Bond Hedging and Risk Management

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The Yield Curve

- Why is the yield curve shaped the way it is?
- Why does its shape change?
- How can a trader profit from this?

Yield Curve Theories

- Expectations Theory
- Liquidity Preference Theory
- Risk Theory

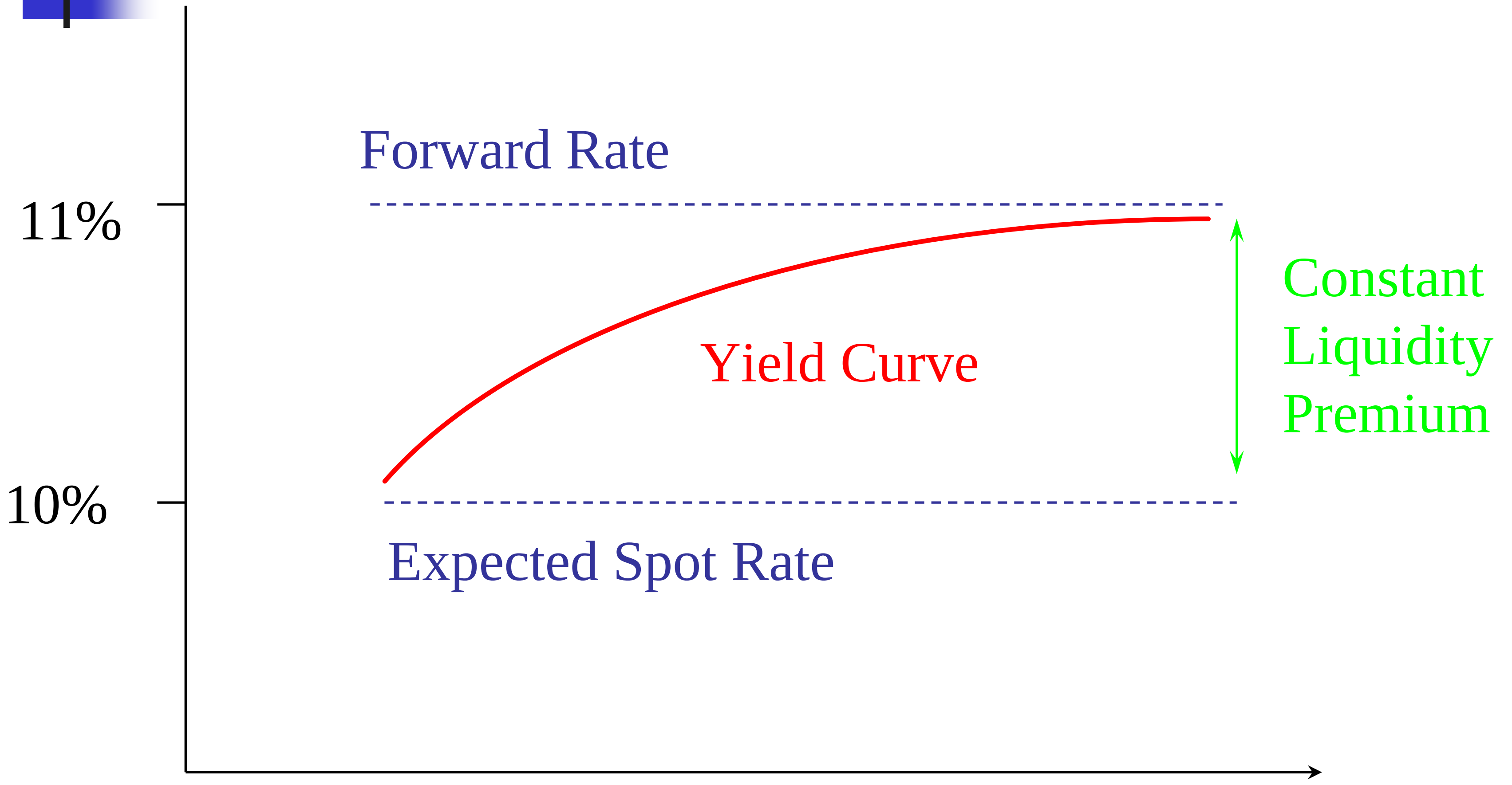
Expectations Theory

- Forward rate = Expected future spot rate
- $F_T = E(S_T)$
- Implications:
  - Bond yields relate to expected future spot rates
    - $(1 + y_2)^2 = (1 + S_1)(1 + f_2) = (1 + S_1)(1 + E[S_2])$
  - Upward sloping yield curve means investors anticipate higher interest rates

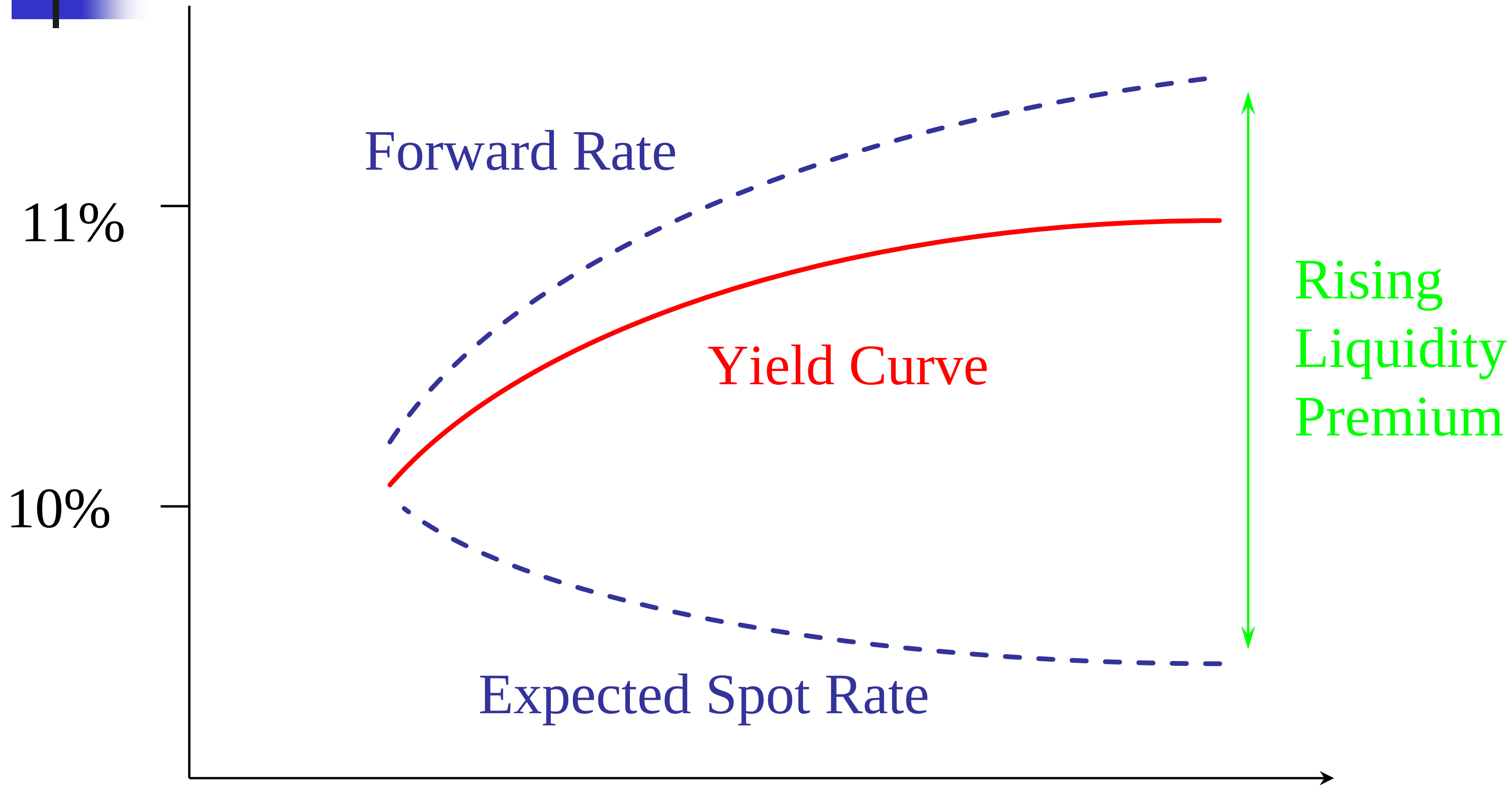
Liquidity Preference Theory

- Investors require a liquidity premium to hold long term securities
- $F_T > E[S_T]$
- Liquidity Premium: $L_T = F_T - E[S_T]$
- Example: $S_1 = E[S_2] = 10\%$
  - Expectations Hypothesis
    - $(1 + y_2)^2 = (1 + S_1)(1 + E[S_2]) \Rightarrow y_2 = 10\%$
  - Liquidity Preference
    - $F_2 = 11\% > E[S_2] = 10\% \; (L_2 = 1\%)$
    - $(1 + y_2)^2 = (1 + S_1)(1 + f_2) \Rightarrow y_2 = 10.5\%$

Constant Liquidity Premium



Rising Liquidity Premium



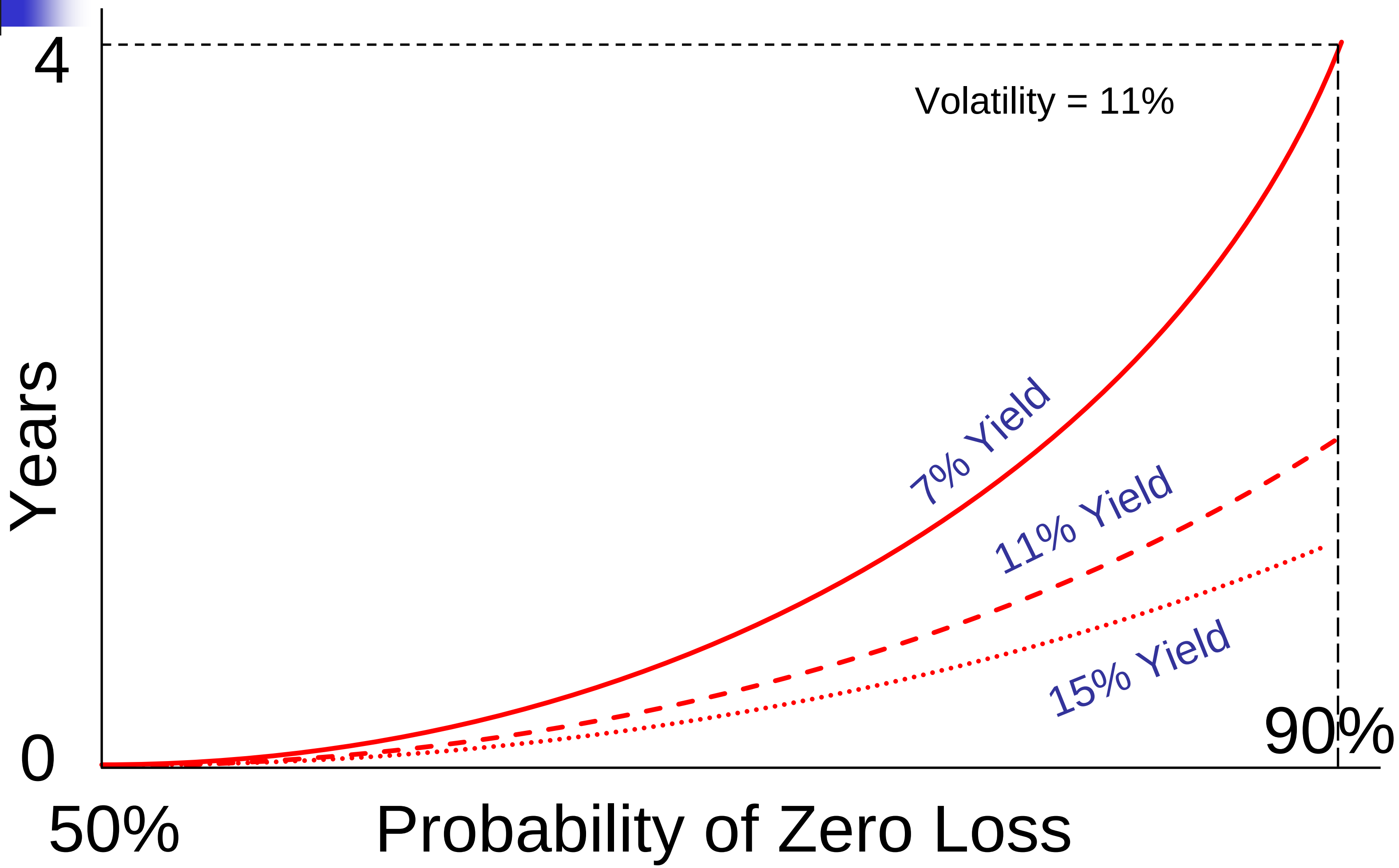
Risk Measures

- Price Risk:
  - Change in price for 1% change in yield (dollar duration or "PV of an 01")
- **Probability of Zero Loss** (over 1 month):
  - Likelihood that price of an issues falls by no more than interest earned (over 1 month)
- Required Holding Period:
  - Period require to hold a security so that the probability of zero loss exceeds a specified level

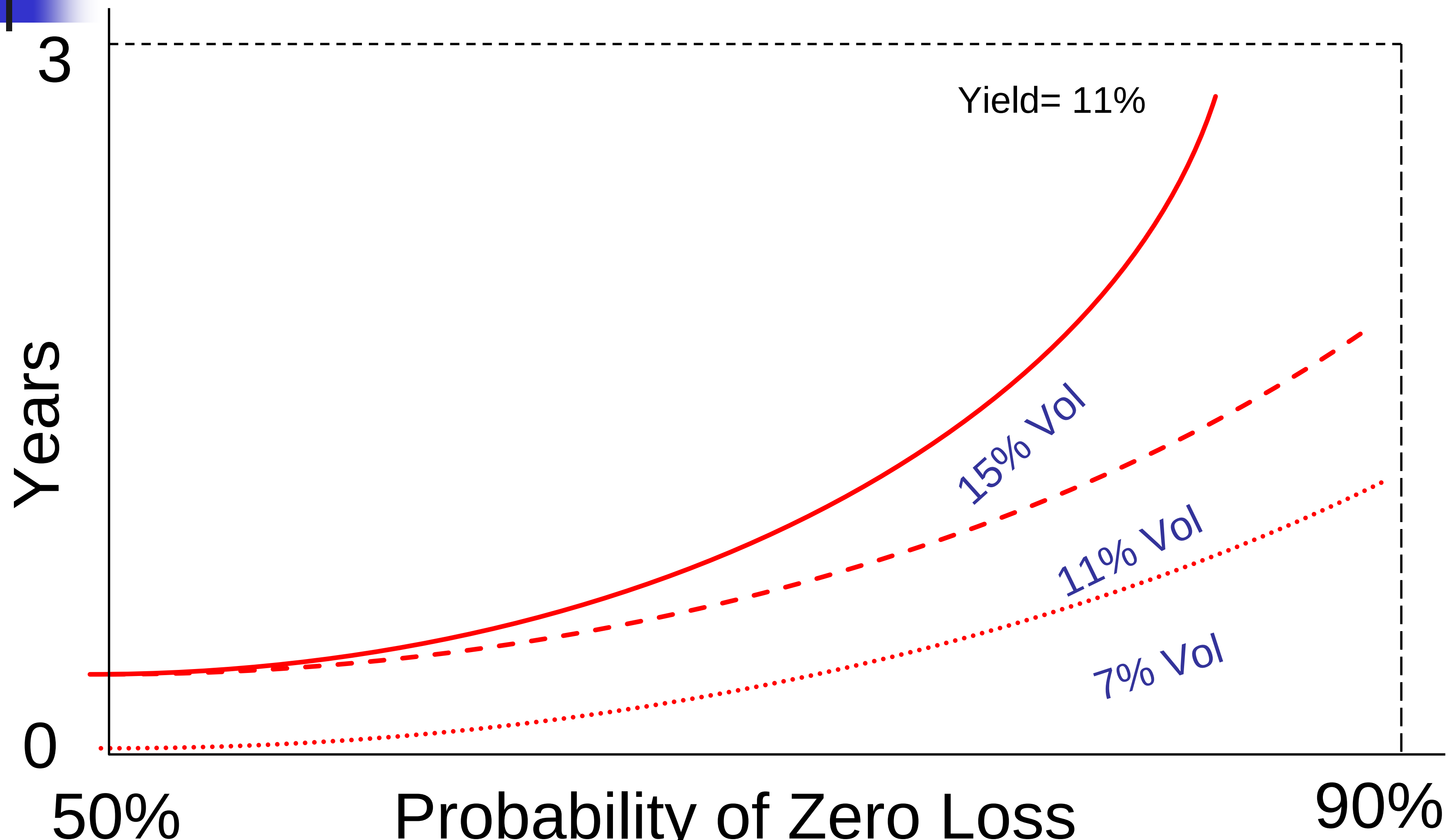
Risk and Yield

- Price risk is proportional to duration
  - 30 year bond has greater price risk than 2 year note
- Higher yield means lower price risk
  - A par bond at 15% yield has a price risk just over half that of a par bond at 7% yield

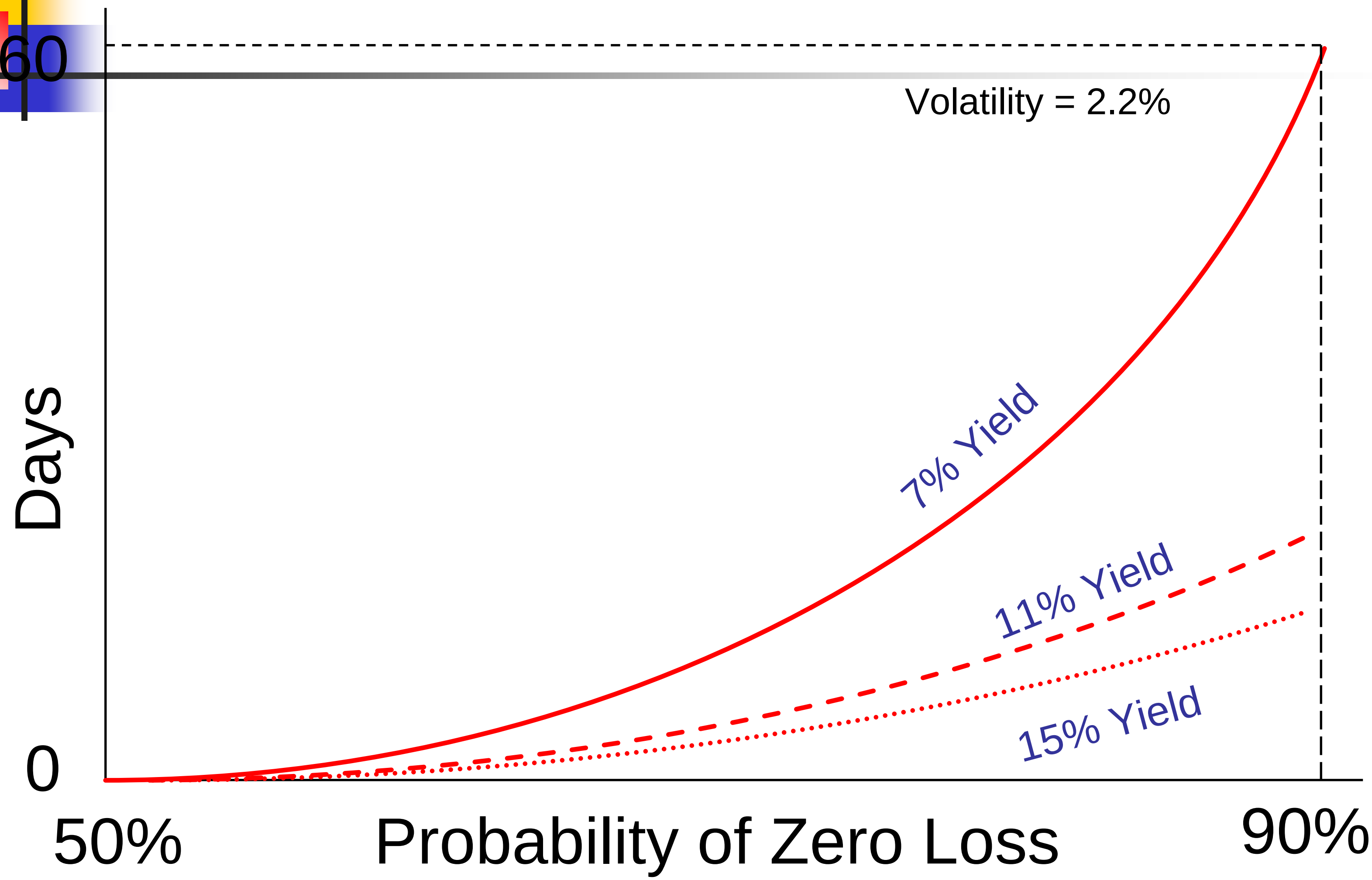
Holding Period for 30Y Bond x Yield



Holding Period for 30Y Bond x Vol



Holding Period for 2Y Note x Yield



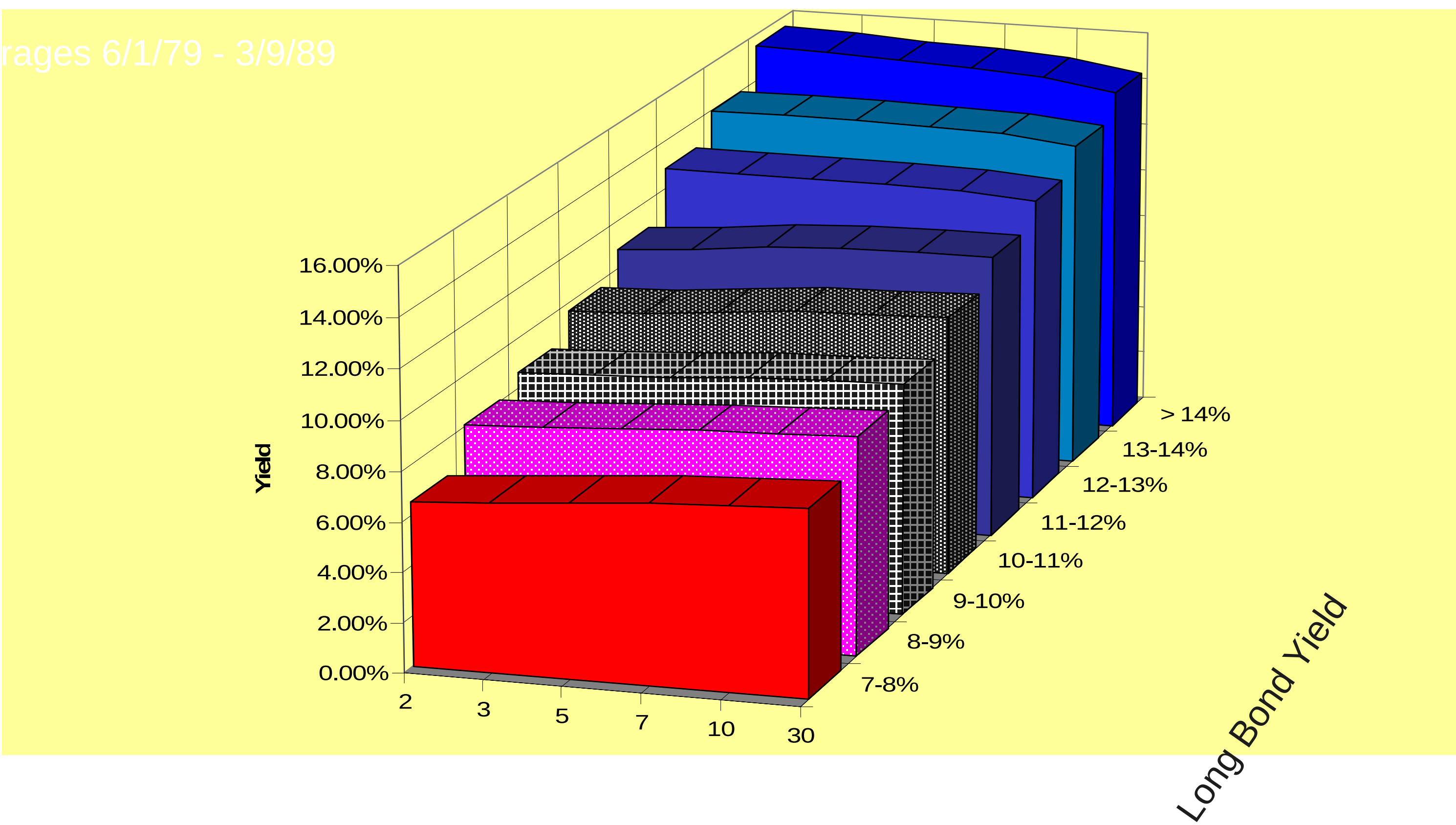
Implications for Yield Curve Shape

- 2y Note much safer than 30y Bond
  - (holding period days rather than years)
- As investor extends along yield curve, probability of losing money rises
- Hence must receive risk premium in higher yields
- CONCLUSION: Yield curve +ve slope

Yield Curve Shape & Yield Level

- Curve has +ve slope at low yields
- Curve has -ve slope at high yields
- Why?
- As yields *increase*:
  - Probability of Zero Loss *rises*
  - Risk of long-maturity issue relative to short-maturity issue *falls*
  - Investors buy the long end, yield curve flattens, then inverts

Shape of Yield Curve Changes with Yield Level



Summary: Yield Curve Theories

- Expectations Hypothesis
  - $F_T = E(S_T)$
  - Empirical evidence suggests otherwise
- Liquidity Preference
  - Investors require a liquidity premium to hold long term securities
  - Liquidity Premium: $L_T = F_T - E[S_T]$
  - Idea: why not try to capture L_T ?
- Risk Theory
  - Probability of zero loss

Interest Rate Risk

- Duration
- Convexity
- Immunization
- Two-factor Duration/Immunization

Duration

- The further away cash flows are, the more their PV is affected by interest rates:
  - $PV = C/(1 + r)^t$
- *Duration* measures weighted average maturity of cash flows:
  - $D = \sum t \times W_t$
    - $W_t = \dfrac{CF_t / (1 + y)^t}{PV}$
    - y is yield to maturity
  - Higher duration means greater risk

Duration and Risk

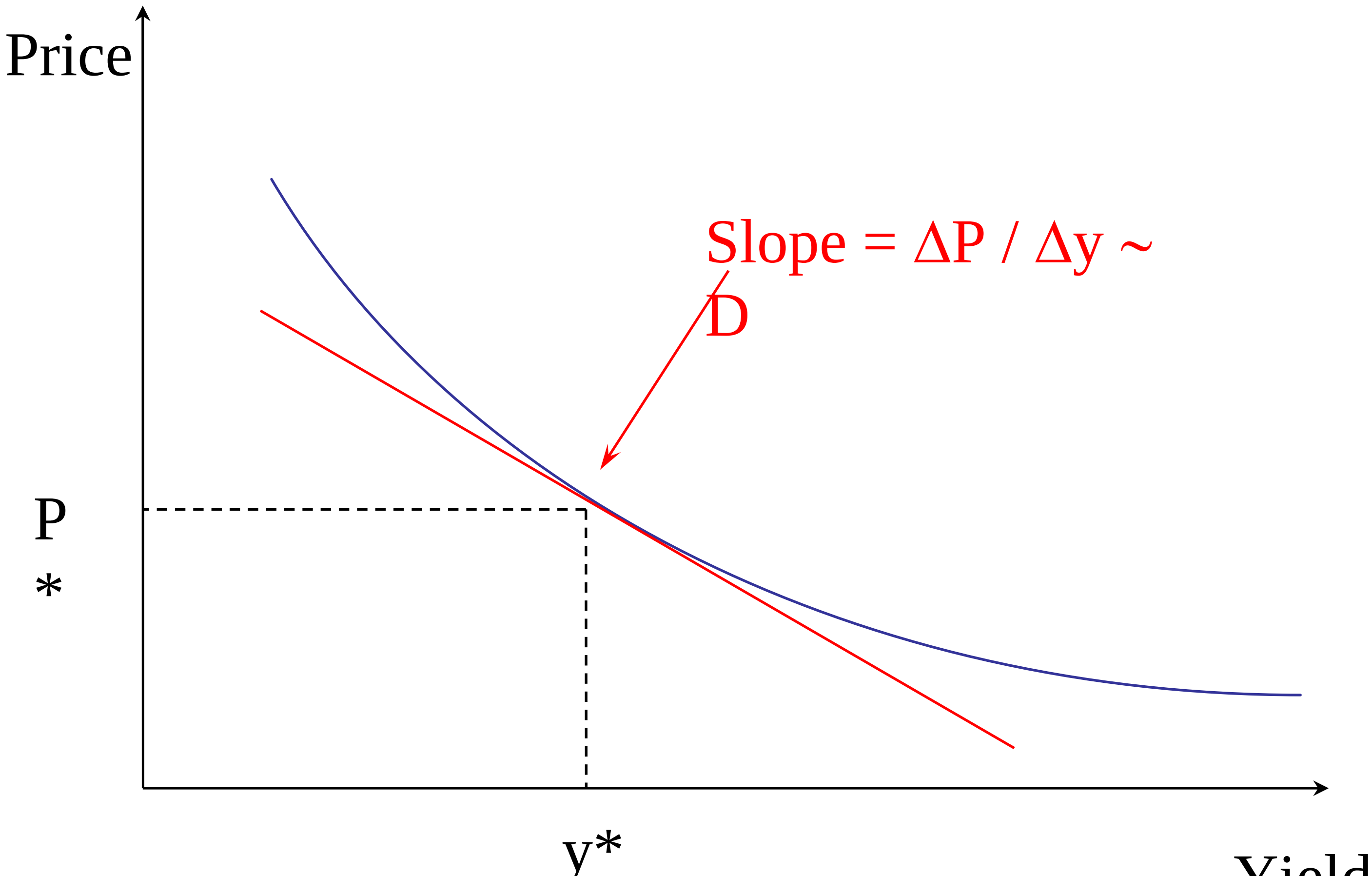
- Impact of changes in YTM:
  - $\Delta P = -[D / (1 + y)] \times P \times \Delta y$
  - $D / (1 + y)$ is known as *modified duration D**
  - $D^* = [\Delta P / P] \times (1 / \Delta y)$
  - Percentage price change $[\Delta P / P] = D^* \times \Delta y$
- Limitations:
  - Small changes in y
  - Parallel changes in yield curve

Example (rates = 10%)

Time	Cash Flow	Discount Factor	PV of Cash Flow	PV Weight	PV Weight x Time
1	100	0.9091	90.91	0.2398	0.2398
2	100	0.8264	82.64	0.2180	0.4360
3	100	0.7513	75.13	0.1982	0.5946
4	100	0.6830	68.30	0.1802	0.7207
5	100	0.6209	62.09	0.1638	0.8190
TOTAL			**379.07**	**1.0000**	**2.8101**

- Duration = 2.81 Years
- Modified Duration = 2.81 / 1.1 = 2.55 years

Duration and Price-Yield Relationship



Two Ways to Think About Duration

- Weighted Average Time to Maturity
  - Weight the time of each cashflow by proportion of total NPV it represents

- As the *sensitivity* of a security's PV to change in interest rates
  - Sensitivity = $\delta P / \delta y = -\Sigma t\ [CF_t / (1 + y)^t] \times 1/P$

Duration as Measure of Rate of Return Volatility

- $D^* = [\Delta P / P] / \Delta y$
- Modified Duration = $\dfrac{\text{Proportional change in value}}{\text{Change in Interest Rate}}$
- Proportional change in value = Modified Duration x Change in Interest Rate
- $\sigma_A = D^* \times \sigma_r$

  σ_A : Standard deviation of asset return

  σ_r : Standard deviation of interest rate changes

Immunization

- If:
  - Duration of Assets = Duration of Liabilities
  - Value of Assets = Value of Liabilities
- Then the portfolio is hedged (“immunized”)
  - For small changes in yield, changes in asset value will offset changes in liability value

Duration and Immunization: Lab Trading Case B04

- Worked Exercise: Duration
  - See worked exercise notes
- Trading Case: B04
  - Flat yield curve 25%
  - Can move to: 5% to 45%
  - You have asset / liability which you cannot trade
  - Must try to preserve value of position

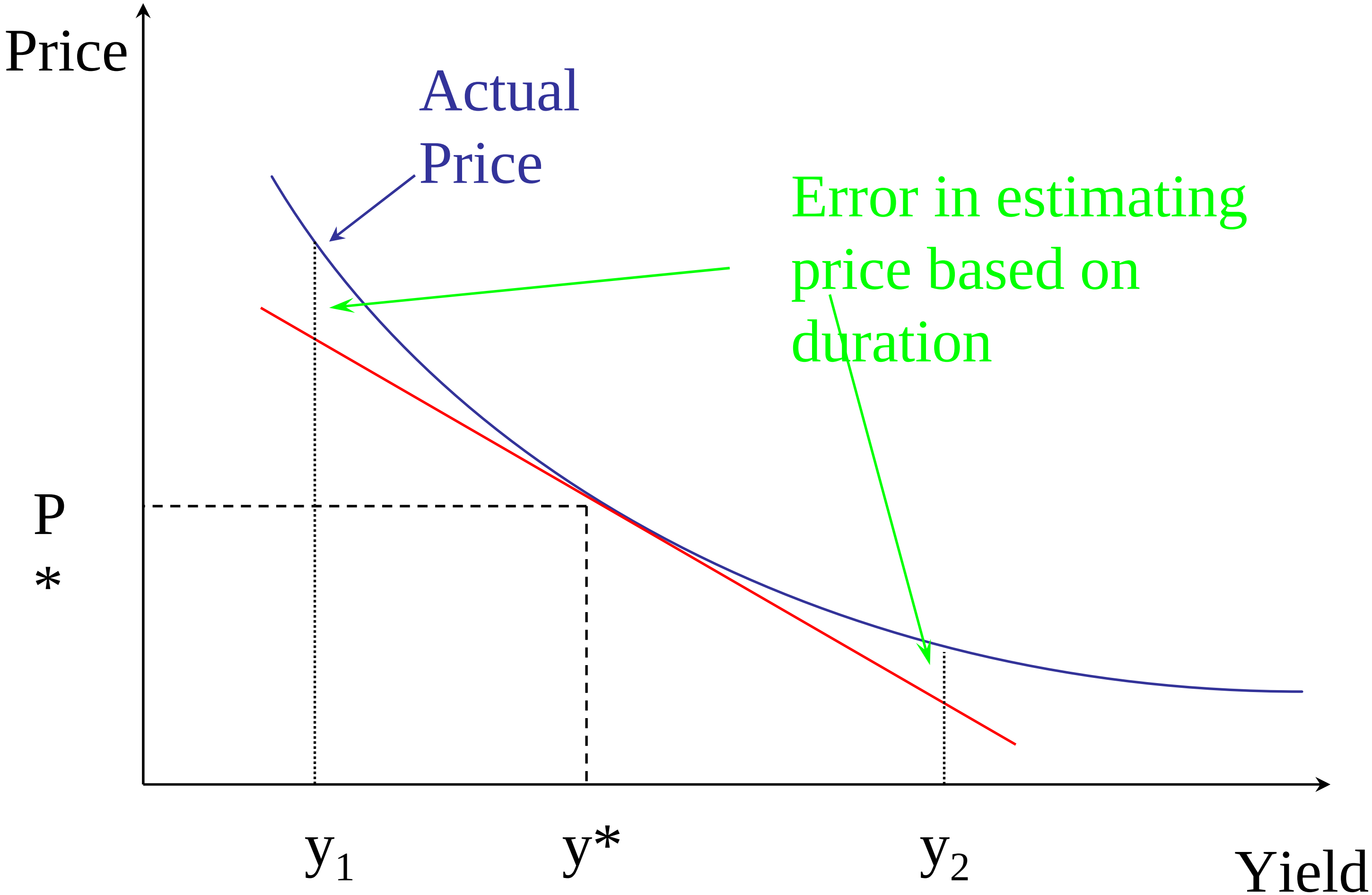
Analysis of Case B04

- Position 1
  - 3200 cash
  - 14 of security worth 307
  - -51 of security worth 64
- How to hedge:
  - Sell 14 @ 307
  - Buy 29 @ 112
    - Asset value = 29 * 112 = 3250
    - Liability value = 51 * 64 = 3264
    - Net cash = +1050

Problems with Conventional Immunization

Assumption	Empirical Evidence
Yield curve shifts are parallel	Short rates move more than long rates
Yield curve changes perfectly correlated along the curve	Correlation between short and long rates much less than 1.0

Price Approximation Using Duration



Convexity

- Duration assumes linear price-yield relationship
  - Duration proportional to the slope of the tangent line
  - Accurate for small changes in yield
- Convexity recognizes that price-yield relationship is curvilinear
  - Important for large changes in yield

Convexity Formula

- Dollar Convexity:
    - $\delta^2 P / \delta y^2 = \sum CF_t \times t(t+1) / (1 + y)^{t+2}$
  - Price change due to convexity:
    - $\Delta P = \text{Dollar Convexity} \times (\Delta y)^2$
- Convexity = $[\delta^2 P / \delta y^2] \times (1 / P)$
  - Percentage price change due to convexity:
    - $\Delta P / P = 0.5 \times \text{Convexity} \times (\Delta y)^2$

Convexity Adjustment Example

- Straight Bond
  - 6% coupon, 25yr, yield 9%
  - Modified Duration =10.62
  - Convexity = 182.92
- % Price Change:

Yield Move	Duration ($D^* \Delta y$)	Convexity $0.5 \times C (\Delta y)^2$	Total
200bp	-21.24%	3.66%	-17.58%
-200bp	+21.24%	3.66%	+24.90%

When Conventional Duration Works

- In most cases using YTM rather than zero coupon yields to compute duration is adequate
- Problems arise with:
  - Short positions
  - Positions with irregular cash flows
- Example:
    - Long $100mm in 10 year zero coupon bonds
    - Short $200mm in 5 years zero coupon bonds
    - Duration = 0
    - BUT: very sensitive to relative movements in 5 and 10 year rates

A Two-Factor Model of Yield Curve Changes

Change in spot rate $= A_t \times$ Change in short rate $+ B_t \times$ Change in long rate

$= \alpha_t \times$ Change in spread $+ \beta_t \times$ Change in long rate

- Spread: (Long rate - Short rate)
- Two factor Model:

α_T : sensitivity of T-period spot rate to changes in spread

β_T: sensitivity of T-period spot rate to changes in long rate

Immunization with Two Factor Model

- Factors
  - Long rate
  - Spread = long rate - short rate

- Durations: each asset has two durations
  - Long Duration: sensitivity to change in long rate
  - Spread Duration: sensitivity to change in spread

Computing Two-Factor Durations

- Duration formula:
  - $D_S = -\Sigma T_i \, \alpha_{Ti}[c_i e^{-RTi}/PV]$
  - $D_L = -\Sigma T_i \, \beta_{Ti}[c_i e^{-RTi}/PV]$

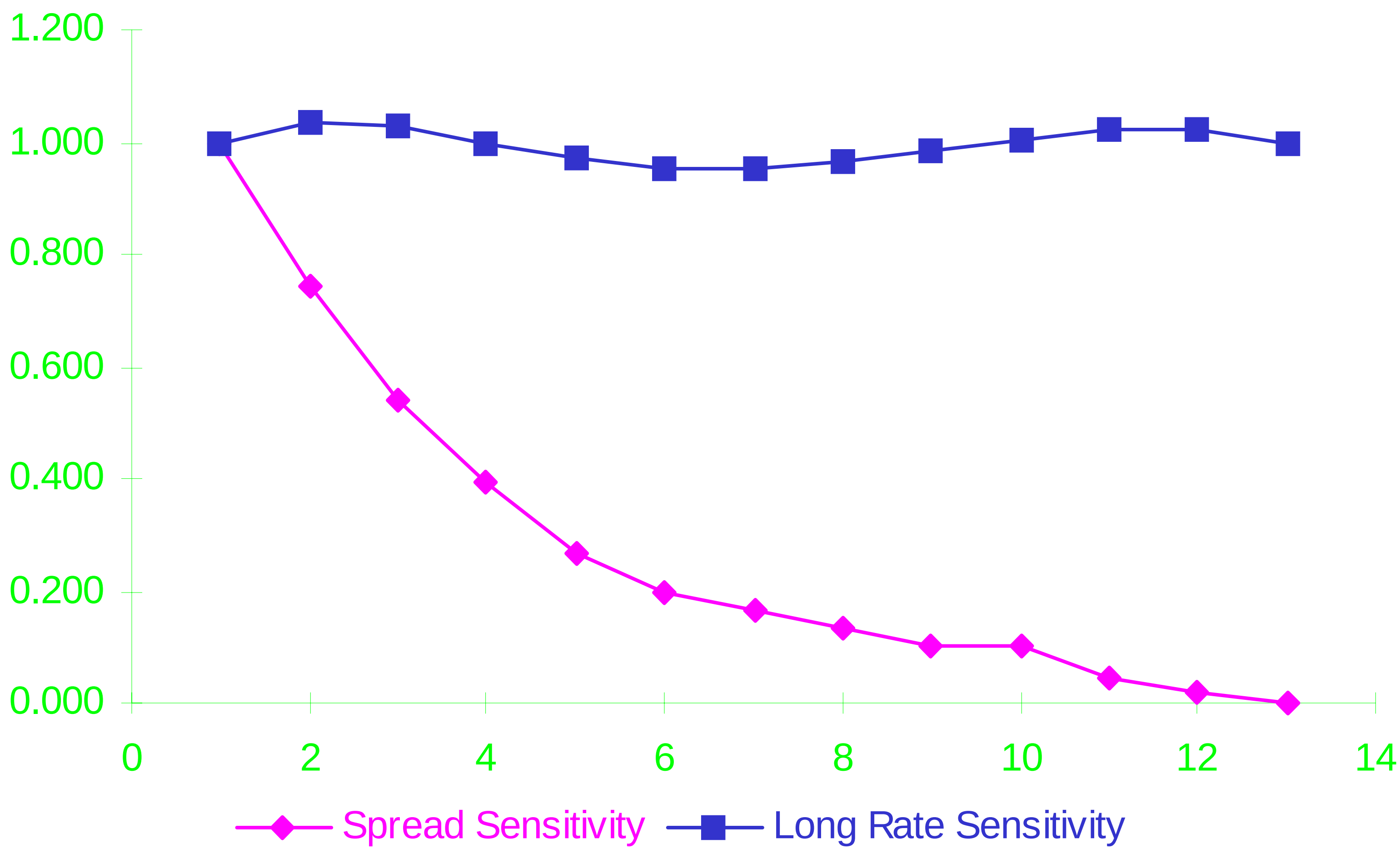
- Regression Analysis

$$\Delta R_T = A_T + \alpha_T \Delta S + \beta_T \Delta L + \varepsilon_T$$

Estimated Long Rate and Spread Sensitivities (Schaefer)

Maturity (Years)	Spread Sensitivity	Long Rate Sensitivity
1	1.000	1.000
2	0.743	1.036
3	0.542	1.026
4	0.391	0.997
5	0.269	0.970
6	0.200	0.953
7	0.163	0.950
8	0.131	0.962
9	0.100	0.983
10	0.100	1.005
11	0.043	1.022
12	0.019	1.022
13	0.000	1.000

Spread and Long Rate Sensitivities



Implied Spot Rates: relative Importance of Factors

Maturity	Total Variance Explained	% of Total Explained Variance Accounted for by		
		Factor 1	Factor 2	Factor 3
6 Months	99.5	79.5	17.2	3.3
1 year	99.4	89.7	10.1	0.2
2 years	98.2	93.4	2.4	4.2
5 years	98.8	98.2	1.1	0.7
8 years	98.7	95.4	4.6	0.0
10 years	98.8	92.9	6.9	0.2
14 years	98.4	86.2	11.5	2.2
18 years	93.5	80.5	14.3	5.2
Average	**98.4**	**89.5**	**8.5**	**2.0**

Source: Journal of Fixed Income, “Volatility and the Yield Curve”, Litterman, Scheinkman & Weiss

Example: Calculating Spread Duration

- 8% 4-year bond
- Spot rates 10% flat

Time	Cash Flow	DF	PV	Spread Sensitivity	Time x PV x Spread Sensitivity
1	8	0.9091	7.27	1.000	7.27
2	8	0.8264	6.61	0.743	9.82
3	8	0.7513	6.01	0.542	9.77
4	108	0.6830	73.77	0.391	115.37
TOTAL			93.66		142.24

Spread Duration = 142.24 / 93.66 = 1.52

Immunization Conditions

- Portfolio Weights add to One
- Match Spread Duration
  - Weighted average of spread duration of assets = spread duration of liabilities
- Match Long Duration
  - Weighted average of long duration of assets = long duration of liabilities
- Equations
  - $w_1 + w_2 + w_3 = 1$
  - $w_1 D_{1S} + w_2 D_{2S} + w_3 D_{3S} = D_S$
  - $w_1 D_{1L} + w_2 D_{2L} + w_3 D_{3L} = D_L$

When One Asset is Cash

- Sensitivity of cash to all interest rates is zero
  - $w_1 D_{1S} + w_2 D_{2S} = D_S$
  - $w_1 D_{1L} + w_2 D_{2L} = D_L$
- Cash holding is residual
  - $w_3 = 1 - w_1 - w_2$

Lab: Bond Hedging

- Worksheet: Bond Hedging
- Scenario:
  - You have a short position in 8-year bonds
  - Have to hedge using 3 and 15 year bonds
- Hedging
  - Create conventional duration hedge
  - Test under 4 scenarios
  - Create 2-factor duration hedge
  - Repeat test & compare
- See Notes & Solution

Solution: Bond Hedging

- Hedge Structure

Method	Holdings			
	Cash	3yr	8yr	15yr
Conventional	0.00	0.3538	-1.000	0.6462
Two-Factor	-.0089	0.4599	-1.000	0.5490

- Hedge Performance (Profit/Loss)

Scenario	Conventional	2-Factor
I	-27bp	3bp
II	-29bp	3bp
III	28bp	2bp
IV	25bp	2bp

