customer gets funds at repo rate, re-invests in higher yield securities at e.g. X% + 50bp
- At maturity dealer offsets amount lent to customer (plus interest) against face value of securities he has reversed in (plus final coupon)

# Reverse to Maturity



# Tails

Purchase 90-day bill

0 → 90

Discount rate 5.95%

Finance purchase with 30-day term repo

0 → 30

Repo rate 5.75%

60-day forward bill

30 → 90

Effective discount rate ??

(current 60-day bill yield is 5.80%)

# Lab: Figuring the Tail

- Current 90-day bill yield is 5.95%
- 30-day term repo rate is 5.75%
- Earn 20bp carry by repo-ing the 90-day bill
- Effectively creates a 60-day bill in 30-days time
- What is the effective discount rate on this forward bill?
  - Current 60-day bill yield is 5.80%

# Figuring the Tail

- Effective yield on future security =

  Yield on cash security purchased +

  (Carry x Days carried / Days left to maturity)

- Yield = 5.95% + (0.20% x 30 / 60) = 6.09%
- Profit = 6.09% - 5.80% = 0.29%
- Will do trade if Fed doesn't tighten or spreads don't change unfavorably by more than 29bp

# Cash and Carry Trade

- Create the tail as before
  - Buy cash bill
  - Finance with term repo
- Sell the tail forward using *bill futures*
- Break-even repo rate is called the *implied repo rate*
- Trade is profitable when current repo rate is less than the implied repo rate.

# Cash & Carry Trade - Example

- March ‘98 T-Bill
  - 147 days to maturity
  - Discount rate is 4.93%
- Dec ‘97 T-Bill futures contract
  - Expiry in 56 days
  - Futures price 95.09
- What is the implied repo rate?
- If the 56-day repo rate is 4.83%, calculate the $ profit per $1MM on the cash and carry trade

# Cash & Carry Trade - Solution

- Purchase 147-day bill at $979,869
- Sell Dec futures contract at $987,589
- Implied repo rate:
  - (979,869 - 987,589) x 360/56 = 5.06%
- Profit on C&C Trade:
  - (5.06% - 4.83%) x $1MM x 56/360 = $357

# Interest Rate Futures

- Contract for future delivery of specific security
- Standard:
  - Contract size
  - Maturity dates
- Exchange traded
  - Market to market daily
  - Traded on margin
- Short term Euro-currency futures are cash settled
- Often liquid in near months

# Eurodollar Futures


- Contract Size = $1,000,000
- 90 day Eurodollar rate
- Price = 100 - f  (futures interest rate)
  - Futures price = 94.5,  rate = 5.5%
    - This is not exactly the forward rate
- Minimum price move = 0.01 = one tick
- A move of one tick represents a gain/loss of $25:
    - (0.01%  x  $1,000,000 x 90/360) = $25

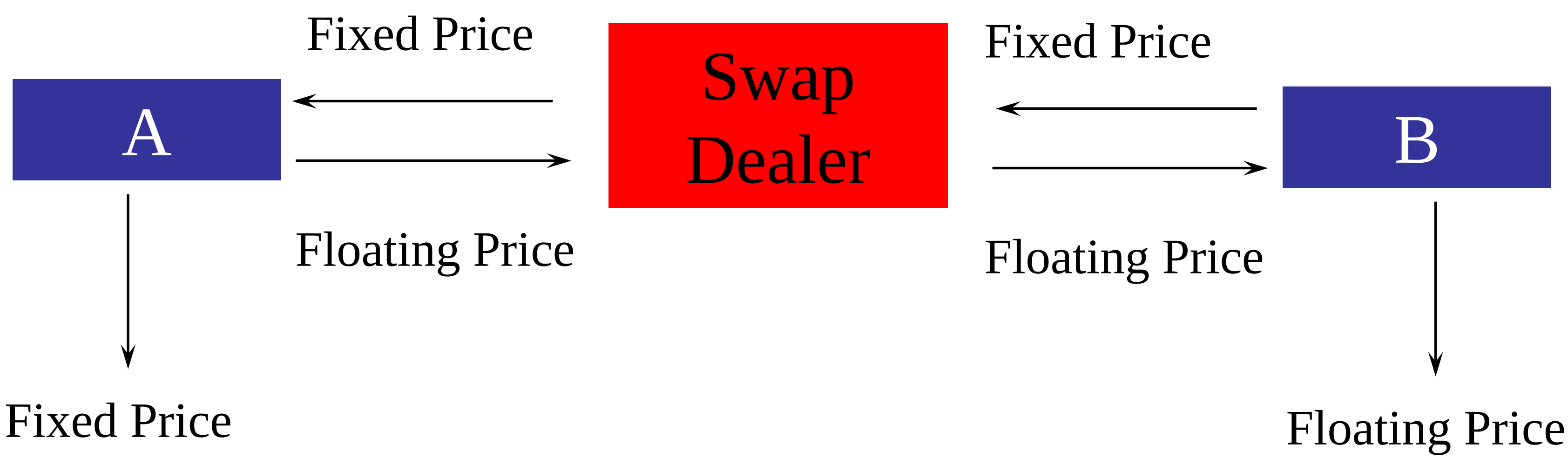
# Trading Case B03

- See if you can value forward contracts
  - Use a spreadsheet for calculations
  - Then, trade

# Swaps

- Basic Structure
- Pricing
- Applications

# A Generic Swaps Structure




- Counterparty A converts from fixed to floating
- Counterparty B converts from floating to fixed

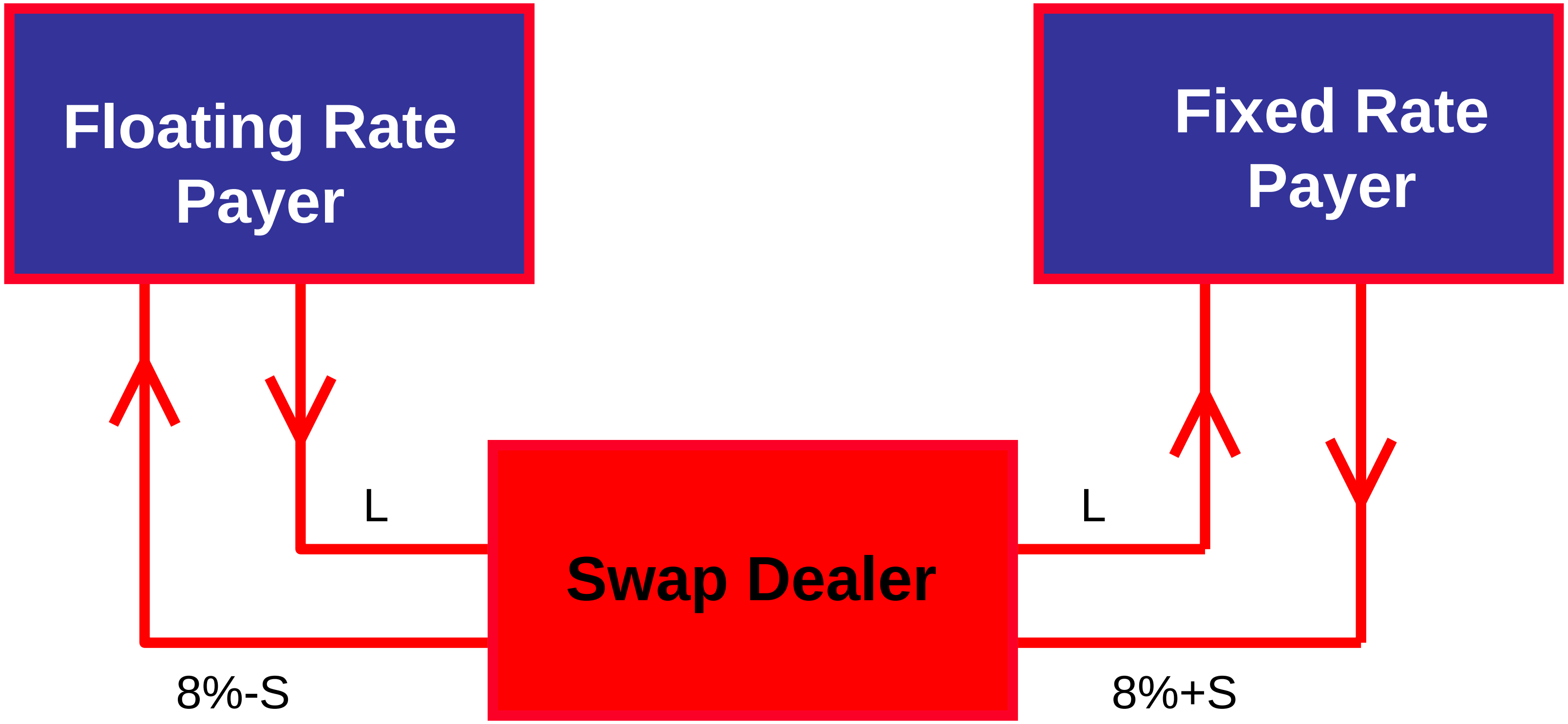
# Vanilla Interest Rate Swap

- Notional principal  $100m
- Fixed rate : 8%, quarterly
- Floating rate: LIBOR, quarterly
- Tenor: 2 years
- One side pays fixed, the other pays floating
  - Betting on movements in LIBOR

# Vanilla Interest Rate Swap

- Every quarter, fixed payer owes approx. \$200k = \$100m*.08/4
- Every quarter, if LIBOR is L, floating payer owes approx. \$100m*(L/4)
- Only net cash flows are exchanged
  - Through an intermediary who charges a spread
  - Payments are in arrears:
    - interest rate known in advance
    - interest due is paid at end of each period

# Vanilla Swap Structure



# Vanilla Swap Quotes

- Quotations
  - Fixed rate usually quoted
    - set so present value of swap is zero
    - called the “swap coupon” or “swap rate”
  - Quoted as a rate, e.g. 8%
    - Swap curve
  - Quoted as spread over a “reference rate”
    - e.g. Treasury of same maturity plus a spread

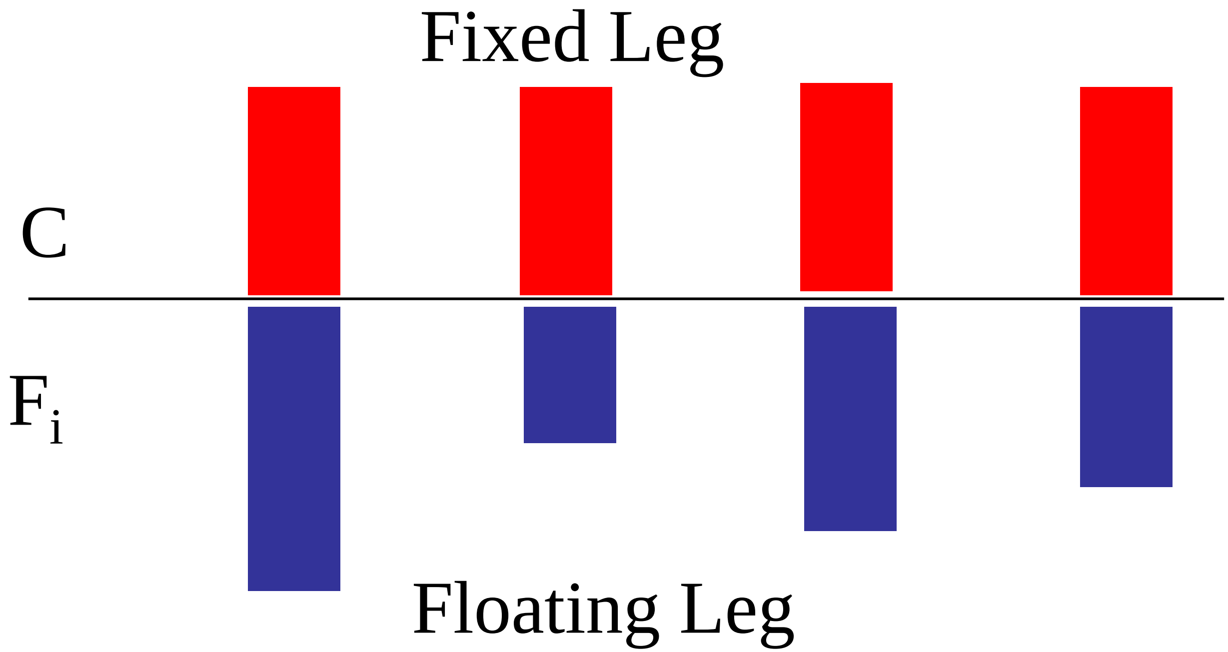
# Swap Quotes

| Term | Bid-Offer | Offer Side Yield | Swap Spread |
|---|---|---|---|
| 2 yr | 99.16-17+ | 8.252 | 68 - 72 |
| 3 yr | 99.08-09+ | 8.402 | 68 - 73 |
| 4 yr | 98.30-31+ | 8.556 | 68 - 73 |

- Swap coupon = Treasury + Spread
  - e.g. 2 yr: 8.252 + 68-72 = 8.932 - 8.972
  - Dealer pays bid coupon (8.932%), receives offer side coupon (8.972%)
  - Other leg is 6-month US$ LIBOR

# Swap Pricing

- Find fixed coupon rate c, such that PV of fixed payments = PV of floating leg cash flows



# Swap Pricing Formula

- Fixed Rate Payments:
  - C = NP x c x n / 360
    - c is the *swap coupon* %
- Floating Leg Payments:
  - Variable payments $F_i$ = NP x $f_i$ x n / 360
    - $f_i$ is the *forward rate* for period i
- Determine Swap Coupon, c, by:

$$NPV = \sum_{1}^{N} \frac{(F_i - C)}{(1 + r_i)} = 0$$

# Lab: Pricing a Vanilla Swap

| | |
|---|---|
| Notional principal amount | $100,000,000 |
| Effective date | September 22, 1994 |
| Day count between each reset date: | |
| December 22, 1994 | 91 days |
| March 22, 1995 | 90 days |
| June 22, 1995 | 92 days |
| September 22, 1995 | 92 days |
| Maturity date | September 22, 1995 |
| Interest settlements are in arrears. | |
| Fixed Side (Leg): | |
| Fixed-rate (Swap Coupon) | 6.1220% |
| Compounding frequency | quarterly |
| Day count | 90/360* |
| Floating Side (Leg): | |
| Reference Rate | 3-month LIBOR |
| Payment frequency | quarterly resets |
| Day count | actual/360 |
| First Coupon | 5.25% |

* Assumption: Fixed Side Cash Flows Equal over Time

# Key Steps

- Step 1:  Project cash flows
  - Contract specifies timing and magnitude of cash flows
- Step 2:  Value cash flows
  - Apply time value of money principles

# Step 1: Cash Flow Projections

| Quarter | LIBOR | Forward Rate* | Expected Variable Interest** |
|---|---|---|---|
| December | 5 1/4 | 5.25 | $1,327,083 |
| March | 5 11/16 | 6.0496% | $1,512,395 |
| June | 5 15/16 | 6.2506% | $1,597,378 |
| September | 6 3/16 | 6.6308% | $1,694,535 |

*LIBOR  Forward Rates computed using actual/360 day count.
**Unbiased Expectations

# Step 2: Discounting Cash Flows

- LIBOR is quoted in an add-on form
- Must use LIBOR spot discount factors

# PV Floating Side

| Quarter | LIBOR Yield Curve | Effective Annual Yield (360 days)* | Expected Floating Rate Payments | Present Value |
|---|---|---|---|---|
| December | 5 1/4 | 1.053539 | $1,327,083 | $1,309,702.4 |
| March | 5 11/16 | 1.057679 | $1,512,395 | $1,470,349.2 |
| June | 5 15/16 | 1.059797 | $1,597,378 | $1,528,553.1 |
| September | 6 3/16 | 1.061849 | $1,694,535+ $100,000,000 | $95,691,395.3 |
| Total PV | | | | $100,000,000 |

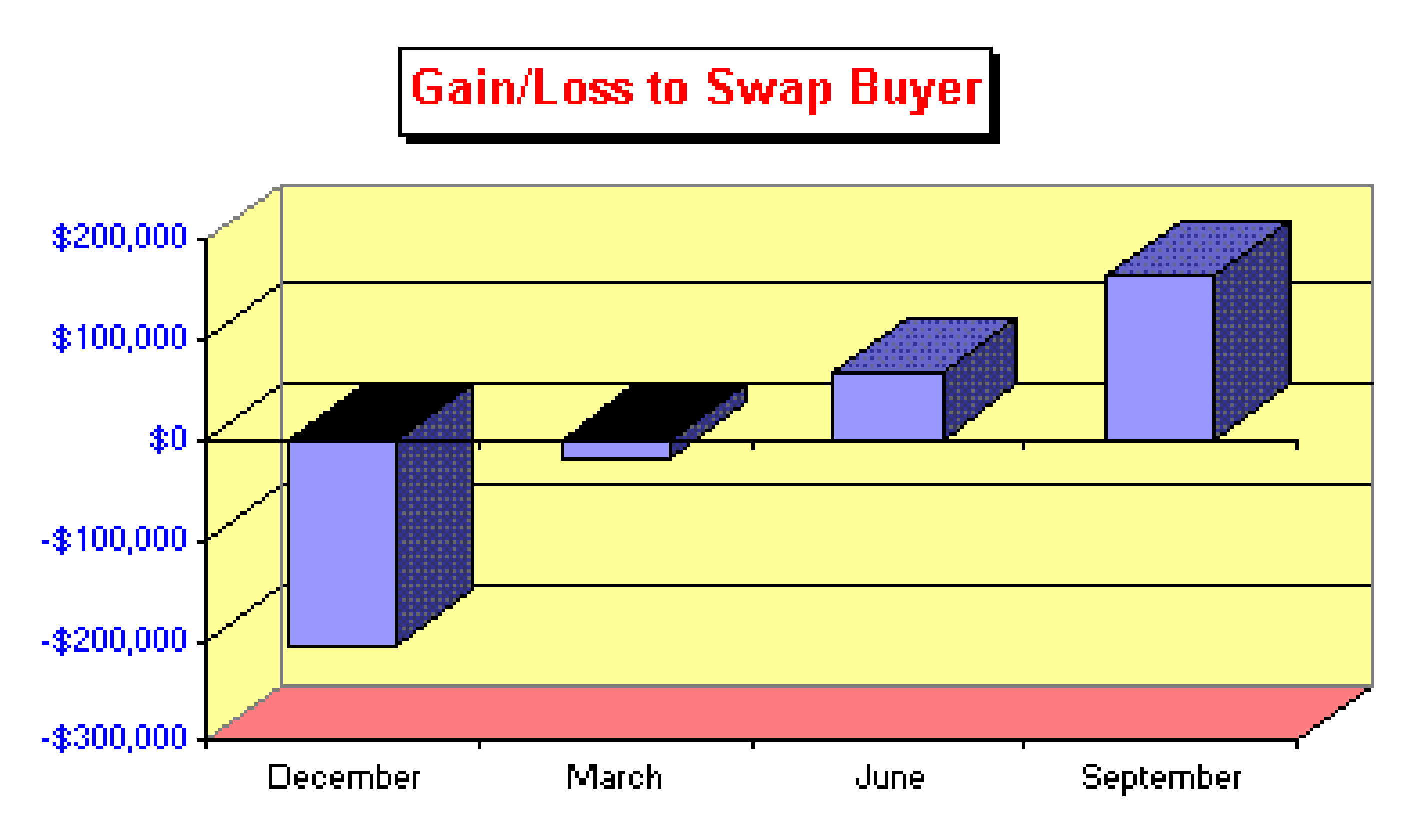
P.V. at EAY = Notional

*Actual/360 Daycount

# Swap Price

| Qtr | LIBOR Yield Curve | Fixed Interest @ 6.1219933% | Present Value @ EAY |
|---|---|---|---|
| Dec | 5 1/4 | $1,530,498 | $1,510,453 |
| March | 5 11/16 | $1,530,498 | $1,487,950 |
| June | 5 15/16 | $1,530,498 | $1,464,555 |
| Sept | 6 3/16 | $101,530,498 | $95,537,042 |
| Total | Both legs exactly equal | | $100,000,000 |

# Buyer: Net Exposure



# Swap Spreads

- Short term: reflects Eurodollar yield curve
  - Hedge/arbitrage using FRA's or futures
  - Some variation due to counterparty risk of swap
- Long term
  - Liquidity in FRA's & futures lower
  - Hedging/arbitrage no longer possible
  - Swap spreads determined by cost of borrowing alternatives

# Long Term Swap Spreads

- Weak Credit
  - Could issue 10 year note
  - Alternative: borrow floating rate @ (LIBOR + spread), then swap
  - Swap Rate + Loan Spread < $\text{Note}_{\text{weak credit}}$
- Strong Credit
  - Can raise short term funding through CP
  - Alternative: issue 10 year note, receive swap rate from weaker counterparty, pay LIBOR
  - Swap Rate - $\text{Note}_{\text{strong credit}}$ > LIBOR - CP Rate

# Long Term Swap Spreads

$$\text{Note}_{\text{strong credit}} + (\text{LIBOR} - \text{CP Rate}) <$$

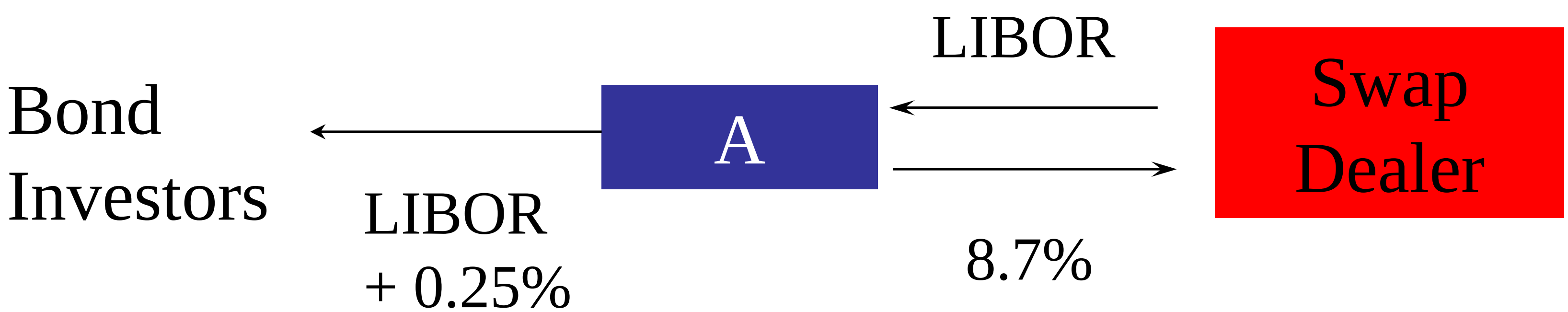
$$\text{Swap Rate}$$

$$< \text{Note}_{\text{weak credit}} - \text{Loan Spread}$$

# Uses of Swaps

- Financing Tool/Swap Arbitrage
  - New Issue Arbitrage
- Tailoring Portfolio to Expectations
  - Market timing
- Hedging and Risk Management
  - ALM

# Lower Fixed Rate Financing




- Example: Cost of fixed rate finance, F = 9.0%, Swap coupon, C = 8.70%, + spread, S = .25%

| | | |
|---|---|---|
| Swap fixed leg | . . . | 8.70% |
| Floating rate bond | . . . | LIBOR + 0.25% |
| Less: Swap floating leg | . . . | (LIBOR) |
| Net fixed cost | | 8.95% |

# Sources of Arbitrage

- Financial Arbitrage
  - Credit spreads
    - Match two parties; one can lower fixed rate, one can lower floating rate.
    - If there are enough gains, then both sides (and the intermediary!) can benefit.
- Restrictions on investments
  - Fund can only invest in AAA bonds, creates yield differentials
- Intermarket arbitrage
  - AAA-BBB spread greater in USA than Euromarket

# Sources of Swap Arbitrage

- Tax and Regulatory arbitrage
  - e.g. capital gains tax applied to Japanese Zeros
  - Also, restrictions on holdings of foreign ZCBs
  - Dual currency bonds not restricted
  - So issued DCB's , principal in US$, coupon in Yen
  - Swap Yen coupons into US$
  - Attractive yield since Japanese institutions willing to pay premium to overcome restrictions

# Yield Curve Plays

- Expectations of lower rates
  - Buy an Inverse Floater
    - e.g., floating rate = 12% - LIBOR
- Protection against higher rates
  - Super floater: pay fixed, receive multiple x LIBOR
- Swap between short-term and long-term if curve expected to flatten or steepen
- Basis Swaps: Swap different floating rates
    - if spreads expected to narrow or widen
    - Treasury-Euro$ (TED) spread

# Asset Liability Matching

- Short term assets, long term liabilities
  - Maturity gap
- Yield curve swap: receive short, pay long
  - Maturity gap is more stable
- Applies to banks, pension plans, life insurance companies, finance companies

# Swaps - Summary

- Structure
- Pricing
- Applications
- Next: Yield curve modeling