



Derivatives Risk Management

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Investment Analytics



Agenda

- Sensitivity factors
- Delta
 - Delta hedging
- Option time value
- Gamma and leverage
- Volatility sensitivity
- Gamma and Vega hedging



Option Sensitivity Factors

- What affects the price of an option
 - the asset price, S
 - the volatility, σ
 - the interest rate, r
 - the time to maturity, t
 - the strike price, X



Greeks

- Delta (“price sensitivity”)
 - change in option price due to change in stock price
- Gamma (“leverage”)
 - change in delta due to change in stock price
- Vega (“volatility sensitivity”)
 - change in option price due to change in volatility
- Theta (“time decay”)
 - change in option value due to change in time to maturity



Option Delta

- Key sensitivity
 - Change in option value for \$1 change in underlying stock
 - Range -1 to $+1$
- Option Delta
 - Put options: negative delta
 - Call options: positive delta



Delta Example

- Call option with Delta 0.5
- Delta tells us how the call price changes
 - If stock moves up by \$10, call price increases by \$5
 - If stock drops by \$10, call price decreases by \$5



Delta Position

- Delta Position

- The call 'behaves' like 0.5 units of stock
- If we hold 10 calls, our position behaves like 5 units of stock
- we say we are "long 5 deltas"



Position Deltas

- Call options
 - have positive delta
 - like being long stock
- Put options
 - have negative delta
 - like being short stock
- Stock - has a delta of 1!
- Bonds - have a delta of zero
- Combinations
 - may have positive, negative or zero delta



Delta Hedging

- If you know the position's delta, you can hedge it
- Example, 10 calls, each of delta 0.5
- How to hedge: Sell Delta units of stock
 - This creates a portfolio of 10 calls plus -5 units of stock
 - The combined position has a delta of $+5 - 5 = 0$
 - Like being long 5 stock & short 5 stock = net zero stock
 - The value of the portfolio will be unchanged, no matter whether the stock moves up or down



Delta Neutral Positions

- A portfolio is *hedged* when its position delta is zero
- We say we have a *delta-neutral* position
- Examples: assume call delta 0.5, put delta -0.5
 - Long 10 calls, short 5 stock
 - Long 10 calls, long 10 puts
 - Short 10 calls, long 5 stock
 - Short 10 puts, short 5 stock
 - Short 10 puts, short 8 calls, short 1 stock

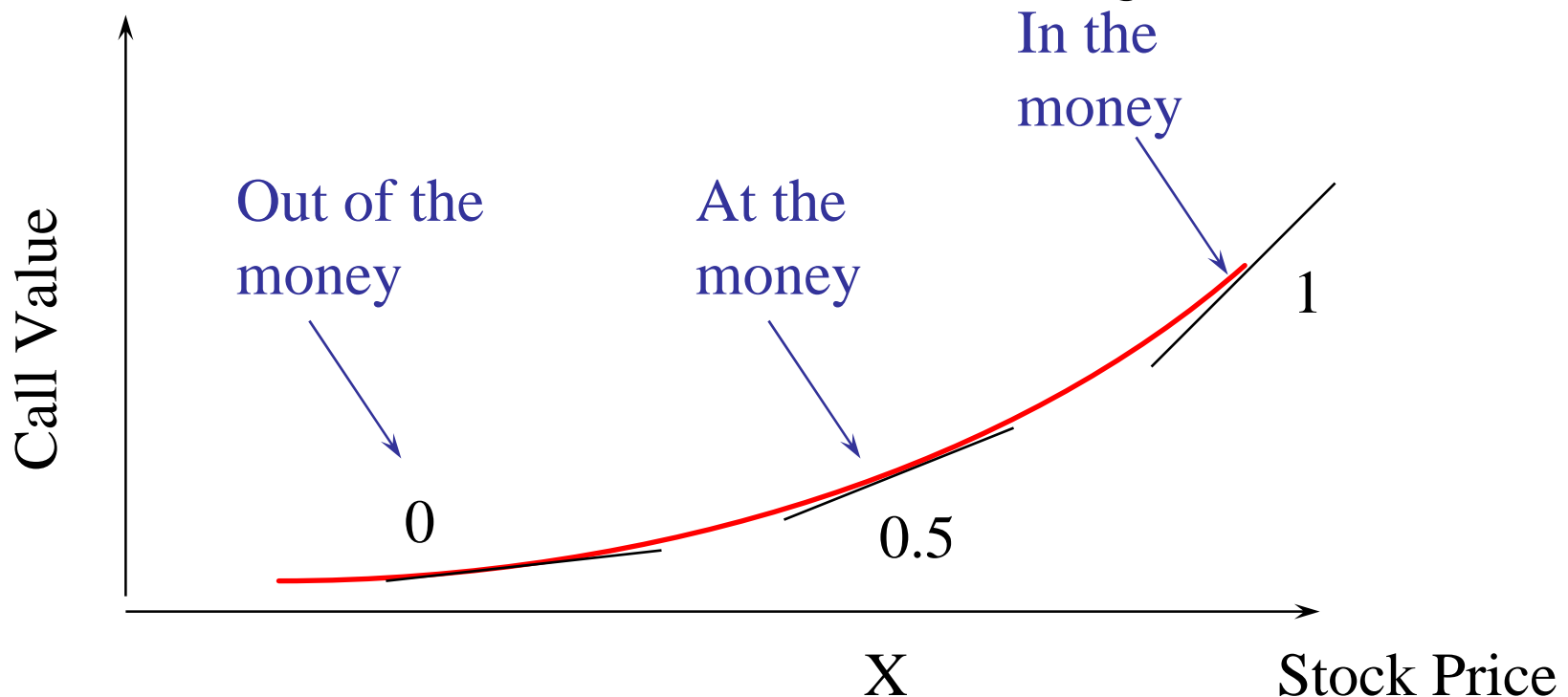


Delta Neutral Strategies

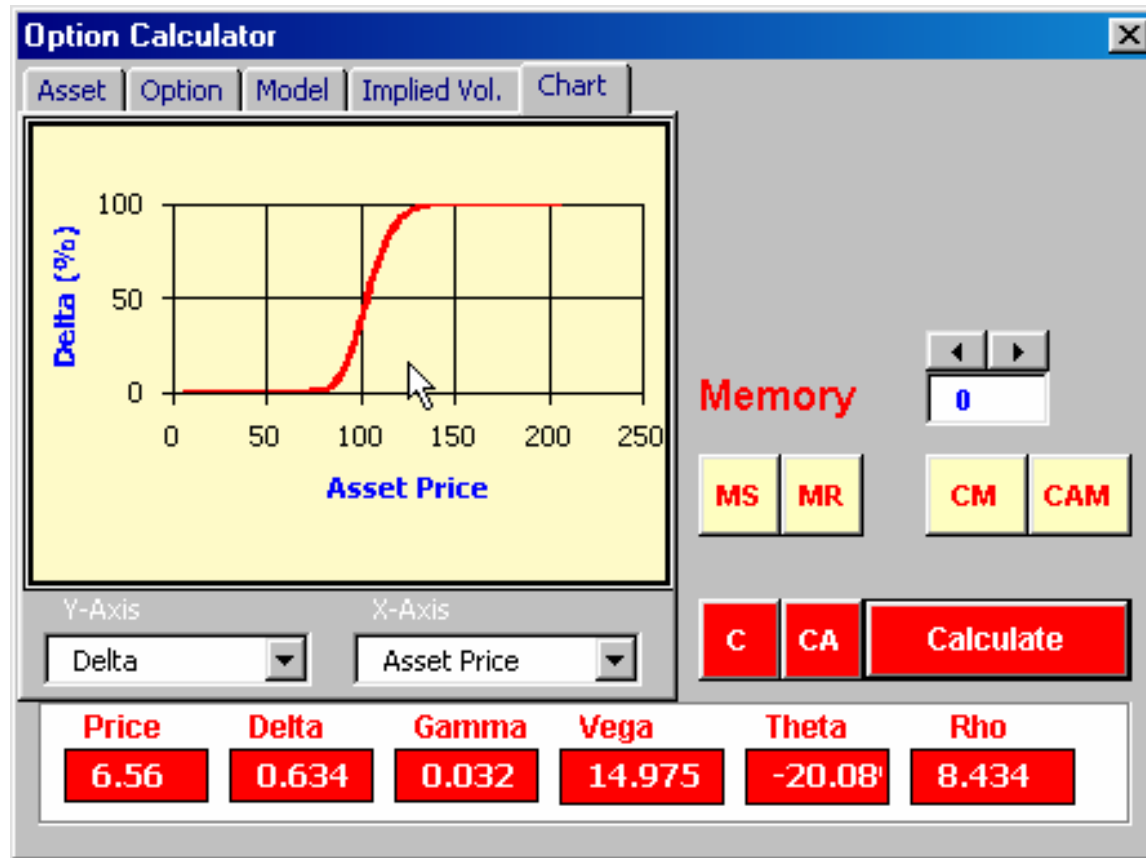
- *Delta neutral strategies* are non-directional
 - Often make money while market doesn't move
- Examples:
 - Butterflies, straddles, strangles are typically delta neutral
- *Directional* strategies are typically not delta-neutral
 - Call spreads have +ve delta
 - Put spreads have -ve delta

The Delta of a Call Option

- Delta changes as stock price changes
 - *Gamma* measures rate of change of delta



IBM Option Delta

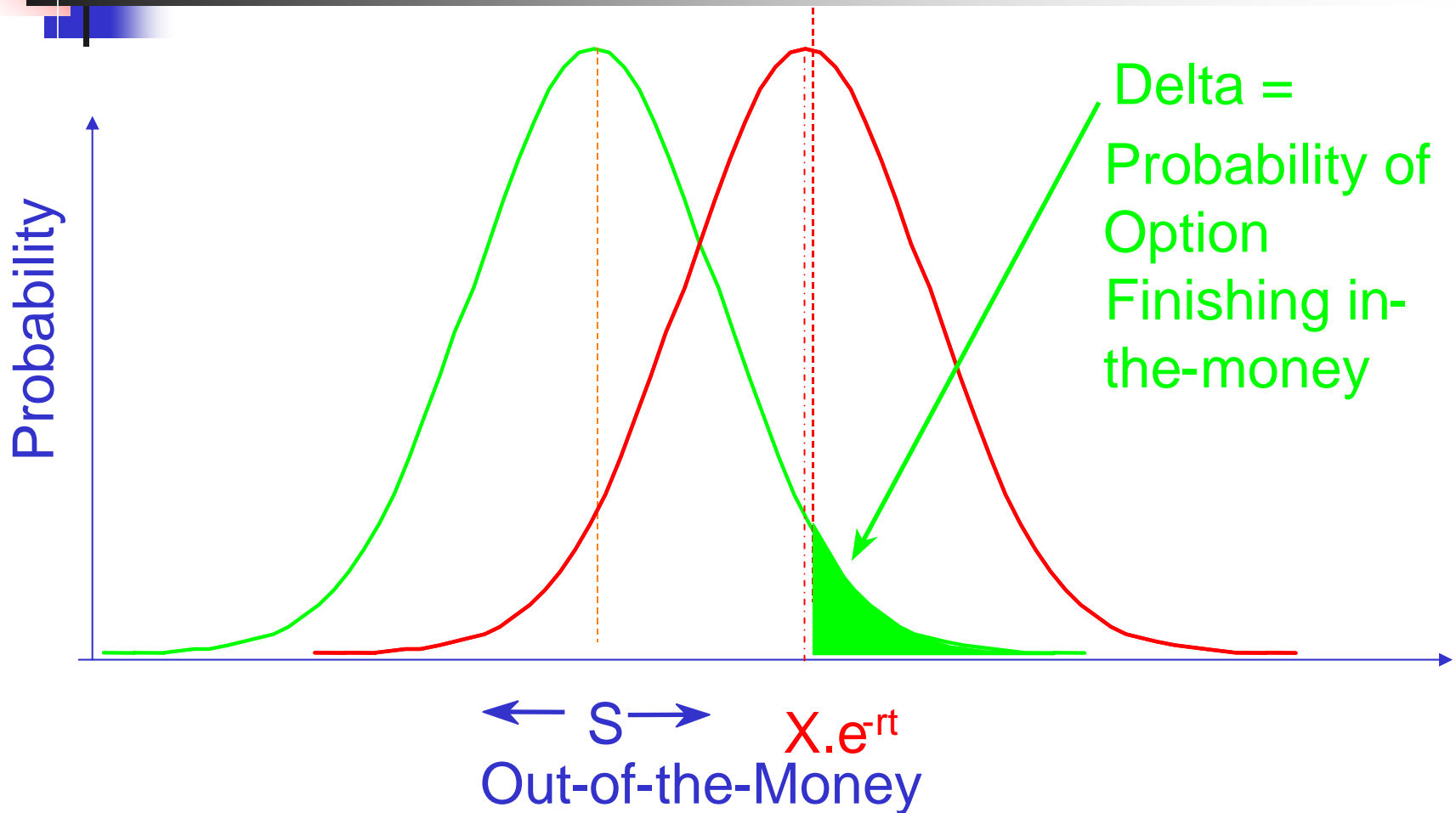




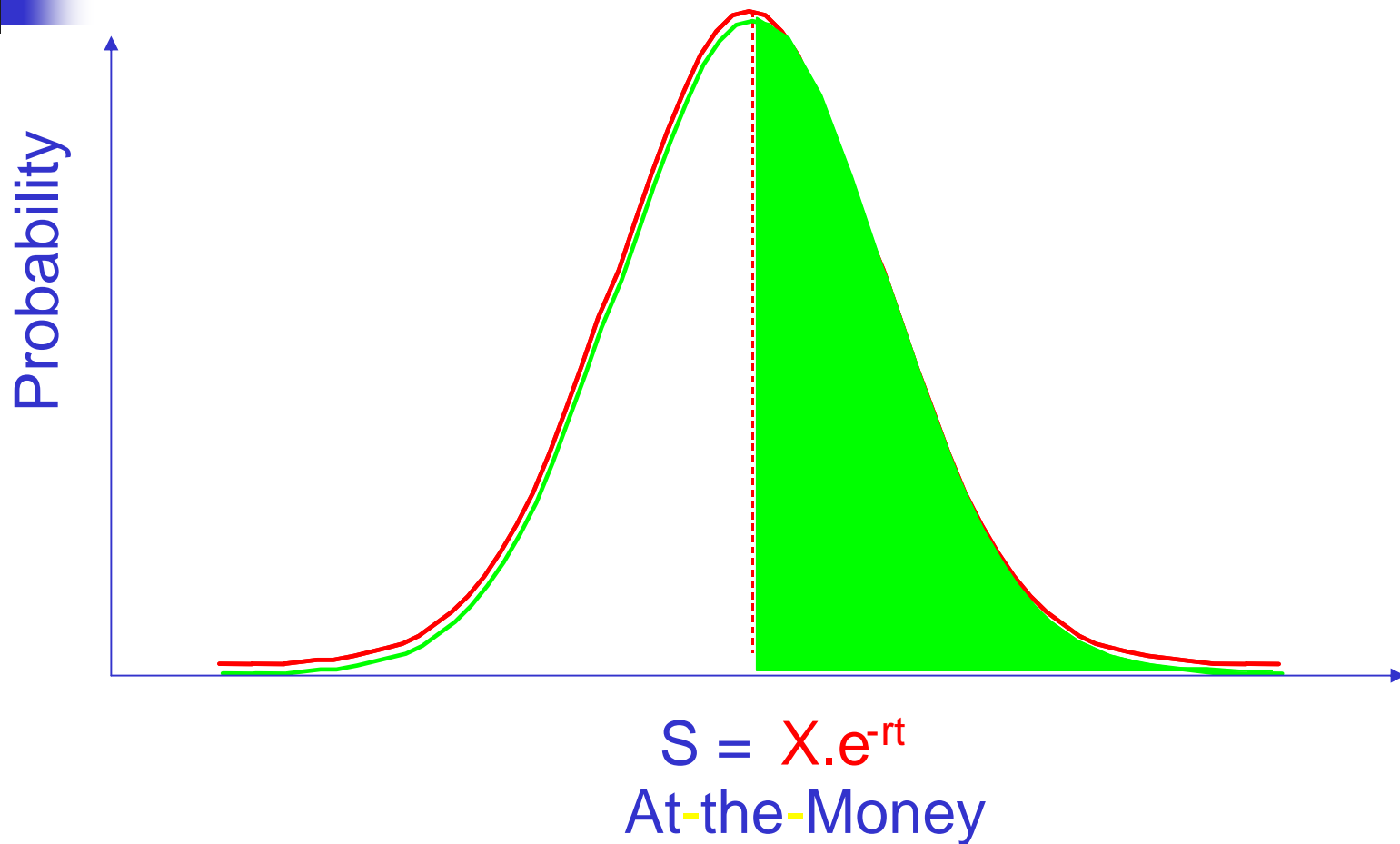
Worked Exercise: Delta Hedging

- Option Tutor
- Subject: Binomial delta hedging
- Worked Exercise

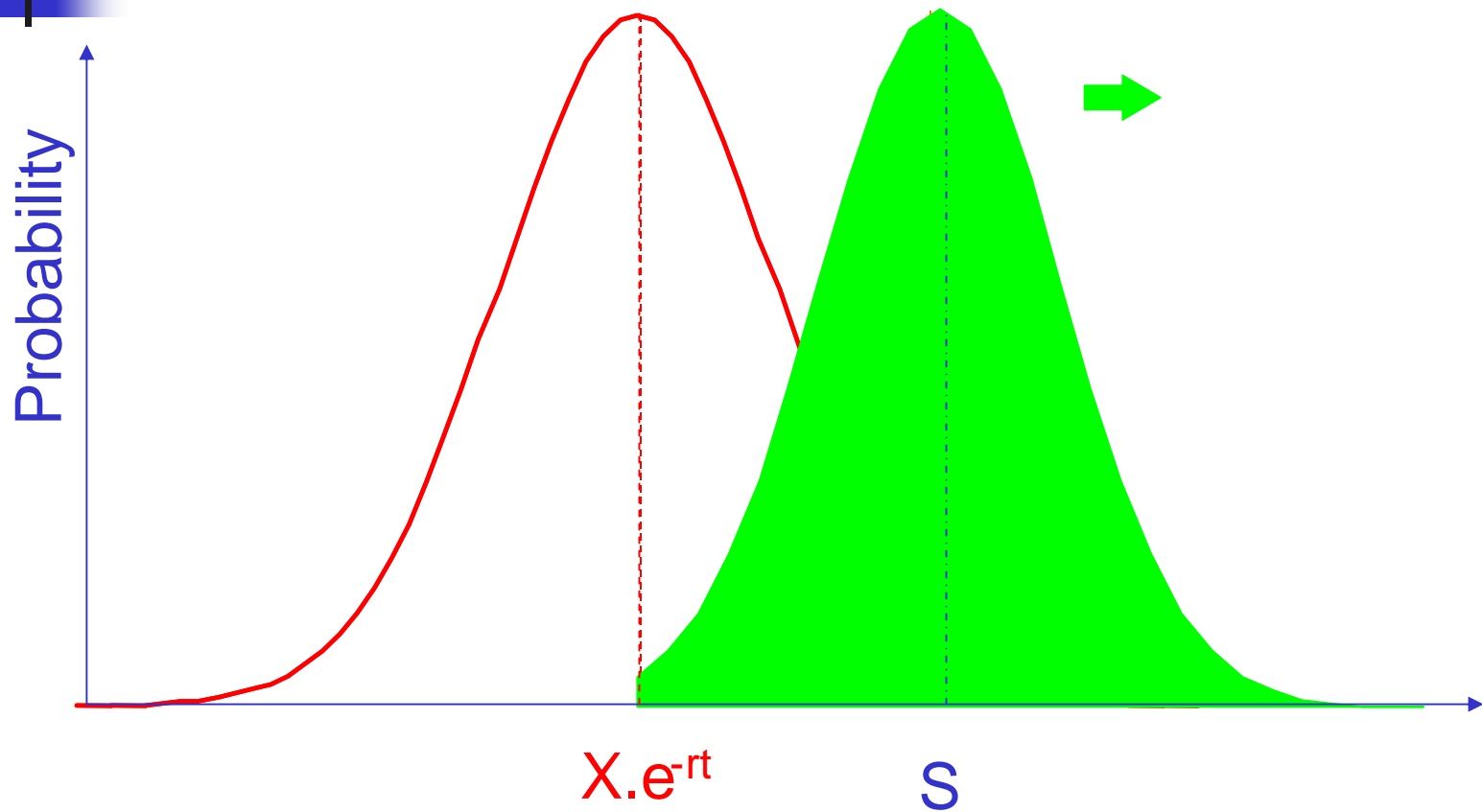
A New Look at Delta



Delta: At-the-Money



Delta: In-The-Money



In-the-Money



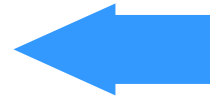
Delta: Conclusions

- Delta “measures probability of finishing in the money”
- A deep out-of-the money option has a low delta
- An at-the-money has a delta of around 0.5 as it approaches expiry
 - i.e. a 50-50 chance of ending up in the money
- As an option moves deep into the money, its delta rapidly approaches 1
 - Gamma measures rate of change of Delta



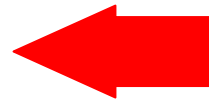
Option Value

■ Intrinsic Value



- Asset Price
- Strike Price
- Interest rates

■ Time Value

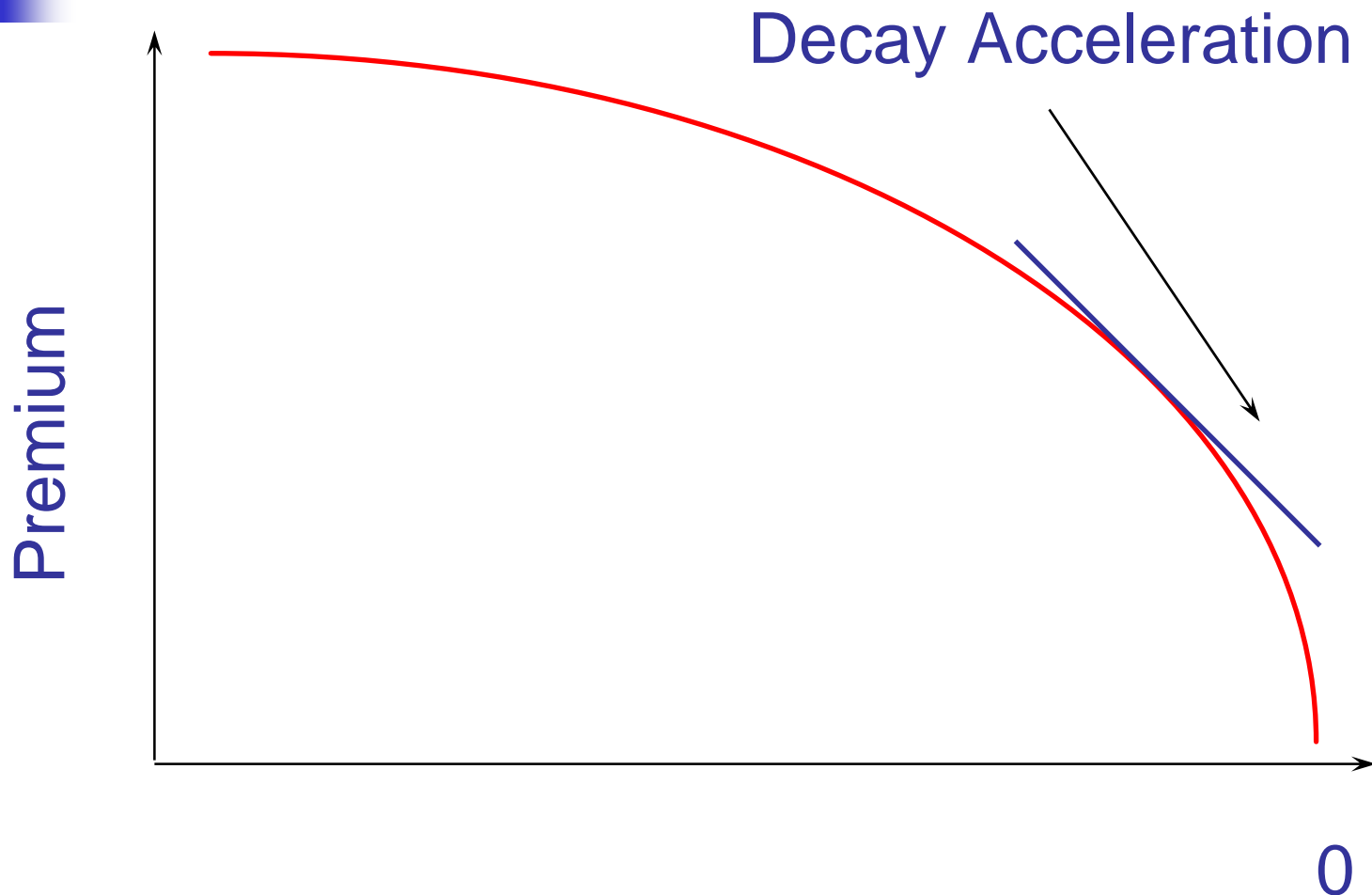


- Time to Expiration
- Volatility

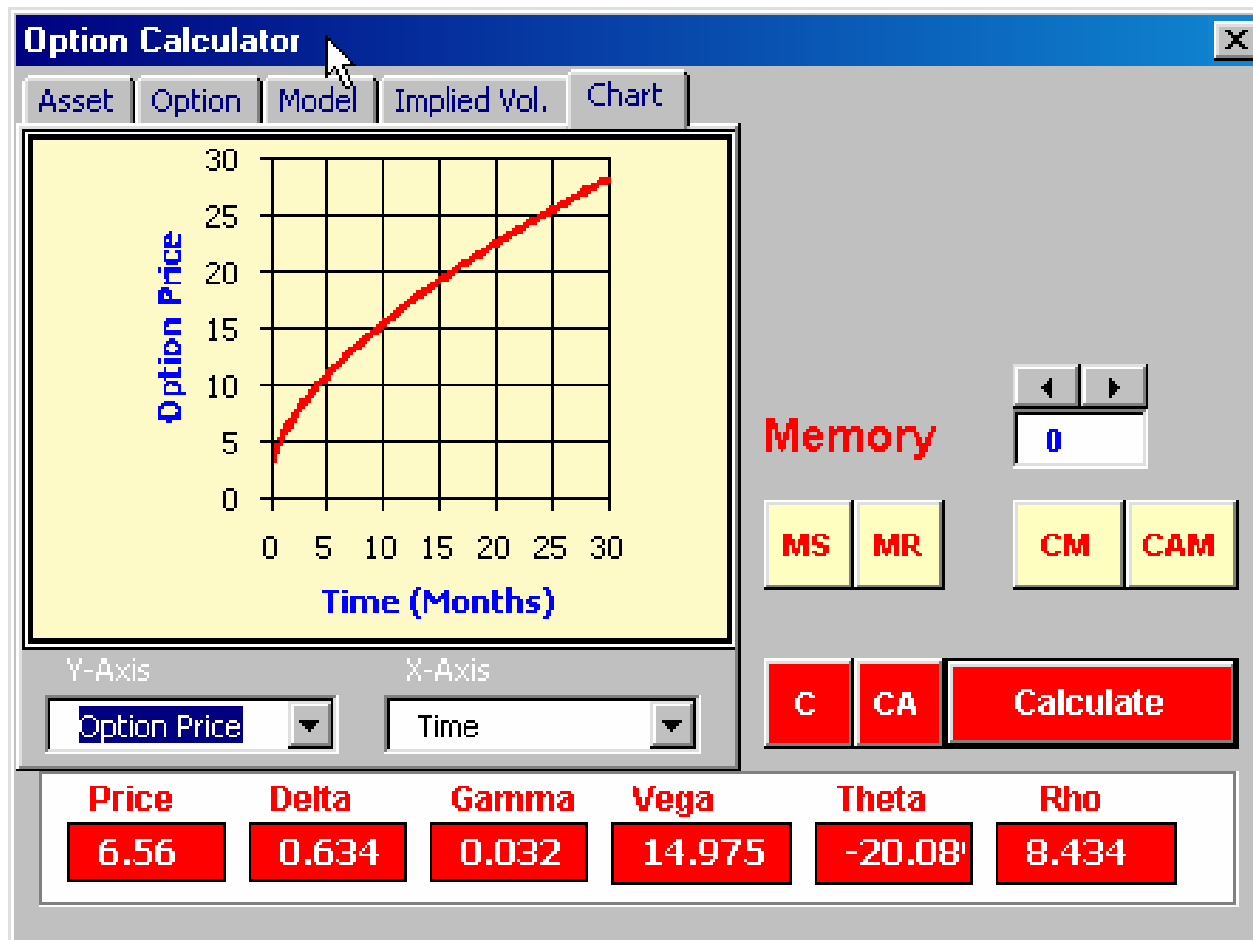
How Time Value Decays



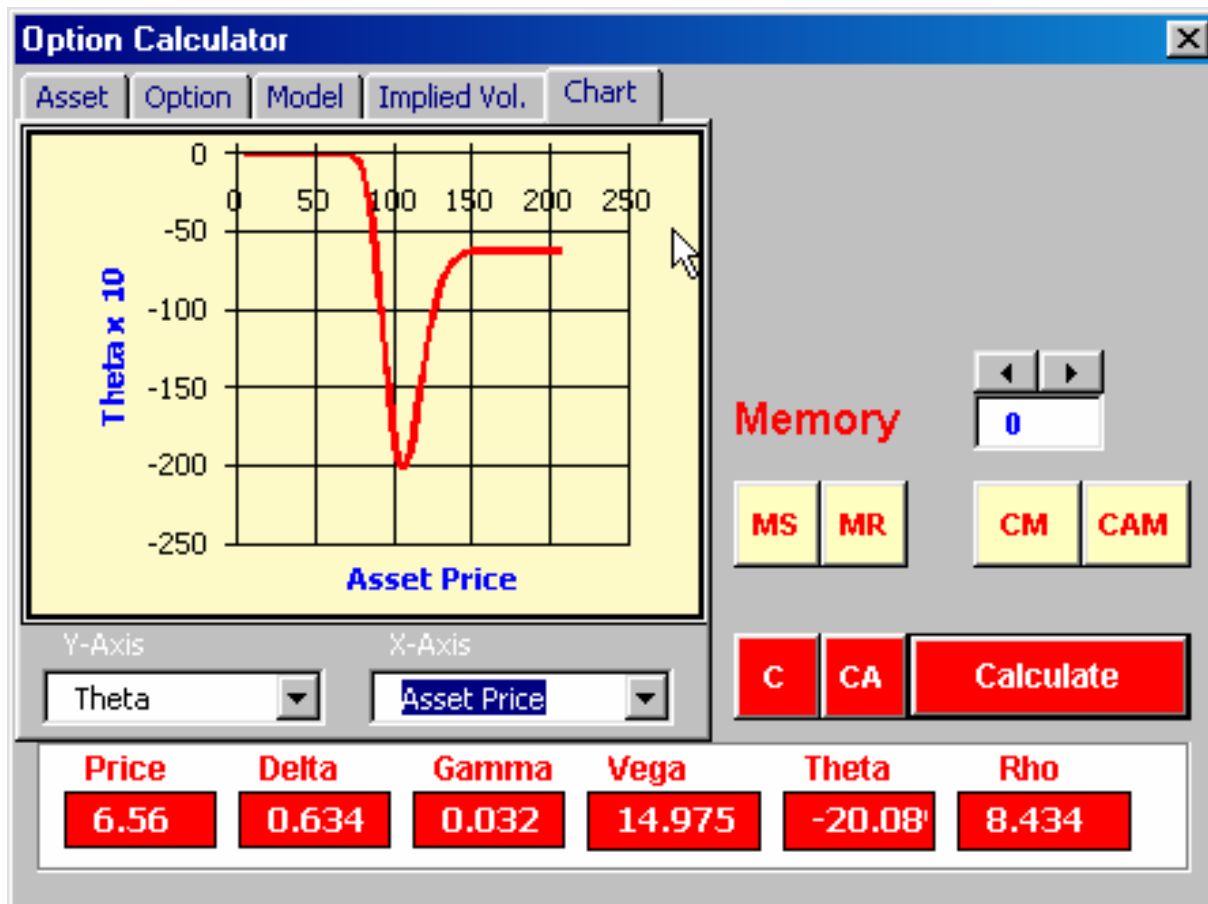
Time Decay (Theta)



Time Value of IBM Option



IBM Option Theta

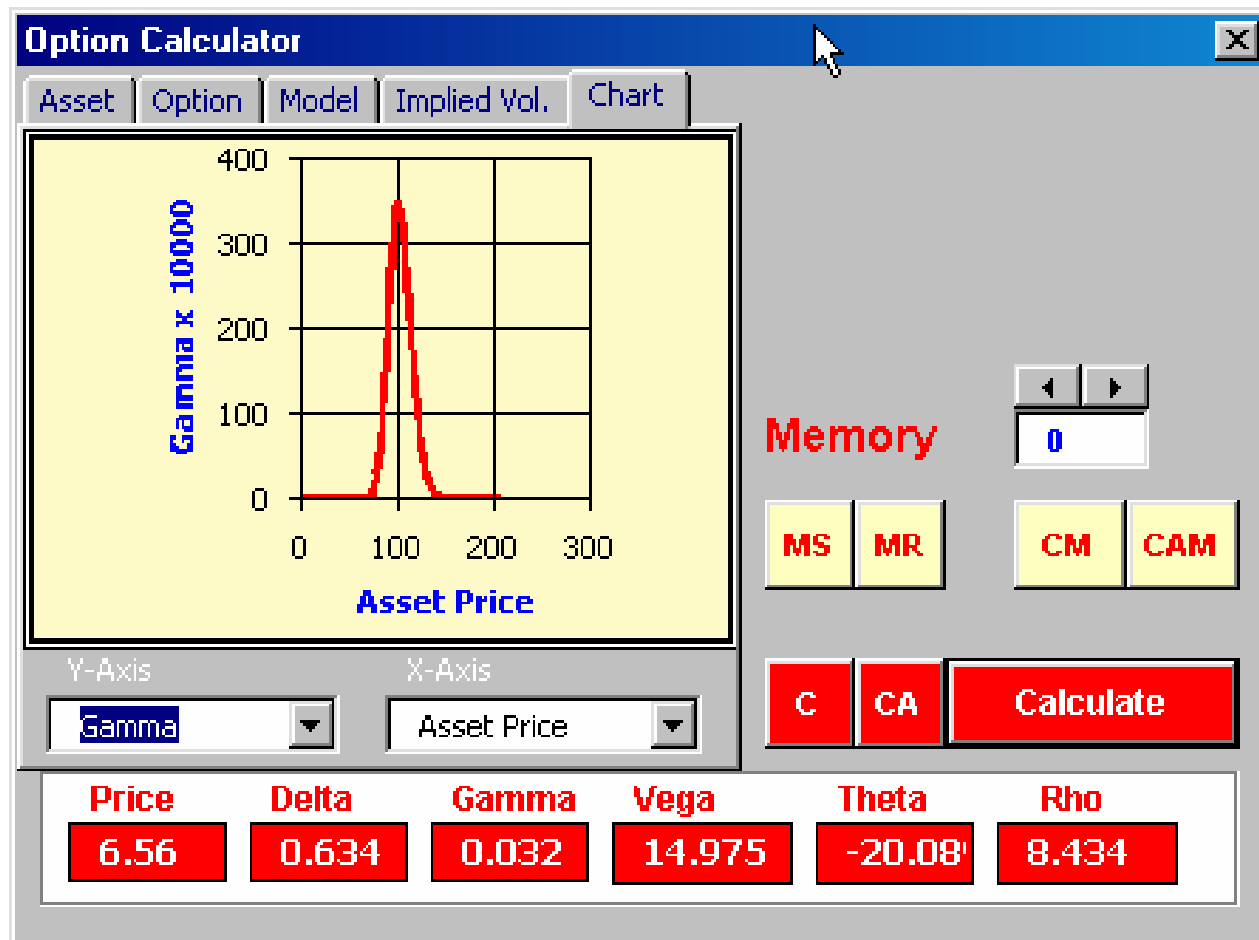




Option Gamma

- Measures change of Delta for \$1 change in stock
- “Rate of Acceleration” of option value with stock price
- Call and put options have positive gamma
 - Option delta becomes larger as stock appreciates
 - Becomes smaller (more negative) as stock declines
- Gamma changes as underlying stock moves
 - Highest for ATM options

IBM Option Gamma





Lab: Leverage

- An Experiment:
 - Assume stock \$100
 - Risk free rate 10%, volatility 25%
 - Call option, strike price 100 (at the money)
- Leverage:
 - If stock moves by \$5, how much does option value change?



Solution: Leverage

<u>Maturity</u>	<u>Call Prices</u>		<u>Return</u>	<u>Gamma</u>
	<u>S=100</u>	<u>S=105</u>	<u>(%)</u>	
1.00	12.34	15.66	27%	0.015
0.50	8.26	11.48	39%	0.022
0.25	5.60	8.80	57%	0.032
0.10	3.40	6.69	97%	0.050
0.01	1.02	5.07	396%	0.160

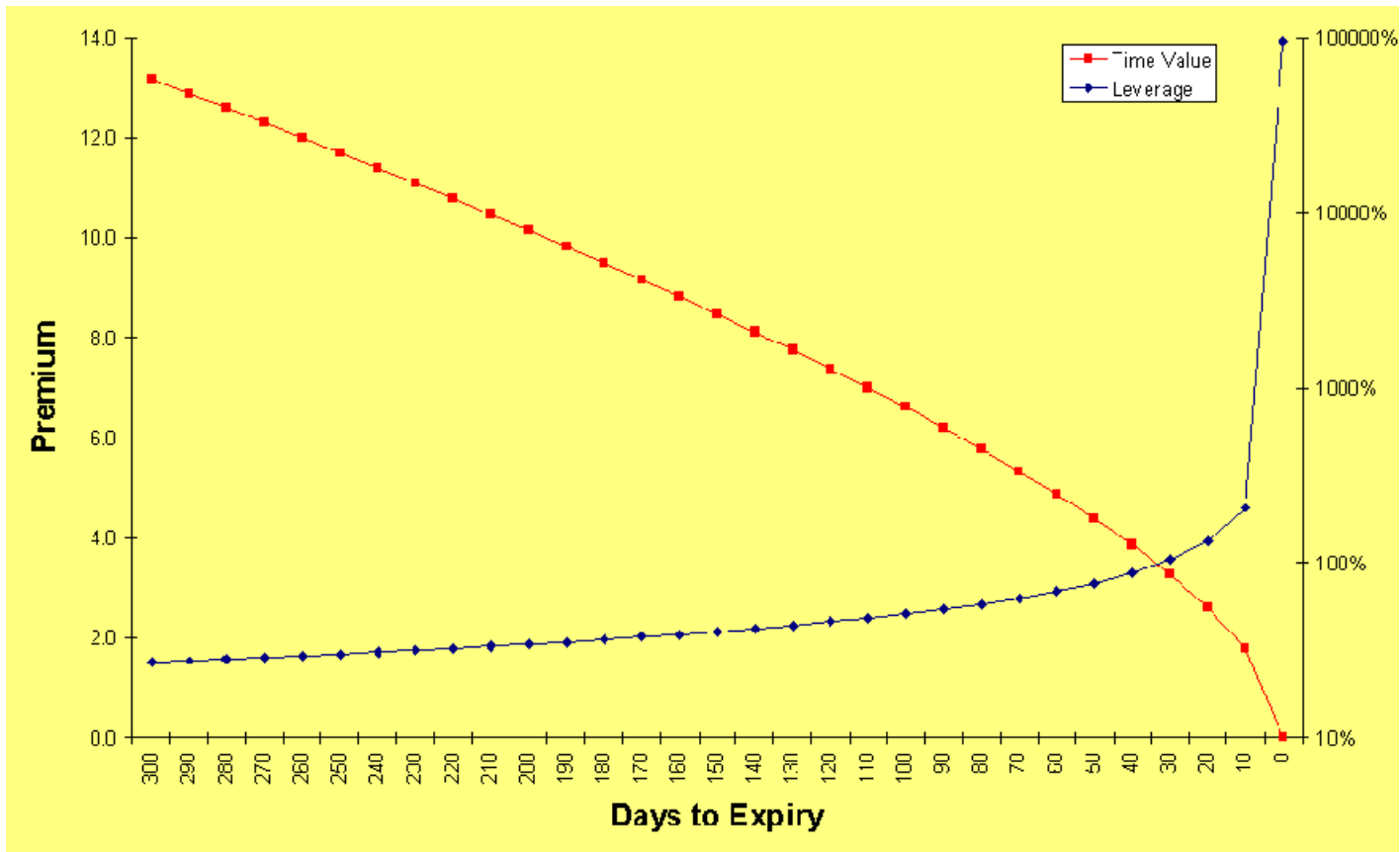
NOTES:

Col 2 is option value with stock price = 100

Col 3 is option value with stock price = 105

Col 4 is 'Leverage': $(C1 - C2)/C1$

Time vs Leverage





Conclusions: Time Value & Leverage

- Gamma is a measure of leverage
- An option with higher gamma gives “more bang for the buck”
- Gamma measures how option value “accelerates”
- Gamma /Leverage increase as time to expiry falls
- Gamma and Theta are “opposites”:
 - High leverage means rapid time decay



Volatility Greeks

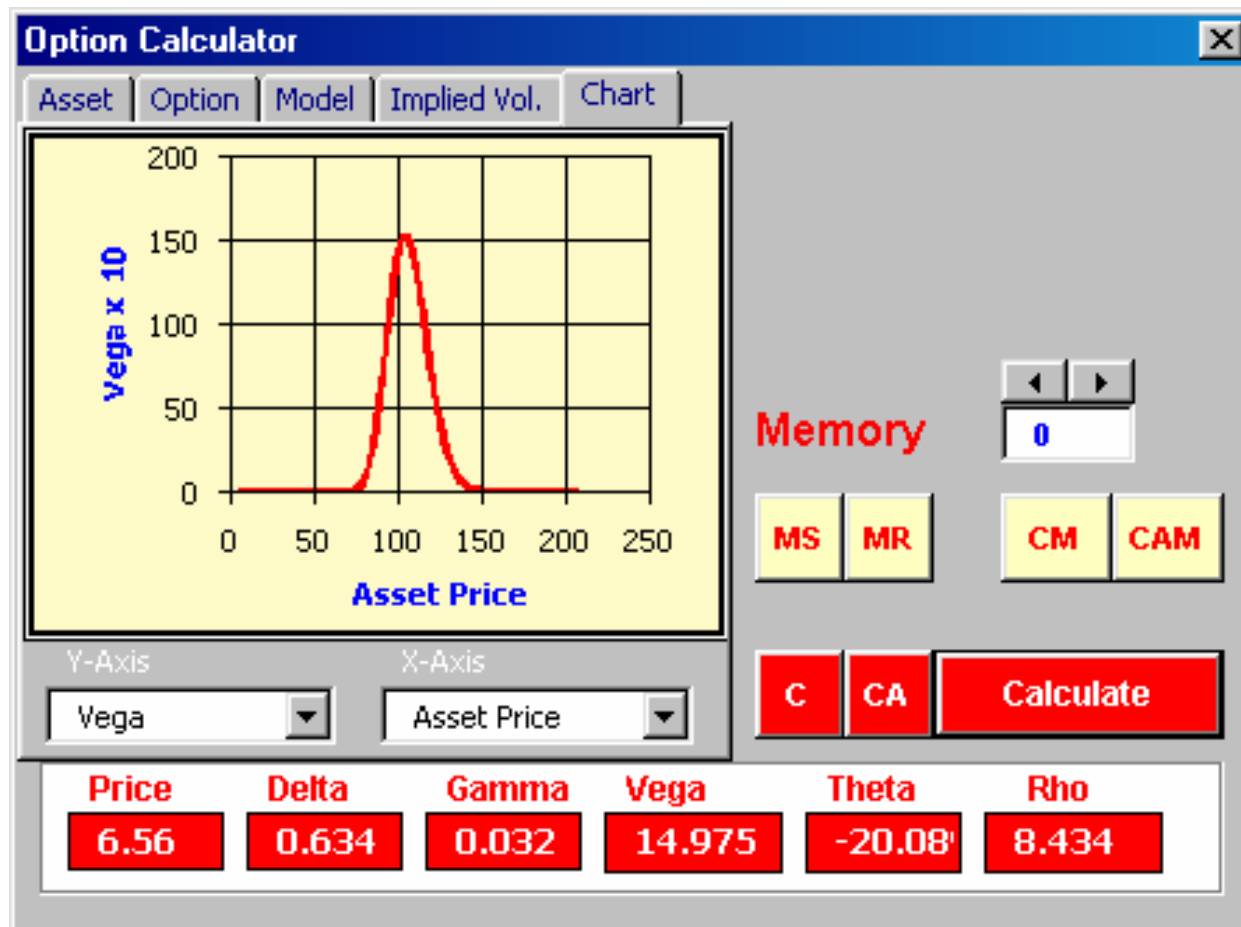
- Vega - sensitivity to implied volatility
- Gamma - sensitivity to actual volatility
- Example: Weather
 - People carrying umbrellas (implied risk of rain) = Vega
 - Rain (the wet stuff) = Gamma
- Implied volatility
 - estimated s.d. implied by option prices by B-S model
 - “market’s” estimate of current volatility



Vega

- Measures change in option value for 1% change in implied volatility
- Call and put options have positive Vega
 - Increase in value as implied volatility increases
 - Option positions may have positive or negative Vega
- Vega changes with underlying stock
 - Highest for ATM options
 - Most sensitive to changes in implied volatility

IBM Option Vega



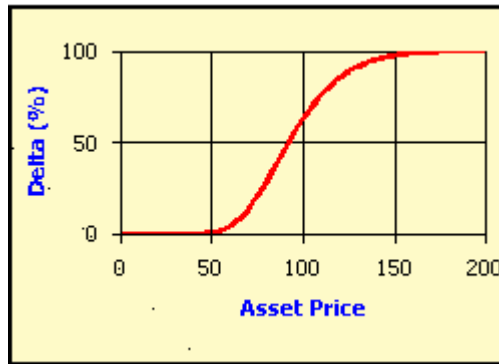


Lab: Option Sensitivities

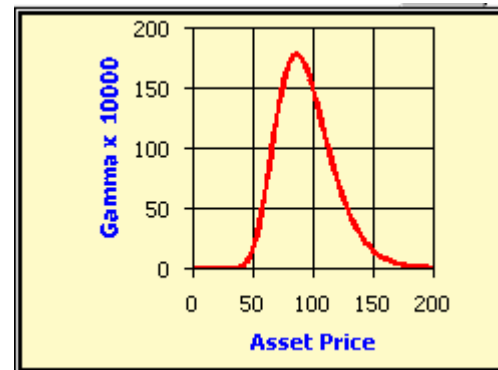
- Examine Delta, Gamma, Vega & Theta
 - Characteristic functions
- How do they vary over time?
- How do they vary with volatility?

Solution: Option Sensitivities

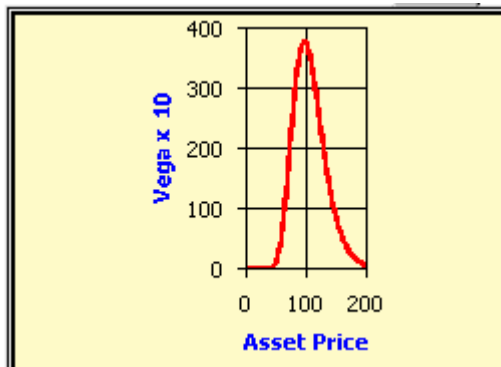
■ Delta



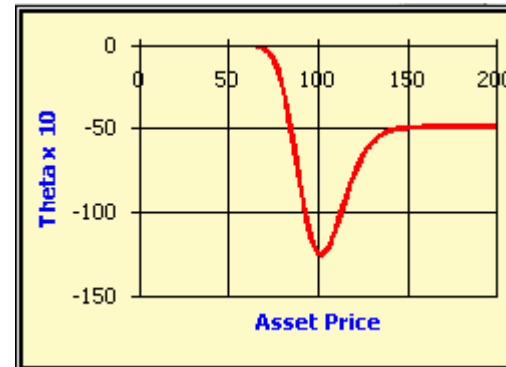
■ Gamma



■ Vega



■ Theta





Solution: Option Sensitivities

- Delta
 - Short-dated, ATM options on less volatile stock are more sensitive
- Gamma
 - Greatest for short-dated ATM options on less volatile stock



Solution: Option Sensitivities

- Vega
 - Long-dated, ATM options on less volatile stock more volatility sensitive
- Theta
 - Short-dated ATM options on more volatile stock experience greatest rate of decay



Volatility Types

- Historical
- Implied
- Forecast
- Seasonal



Historical Volatility

- Fama & French study (1965) of US stock prices, tested:
 - A: Volatility between consecutive trading days
 - B: Volatility over weekend (close Friday to close Monday)
 - Expected $B = 3 \times A$
 - Found B was only 20% higher
- Volatility is much higher on trading days
- Use daily data from trading days (252 per year)



Estimating Historical Volatility

- Standard Deviation

$$X_i = \text{Ln}(P_{i+1} / P_i)$$

$$\sigma = \sqrt{\sum (X_i - \bar{X})^2 / (N - 1)}$$

- Parkinson (5x times more efficient)

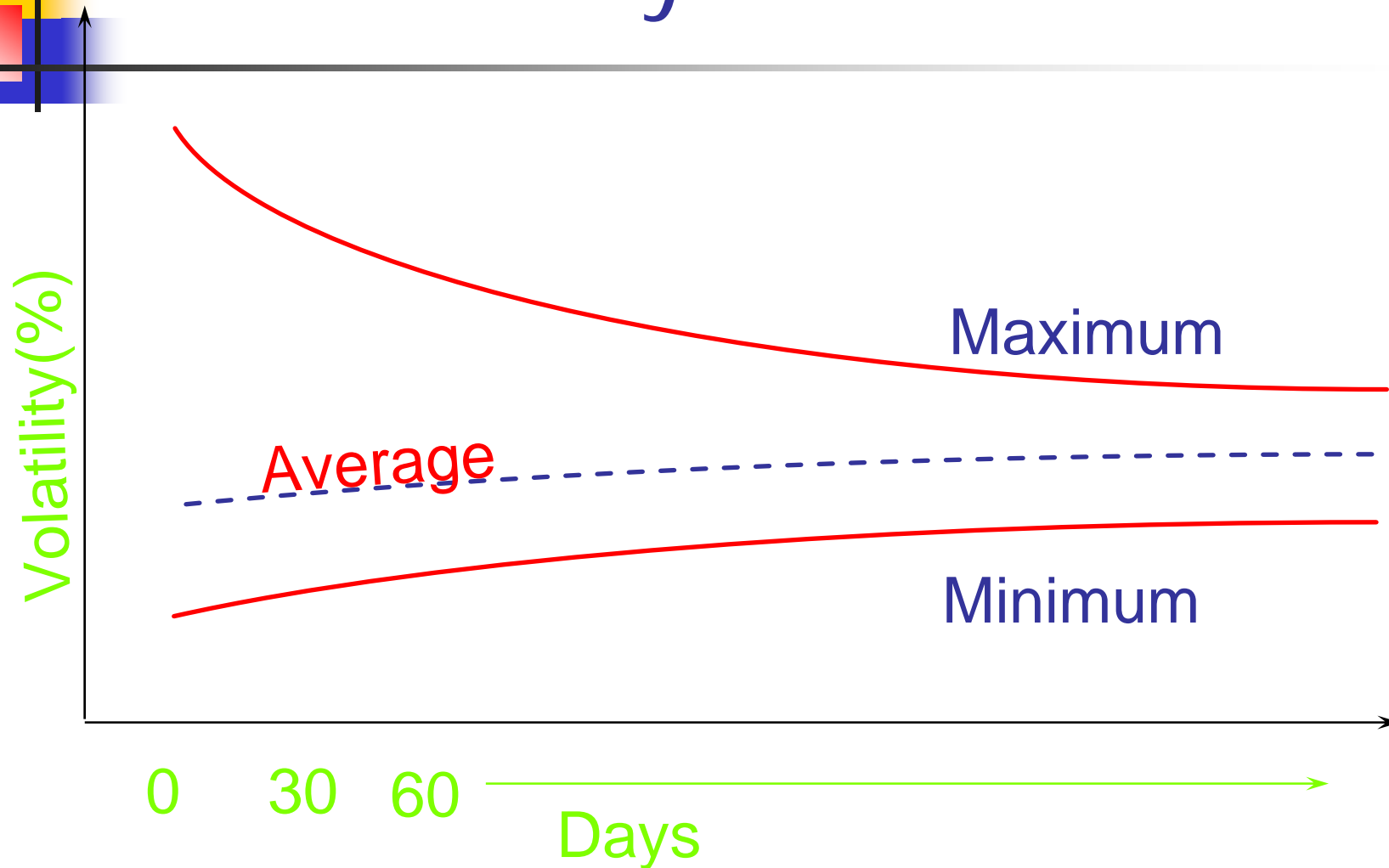
$$\sigma = \frac{1}{2N\sqrt{\text{Ln}(2)}} \sum \text{Ln}(H_i / L_i)$$



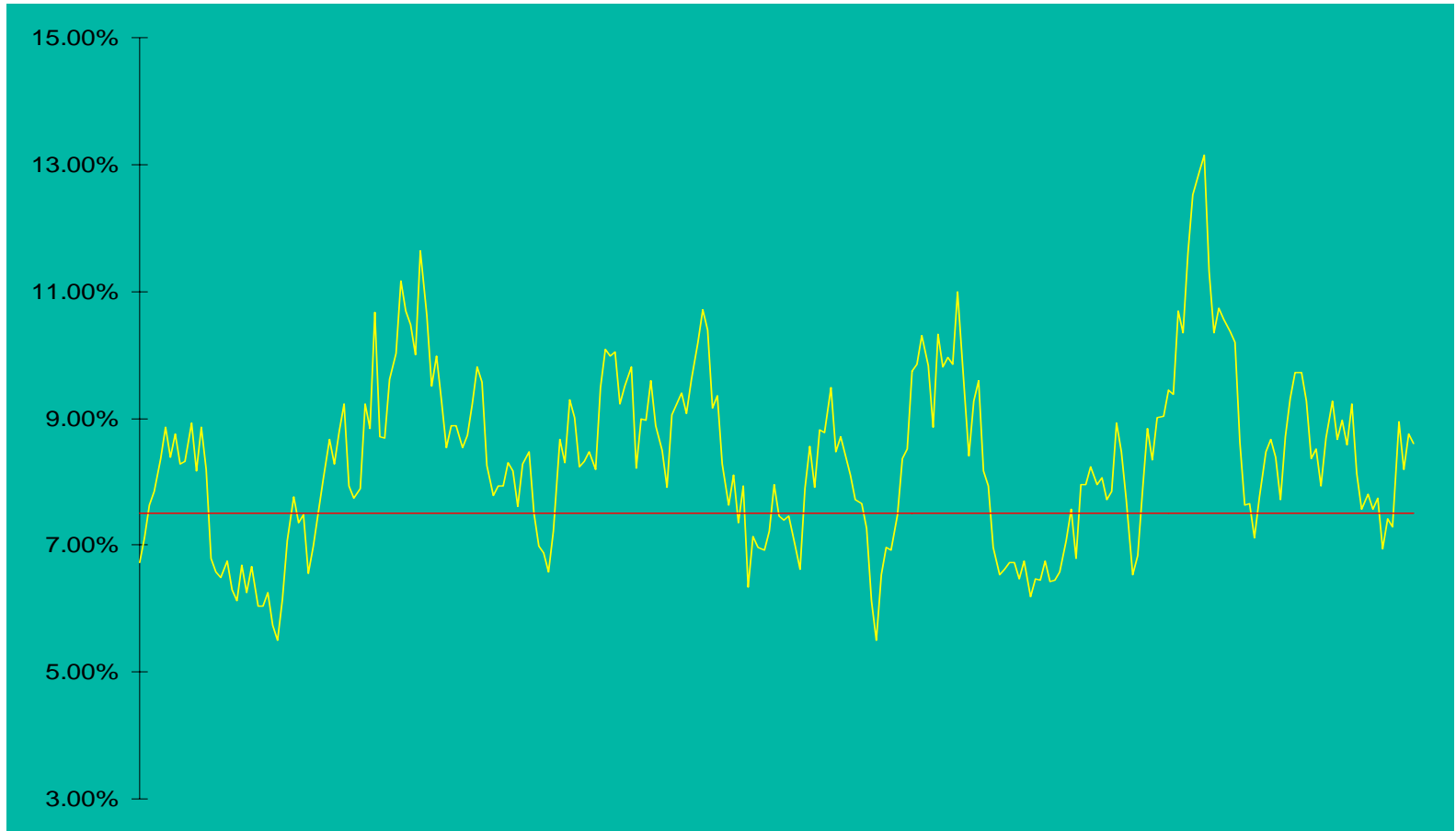
How much historical data?

- How many days?
 - 30, 60, 90, 180, 360, more?
- Does it matter?
 - Yes! Provides different estimates
- General rules
 - The more data the better
 - But not too much!
(there may have been important recent structural changes in the market, e.g. Sterling post ERM)
 - Match to forecast period, e.g. use 30-day historical vol. to estimate volatility over next 30 days

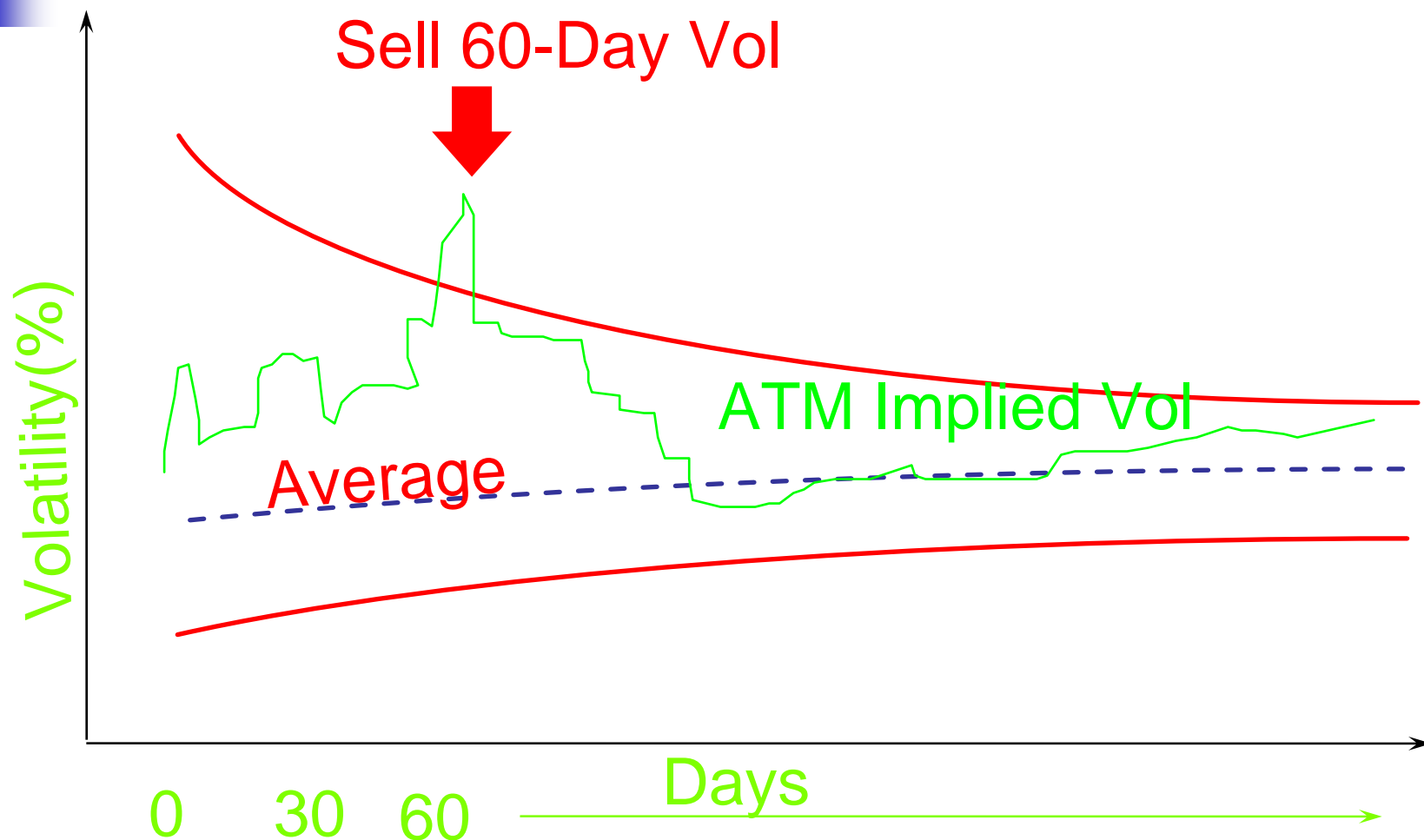
The Volatility Cone



Volatility is Mean Reverting



Trading the Volatility Cone





Short Volatility Strategies

When:

- Implied vol is “rich”, relative to long term mean
- Market is moving sideways
- Trading Strategies
 - Selling Straddles, Strangles, Butterflies
 - Maintain delta-neutral by:-
 - Rebalancing
 - Rollup/rolldown technique



Forward Volatility

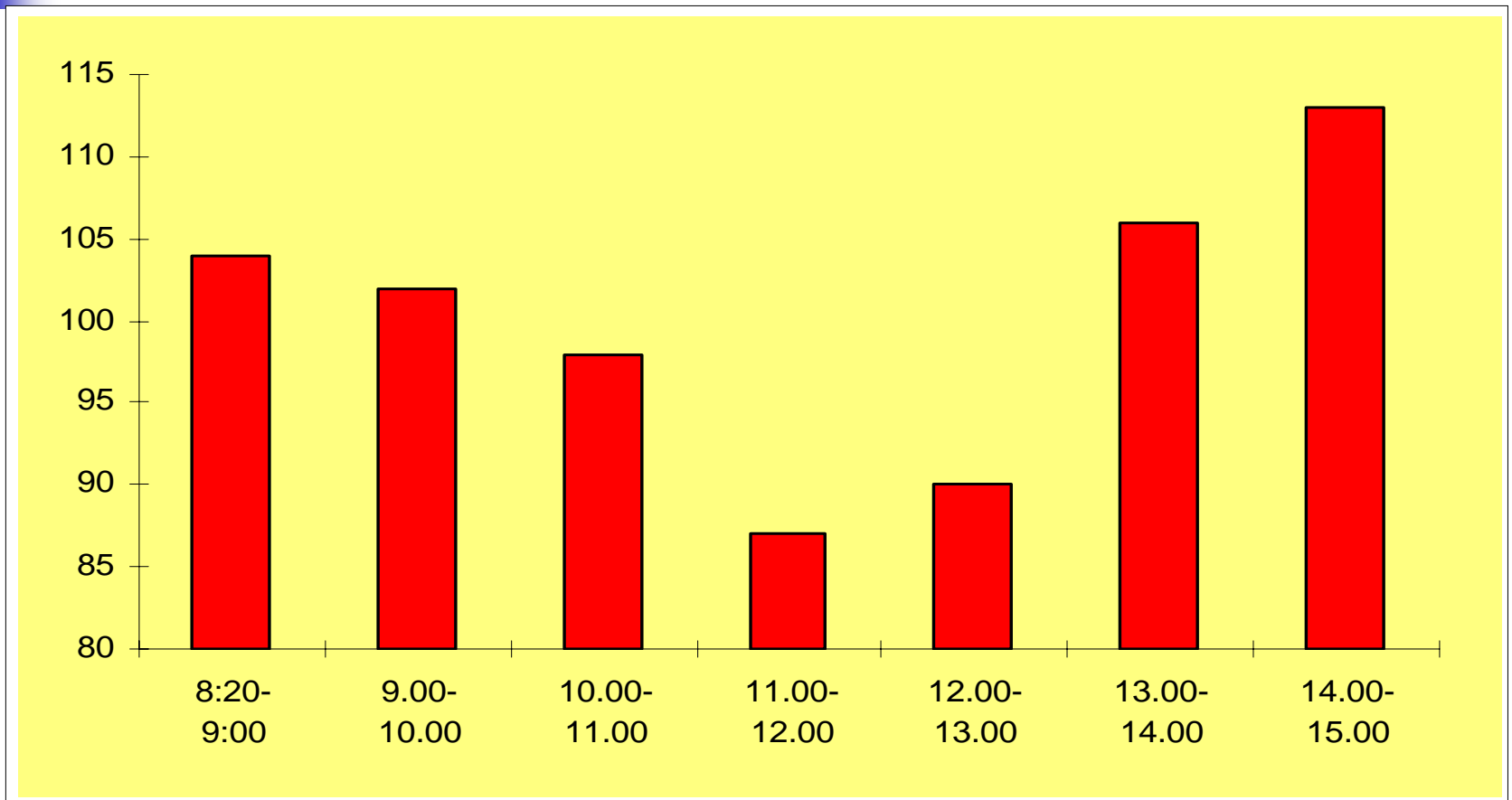
- *Forward volatility term structure*
 - Implied volatility of ATM options with different maturities
 - Plot volatility vs. maturity
 - Analogue of yield curve/term structure of interest rates
- *Forward vol. surface*
 - use all options



Seasonal Volatility

- Some months significantly more/less volatile than the average
- Well-known seasonal effects in agriculturals
- Examples include Wheat, OJ, Hogs
- “Seasonal Effects” in financial markets much less well known

Intra-Day Seasonality in Bonds





Volatility Risk Management

- Hedging exposure to underlying risks
- Delta Hedging
 - Make portfolio Delta-neutral
 - Insensitive to changes in underlying stock
- Gamma Hedging
 - Make portfolio Gamma-neutral
- Vega Hedging
 - Make portfolio Vega-neutral



Volatility Greeks

- Vega - sensitivity to implied volatility
- Gamma - sensitivity to actual volatility
- Example: Weather
 - People carrying umbrellas (implied risk of rain) = Vega
 - Rain (the wet stuff) = Gamma
- Implied volatility
 - estimated s.d. implied by option prices by B-S model
 - “market’s” estimate of current volatility



Gamma Hedging

- Suppose unhedged position = one call
 - You can make it delta neutral
 - But this only works for small changes
 - Need to eliminate gamma risk too
- How do you make it Delta & Gamma neutral?
 - Can't use stock: gamma is zero
 - Must use other options, O_1 and O_2
- Solve:

$$\begin{aligned}m\delta_1 + n\delta_2 &= 0 \\m\Gamma_1 + n\Gamma_2 &= 0\end{aligned}$$



Vega Hedging

- Make a portfolio Delta-Vega neutral
- Again, use other options
- Solve:

$$m\delta_1 + n\delta_2 = 0$$

$$mV_1 + nV_2 = 0$$



Lab: Greek Hedging

- Excel: Workbook
 - Worksheet: Greek hedging
- Sensitivity:
 - Check how position value, delta changes with stock price
 - Check how position value changes with volatility
- Hedging:
 - Gamma hedge the given portfolio
 - Check sensitivity of position to stock price
- Use Solver

Greek Hedging – Using SOLVER

Position					
	Value	Delta	Gamma	Vega	Theta
Portfolio	-493.9	9.2	-15.9	-1,169.8	282.2
Hedge	493.9	-9.2	15.9	1,168.1	-281.6
Net Position	-0.0	-0.1	-0.0	-1.7	0.6
Target	0.0	0.0	0.0	1,169.8	282.2
Abs. Position	0.0	0.1			

Stock	45
Volatility	14.0%
Risk Free	5.0%

Quantity	Type	Strike
0.0	C	35
0.0	C	40
99.5	C	45
2.0	C	50
0.1	C	55
0.0	P	35
-0.1	P	40
-0.7	P	45
75.3	P	50
0.0	P	55

Solver Parameters

Set Target Cell:

Equal to: Max Min Value of:

By Changing Cells:

Subject to the Constraints:

-
-
-
-
-
-

Solution: Greek Hedging

Position						Stock	45
	Value	Delta	Gamma	Vega	Theta	Volatility	14.0%
Portfolio	-493.9	9.2	-15.9	-1,169.8	282.2	Risk Free	5.0%
Hedge	493.9	-9.2	15.9	1,168.1	-281.6		
Net Position	-0.0	-0.1	-0.0	-1.7	0.6		
Target	0.0	0.0	0.0	1,169.8	282.2		
Abs. Position	0.0	0.1	0.0	1.7	0.6		

Quantity	Type	Strike	Maturity	Price	Delta	Gamma	Vega	Theta
0.0	C	35	0.25	10.44	1.00	0.000	0.01	-1.73
0.0	C	40	0.25	5.53	0.97	0.020	1.49	-2.33
99.5	C	45	0.25	1.55	0.59	0.119	8.77	-3.70
2.0	C	50	0.25	0.14	0.10	0.053	3.90	-1.31
0.1	C	55	0.25	0.00	0.00	0.004	0.27	-0.08
0.0	P	35	0.25	0.00	0.00	0.000	0.01	0.00
-0.1	P	40	0.25	0.04	-0.03	0.020	1.49	-0.35
-0.7	P	45	0.25	0.99	-0.42	0.119	8.77	-1.47
75.3	P	50	0.25	4.52	-0.90	0.053	3.90	1.16
0.0	P	55	0.25	9.32	-1.00	0.004	0.27	2.63



Summary:

Derivatives Risk Management

- Greeks
 - Measures of option sensitivity
 - Delta hedging
 - Gamma & Vega hedging