



Bonds with Embedded Options

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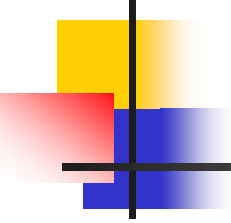
Bonds with Embedded Options

- Callable & Puttable Bonds
- Yield sensitivity
 - Price compression
- Option-adjusted convexity & duration
- Yield to call & yield to worst
- Yield spread
- Static spread
- Option adjusted spread
- Monte-Carlo techniques



Callable Bonds

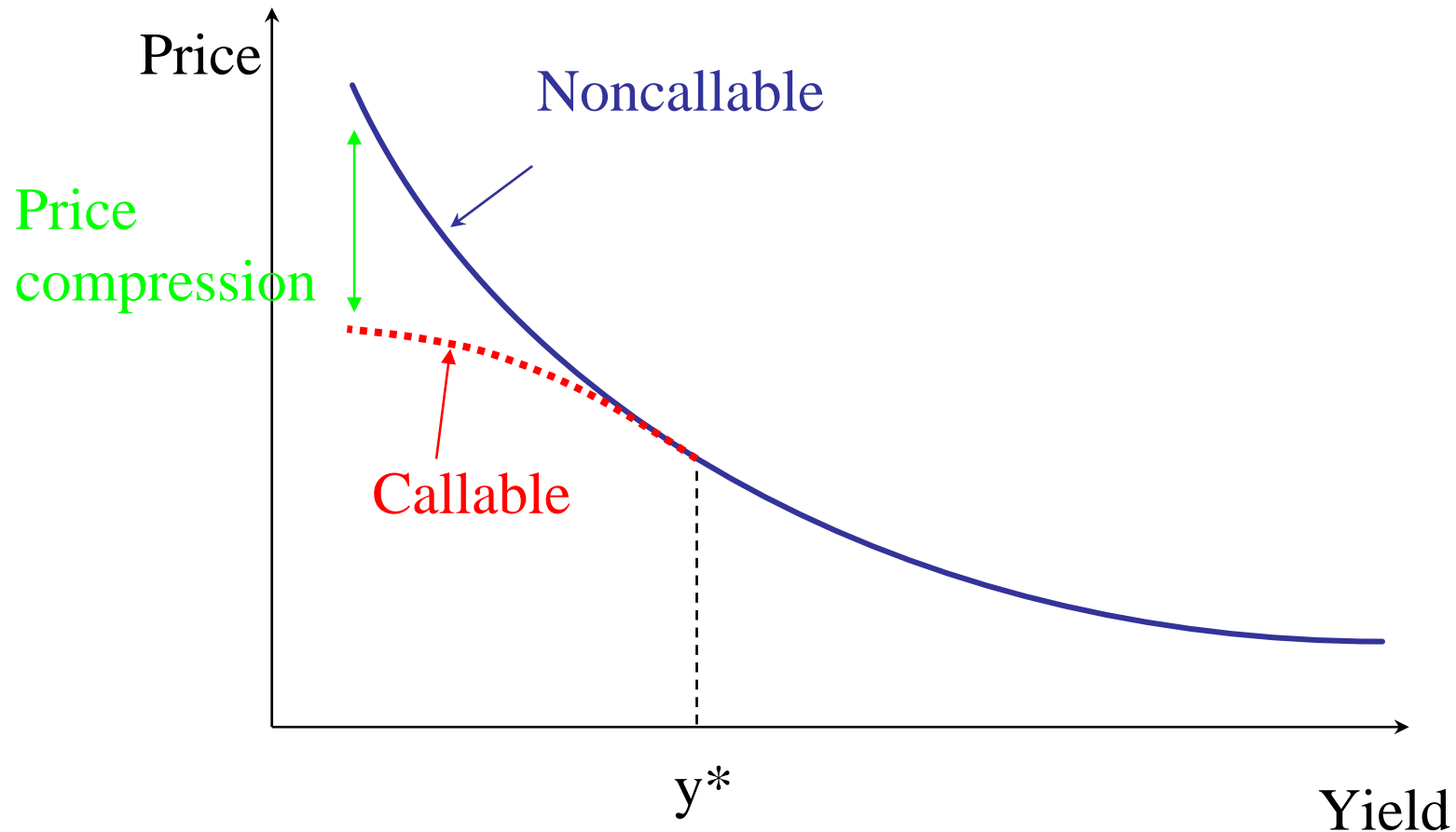
- Many bonds have embedded option features
 - Callable bonds, puttable bonds, etc
- Callable bonds
 - Give issuer right to redeem the issue call price (par)
- Call risk
 - As yields fall likelihood increases that issuer will call
 - Investor faces reinvestment risk
 - Compensated by higher potential yield



Callable Bonds – Traditional Analysis

- Yield to call
 - Calculate yield of bond assuming its expected cash flows are coupon payments to the first call date plus call price
- Yield to worst
 - Calculate yield to call for all call dates and pick the lowest
- Assumptions
 - Reinvestment
 - Issue will be called on call date

Price-Yield Relationship for Callable Bonds





Features of Callable Bonds

- Price compression
 - Limited price appreciate as yields decline
- Negative convexity
 - As yields fall:
 - Duration *increases* (as for non-callable)
 - Then duration *decreases*



Components of a Callable Bond

- Callable Bond = Straight Bond - Call Option
 - Higher yield due to call option premium received
 - As yields decline, value of call option increases, hence price compression
- Pricing of callable bonds
 - Price straight (i.e. non-callable) bond
 - Price call option
 - Interest rate option model
 - $\text{Price}_{\text{CB}} = \text{Price}_{\text{NCB}} - \text{Price}_{\text{CO}}$



Option Adjusted Yield

- Implied Price of Non-Callable Bond:
 - $\text{Price}_{\text{NCB}} = \text{Price}_{\text{CB}} + \text{Price}_{\text{CO}}$
 - Given option price and market price of callable, we can calculate implied price of NCB
- Option-Adjusted Yield
 - YTM of implied non-callable bond
- Rich-Cheap Analysis of Callable Bonds
 - Calculate option-adjusted yield
 - Compare to zero coupon spot yield curve
 - Rich: OAY is low; Cheap: OAY is high



Option-Adjusted Yield Example

- Callable bond
 - 20 years to maturity
 - 8% coupon
 - Price 105 6/32
- Theoretical option value 5.56
- Implied price of straight bond
 - $105.19 + 5.56 = 110.75$
- YTM of 20-yr, 8% coupon straight bond
 - Option-adjusted yield = 6.99%



Using Treasury Calculator

Callable Bond

Face Value	100
Quoted Price	105 6/32
Settlement	6-Nov-97
Maturity	1-Nov-17
Call Date	5-Nov-02
Coupon	8.00%
Option Value	5.56
Option Delta	0.45
Implied NCB Price	110.7500
Yield to Call	5.51%
Option Adjusted Yield	6.99%
Option Adjusted Duration	6.01



Option-Adjusted Duration

- Option Adjusted Duration
 - $D_{OA} = P_{NCB} / P_{CB} \times D_{NCB} \times (1 - \Delta)$
- Example (previous case):
 - $P_{NCB} = 110.75$
 - $P_{CB} = 105.19$
 - $D_{NCB} = 10.39$
 - $\Delta = 0.45$
 - $D_{OA} = (110.75 / 105.19) \times 10.39 \times (1 - 0.45) = 6.01$



Option-Adjusted Convexity

- $C_{OA} = P_{NCB} / P_{CB} [C_{NCB} \times (1 - \Delta) - P_{NCB} \times \Gamma \times D_{NCB}^2]$
- Becomes negative when:
 - $C_{NCB} \times (1 - \Delta) < P_{NCB} \times \Gamma \times D_{NCB}^2$
 - As yields fall, Δ becomes 1 and the left hand term approaches zero



Yield Spread

■ Example:

- 12% coupon Treasury, 7 years to maturity, price 125.575
 - YTM 7.272%
- Corporate Bond, 12% coupon, price 119.45
 - YTM 8.283%
- Yield spread = 101 bp

■ Drawbacks

- Fails to take term structure into consideration
- Doesn't consider yield volatility
 - Will affect cash flows of callable bonds



Static Spread

- Takes account of term structure
 - Spread that investor would realize over entire spot curve if bond is held to maturity
- Procedure:
 - Find spread that will make
PV of cash flows = bond price
 - Cash flows are discounted using:
 - $DF_n = 1 / [1 + (y_n + s)/2]^{2n}$
 - y is period- n spot rate
 - s is static spread

Static Spread Example

Period	Spot	Cash Flow	PV	Cash Flow	PV
0		-125.58		-119.45	
1	5.05%	6.00	5.85	6.00	5.82
2	5.35%	6.00	5.69	6.00	5.64
3	5.88%	6.00	5.50	6.00	5.42
4	6.03%	6.00	5.33	6.00	5.22
5	6.18%	6.00	5.15	6.00	5.03
6	6.33%	6.00	4.98	6.00	4.83
7	6.48%	6.00	4.80	6.00	4.64
8	6.63%	6.00	4.62	6.00	4.44
9	6.78%	6.00	4.44	6.00	4.25
10	6.93%	6.00	4.27	6.00	4.06
11	7.08%	6.00	4.09	6.00	3.88
12	7.23%	6.00	3.92	6.00	3.69
13	7.38%	6.00	3.75	6.00	3.52
14	7.53%	106.00	63.18	106.00	59.00
			<u>125.575</u>		<u>119.450</u>

Face Value	Price	Coupon	YTM	Static Spread (bp)
100	125.575	12.0%	7.273%	0.0
100	119.450	12.0%	8.283%	101.7



Static Spread

- Static spread is 101.7 bp
 - 0.7 bp more than yield spread
- Static spread becomes more significant for
 - Steeper yield curves
 - Longer maturity issues
 - Typical difference may be 10bp for a 25-yr maturity



Option Adjusted Spread

- Static Spread
 - Takes account of term structure
- Option adjusted spread
 - The spread over the Treasury spot curve for which NPV of cash flows = bond price
 - Takes account of yield volatility
 - Affects cash flows of bonds with embedded options



OAS Terms

- Short rate process
 - A path of (6-m) forward rates generated using Monte-Carlo simulation
- Refinancing Spread
 - Spread over treasury forward rate which company must pay to refinance
 - Used to decide whether to call the bond
 - Our example: spread = 100 bp
- Call Rule: when is the bond called?
 - Assume call if refinancing rate < coupon



OAS Procedure

- Generate forward rate for each period using MCS
 - Our model: $F_n = N_0[F_{n-1}, V \times F_{n-1}]$
 - V is volatility parameter (10%)
- Calculate refinancing rate
 - Forward rate + refinancing spread
- Decide in which period bond is called (if any)
 - This determines cash flows
- Calculate Treasury spot rates
- Add OAS spread (guess) & value cash flows
- Check is NPV of cash flows = bond price



OAS Example

- 7 year maturity, 5% semi-annual coupon
- Price 74.50
- Callable at par, anytime
- Assume refinancing spread of 1%
 - Company pays Treasury fwd rate + refinancing spread
- Assume bond is automatically called if refinancing rate falls below Treasury rate

OAS Example

Monte-Carlo Simulation

Face Value	100
Coupon	5.0%
Price	74.50
6-m Fwd Rate	7.00%
Refinancing Spread	1.00%
Volatility	10.0%

Simulations	500
OAS	0.75%
Yield to Maturity	8.91%
Yield to Call	16.04%
Average Value	75.05
Standard Dev.	15.63
T-Test	2.8%

Period	Treasury Forward Rate	Forward Refinancing Rate	Treasury Spot Rate	OAS Spot	Cash Flow	NPV
					-74.50	
1	7.00%	8.00%	7.00%	7.75%	2.50	2.41
2	6.55%	7.55%	6.89%	7.64%	2.50	2.32
3	5.52%	6.52%	6.54%	7.29%	2.50	2.25
4	4.18%	5.18%	6.03%	6.78%	2.50	2.19
5	4.01%	5.01%	5.70%	6.45%	2.50	2.13
6	3.68%	4.68%	5.44%	6.19%	102.50	85.37
7	3.91%	4.91%	5.29%	6.04%	0.00	0.00
8	3.26%	4.26%	5.10%	5.85%	0.00	0.00
9	4.32%	5.32%	5.07%	5.82%	0.00	0.00
10	2.99%	3.99%	4.92%	5.67%	0.00	0.00
11	0.85%	1.85%	4.60%	5.35%	0.00	0.00
12	1.01%	2.01%	4.35%	5.10%	0.00	0.00
13	0.45%	1.45%	4.09%	4.84%	0.00	0.00
14	0.89%	1.89%	3.89%	4.64%	0.00	0.00
15	1.63%	2.63%	3.78%	4.53%	0.00	0.00
16	1.71%	2.71%	3.68%	4.43%	0.00	0.00
17	1.15%	2.15%	3.56%	4.31%	0.00	0.00
18	1.72%	2.72%	3.49%	4.24%	0.00	0.00
19	3.01%	4.01%	3.49%	4.24%	0.00	0.00
20	2.29%	3.29%	3.46%	4.21%	0.00	0.00
						96.67



OAS Results

- Try OAS of 75 bp
- Average simulated value (500 iterations)
 - Average NPV of cash flows = 75.02
- Use T-test
 - check whether difference between actual and simulated price is significant
 - In our case there is only a 2.8% chance
 - Conclusion: Price = NPV
 - Therefore OAS is correct



Example You Can Try

- 7.5% coupon, immediately callable
- Trading at 91.10
- Same maturity, refinancing spread

Second OAS Example

Monte-Carlo Simulation

Face Value	100
Coupon	7.5%
Price	91.10
6-m Fwd Rate	7.00%
Refinancing Spread	1.00%
Volatility	10.0%

Simulations	5000
OAS	1.10%
Yield to Maturity	8.86%
Yield to Call	14.30%
Average Value	91.29
Standard Dev.	12.87
T-Test	1.2%

Period	Treasury Forward Rate	Forward Refinancing Rate	Treasury Spot Rate	OAS Spot	Cash Flow	NPV
					-91.10	
1	7.00%	8.00%	7.00%	8.10%	3.75	3.60
2	8.00%	9.00%	7.64%	8.74%	3.75	3.44
3	5.94%	6.94%	7.20%	8.30%	103.75	91.84
4	3.85%	4.85%	6.45%	7.55%	0.00	0.00
5	3.89%	4.89%	6.03%	7.13%	0.00	0.00
6	3.54%	4.54%	5.69%	6.79%	0.00	0.00
7	2.29%	3.29%	5.27%	6.37%	0.00	0.00
8	4.79%	5.79%	5.28%	6.38%	0.00	0.00
9	5.43%	6.43%	5.36%	6.46%	0.00	0.00
10	4.08%	5.08%	5.30%	6.40%	0.00	0.00
11	4.43%	5.43%	5.29%	6.39%	0.00	0.00
12	4.13%	5.13%	5.26%	6.36%	0.00	0.00
13	2.31%	3.31%	5.10%	6.20%	0.00	0.00
14	1.06%	2.06%	4.86%	5.96%	0.00	0.00
15	0.98%	1.98%	4.66%	5.76%	0.00	0.00
16	-0.03%	0.97%	4.41%	5.51%	0.00	0.00
17	-2.08%	-1.08%	4.06%	5.16%	0.00	0.00
18	-1.93%	-0.93%	3.76%	4.86%	0.00	0.00
19	-0.35%	0.65%	3.57%	4.67%	0.00	0.00
20	1.13%	2.13%	3.48%	4.58%	0.00	0.00
						98.89



OAS Analysis - Technical Issues

- Short rate model
 - Our model allows negative interest rates
 - Has no 'mean reversion' or drift parameter
 - Constant volatility term may not fit vol. term structure
 - Other models may deal with this - e.g. BDT model
- Calibration: use Treasuries (OAS of 0%)
- Refinancing rate
 - Unlikely to be constant over all maturities
- Call Rule
 - Usually much more complex
 - Takes account of tax, transaction costs, etc.



Summary: Bonds with Embedded Options

- Callable & Puttable Bonds
- Yield sensitivity
 - Price compression
- Option-adjusted convexity & duration
- Yield to call & yield to worst
- Yield spread
- Static spread
- Option adjusted spread
- Monte-Carlo techniques