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LONG MEMORY IN FINANCIAL MARKETS

Jonathan Kinlay,
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This quarter we examine one of the current "hot topics" in financial market empirical research: long memory. Characterized by autocorrelations at very high lags, long memory creates persistence in the series over long time horizons. Autocorrelations decay at a slower, hyperbolic rate, rather than at the exponential rate in a standard ARMA process. Hence events which occur in the process today may continue to influence it for months or even years afterwards. Persistent series are "smoother" with more easily discernable trends, and volatility in such processes scales faster than the square root of time. In the case of financial time series this means that risk-return characteristics tend to improve over longer holding periods. Clearly, the ability to detect and model such effects might potentially convey a significant benefit in terms of the ability to forecast the future evolution of the process concerned. Early empirical research has focused on returns processes, with the finding that long term persistence features to some extent in the returns processes of most asset classes. Financial time series typically are fractionally integrated, meaning they are difference stationary order d, where d < 1 is non-integer. Whereas for a white noise process d = 0.5, a persistent., long-memory black noise process will have a fractional inte-

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REVIEW OF RECENT RESEARCH

Pilar Grau-Carles,
Empirical evidence of long term correlations in stock returns,
Physica A 287, 2000, 396-404

Pilar Grau-Carles examines the long-range behavior of various equity markets using Rescaled Range, Detrended Fluctuation Analysis and ARFIMA models. The series examined include the DJIA, SP500, NIKKEI, IGBM and FTSE indices. The paper refers to the work of Ding et al. which examined the autocorrelation structures of $|r_i|^a$ where r_i is the stock market return and a is a positive number. They found that correlation is stronger in the case where a = 1 than for other values of a. Grau-Carles examines long term correlations on r_i , $|r_i|$ and $|r_i|^2$ for all five indices.

Beginning with classical R/S analysis, he points out the lack of robustness in the R/S statistic in the presence of short memory or heteroscedasticity. Lo (1991) suggests replacing the denominator S, the standard deviation, with a consistent estimator of the square root of the variance of the partial sums of x. (see box on next page for details). The next method used by Grau-Carles is the detrended fluctuation analysis (DFA) due to Peng et al. (1994). The advantage of DFA over Hurst analysis is that it avoids spurious detection of long run correlation that is the product of non-stationarities. In this method we work with the integrate series

$$y(t') = \sum_{T=1}^{t'} x(t)$$

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LONG MEMORY IN FINANCIAL MARKETS

"Bollerslev and Mikkelsen (1996), for example, find strong evidence of high persistence and fractional integration in the volatility process of the SP500 index. Grau-Carles (2000) confirms the presence of significant long memory effects in the volatility processes of all major stock market indices . . ."

"A FIGARCH process exhibits the characteristic volatility clustering effect captured by standard GARCH models, but with the difference that shocks to the error process die away at a slower, hypergeometric rate . . ."

(Continued from page 1)

gration parameter $0 < d < 0.5$. Feder (1988) showed that a fractionally integrated random walk may be simulated using the formula:

$$\Delta y_H(t) = \left(\frac{n^{-H}}{\Gamma(H+0.5)}\right) \times \left\{ \sum_{i=1}^{nt} i^{(H-0.5)} E_{(1+n(M+t)-i)} + \sum_{i=1}^{n(M-1)} \left[(n+i)^{(H-0.5)} - i^{(H-0.5)} \right] E_{(1+n(M-1+t)-i)} \right\}$$

Where:

E_t is a strict white noise process
M is the number of periods for which long memory is to be generated
H is the Hurst exponent

In a fractionally integrated process the integration parameter d is linked in a simple way to the Hurst exponent H ($d = H–0.5$) and may consequently be estimated using Hurst's classical rescaled range analysis technique, although alternative procedure exist in the form of quasi maximum-likelihood estimation, Peng's Detrended Fluctuation Analysis, or other methodologies in the frequency domain.
An example of the Hurst exponent estimation technique for the GE volatility process is shown in Fig. 1.

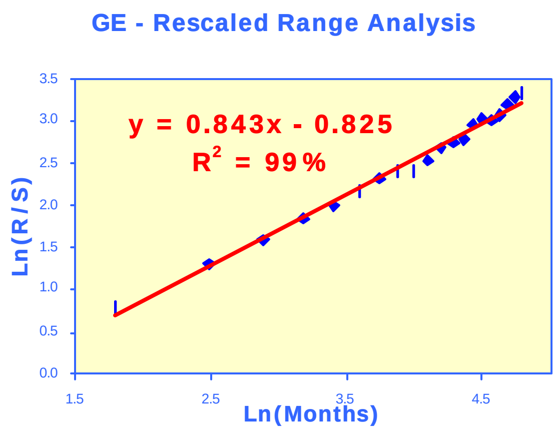

Figure 1

A modeling framework has been developed by Bollerslev and others to handle such processes. ARFIMA (Auto-Regressive Fractionally Integrated Moving Average) models incorporate both long– and short-term memory effects in the returns process. The general form of the model is:

$$f(L)(1-L)^d y_t = q(L)e_t$$

$$(1-L)^d = \sum_{j=0}^{\infty} \frac{\Gamma(j-d)L^j}{\Gamma(-d)\Gamma(j+1)}$$

Where ϕ and θ are polynomials in the lag operator L.

More recently attention has turned to modeling of volatility processes. The empirical evidence is that long memory effects are typically far more pronounced in volatility processes than in returns series, with fractional integration parameters in the region of 0.25—0.40. Bollerslev and Mikkelsen (1996), for example, find strong evidence of high persistence and fractional integration in the volatility process of the SP500 index. Grau-Carles (2000) confirms the presence of significant long memory effects in the volatility processes of all major stock market indices, with the possible exception of the FTSE 100. By way of illustration, Fig. 2 shows long term autocorrelations in the volatility processes of a number of DOW stocks. This clearly indicates the presence of significant autocorrelations in the volatility processes at lags of up to two years or even longer.

Several theories have been developed to explain the presence of long memory effects in volatility processes. Lamoureux and Lastrapes (1990) demonstrated that long memory effects could arise from regime switching in the volatility process. Zin and Bachus (1993) developed a theory to show how long memory effects would spread from other variables, while Anderssen and Bollerslev (1997) showed that similar effects could also arise from the aggregation of a news arrival process with differing persistence levels.

Another important development in this field is the Fractionally Integrated GARCH model developed by Baillie, Bollerslev and Mikkelsen in 1996. The FIGARCH model takes the form

$$f(L)(1-L)^d e_t^2 = w + [1-b(L)]v_t$$

Where f and b are polynomials in the lag operator L, and $V_t = \varepsilon_t^2 — \sigma_t^2$
A FIGARCH process exhibits the characteristic volatility clustering effect captured by standard GARCH models, but with the difference that shocks to the error process die away at a slower, hypergeometric rate rather than the short-term exponential decay typical of a short memory process. Grau-Carles (2000) develops FIGARCH models for a number of major equity indices, pro-

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REVIEW OF RECENT RESEARCH

(Continued from page 1)

The integrate series is divided into non-overlapping intervals, each containing m data points. In each interval a least-squared line is fitted and the y-coordinate $y_m(t')$ is estimated. Next the root mean square fluctuation of the integrated and detrended series is calculated:

$$F(m) = \sqrt{\frac{1}{T}\sum_{t'=1}^{T}[y(t') - y_m(t')]^2}.$$

This calculation is repeated over all intervals. A linear relationship on a double log graph of F(m) indicates the presence of a power law. If there is no correlation, or only short-term memory, then F(m) ~ $m^{1/2}$, but a higher power coefficient indicates the presence of long memory.

Grau-Carles also uses the method of Geweke and Porter-Hudak (1983) to estimate the factional differencing parameter d based on the slope of the spectral density function around the angular frequency w = 0. The spectral regression is defined by :

$$\ln\{I(w_\lambda)\} = a + b\ln\left\{4\sin^2\left(\frac{w_\lambda}{2}\right)\right\} + n_\lambda, \quad \lambda = l, \ldots, v,$$

Where $I(\omega_\lambda)$ is the periodogram of the time series at frequencies $\omega_\lambda = 2\pi\lambda/T$. The least squares estimate of the slope coefficient provides and estimate of d.

The fourth method was developed by Sowell (1992) and is a procedure to estimate stationary ARFIMA models using the autocovariance function in terms of the spectral density function:

$$g(k) = \frac{1}{2p}\int_0^{2p} f(w)e^{iwk}\,dw$$

The parameters of the model are then estimated by maximum likelihood.

Turning now to the results, Grau-Carles finds that for the five index returns series the estimated fractional integration parameter d, while statistically significant, is estimated to be very small indeed by all four methods of analysis. In other words there is little or no long term dependence in the returns processes. The same is not true for the absolute and squared returns. The results from the various methods are quite consistent and show that long memory is found to be stronger in absolute returns rather than squared returns. Typical results are shown in the following table, which shows estimated exponents from DFA analysis of the returns, absolute returns and squared returns series.

	r_i	|ri|	$\|ri\|^2$
DOW [a]	0.490	0.627	0.600
DOW [b]	0.497	0.813	0.650
SP500 [a]	0.528	0.896	0.835
SP500 [b]	0.490	0.825	0.685
FTSE	0.393	0.616	0.601
NIKKEI	0.502	0.830	0.724
IGBM	0.603	0.835	0.797

Table 1

a The whole data b Last 9000 data points

(Continued on page 5)

"... Grau-Carles finds that for the five index returns series the estimated fractional integration parameter d, while statistically significant, is estimated to be very small indeed by all four methods of analysis. In other words there is little or no long term dependence in the returns processes. The same is not true for the absolute and squared returns. The results from the various methods are quite consistent and show that long memory is found to be stronger in absolute returns rather than squared returns."

MODIFIED RESCALED RANGE ANALYSIS

Lo (1991) developed a modified version of the rescaled range statistic to take account of short term memory and heteroscedasticity, which can lead to inflated estimates of the Hurst exponent H. Lo derives the limiting distribution of $Q_T / T^{1/2}$ under the null hypothesis of no memory and shows that the modified rescaled range statistic is robust to short-range dependence.

In a recent paper, Teverlosky et al. (1999) show that the Lo test tends to reject the null hypothesis of no long-range dependence when in fact such dependence is present in the series and that the choice of the lag q is crucial.

$$Q_T = \left\{\max_{1\leqslant i\leqslant T}\sum_{t=1}^{i}(x_t - \bar{x}) \quad \min_{1\leqslant i\leqslant T}\sum_{t=1}^{i}(x_t - \bar{x})\right\} \Big/ s_T(q)$$

$$s_T(q) = \left\{\sum_{i=1}^{T}(x_i - \bar{x})^2/T + \frac{2}{T}\sum_{j=1}^{q}\tau_j(q)\left(\sum_{i=j+1}^{T}(x_i - \bar{x})(x_{i-j} - \bar{x})\right)\right\}^{1/2}$$

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LONG TERM MEMORY IN FINANCIAL MERKETS

"Alexander (2001) points out there are problems with using the standard statistical criteria for performance evaluation. Minimising root mean square error, for instance, is equivalent to likelihood maximization when the likelihood is normal with a constant volatility."

"The investment implication of this is that divergences in fractionally cointegrated volatility processes may be more stable than the processes themselves. Cointegrated baskets of volatility processes potentially offer opportunities for statistical arbitrage at relatively low risk. . . ."

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ducing estimates in the range 0.11–0.42. FIGARCH models can produce significant improvements in volatility forecast accuracy, although as Alexander (2001) points out there are problems with using the standard statistical criteria for performance evaluation. Minimising root mean square error, for instance, is equivalent to likelihood maximization when the likelihood is normal with a *constant* volatility. The excessive variation in squared returns also causes problems for regression models that fit the squared returns to the variance forecast. The R^2 from this regression will be bounded above (the bound depending on the form of the generation process), resulting in very low values.

One criteria of performance that has some validity is sign prediction accuracy: the ability to forecast whether volatility will expand or contract in a future period. The statistical significance of direction forecast accuracy can readily be tested using non-parametric Sign, Wilcoxon or Pesaran-Timmerman tests. Direction forecast accuracy using FIGARCH models typically averages 70% - 75% for most volatility processes.

The univariate FIGARCH model can readily be extended to the multivariate environment, with general form:

$$\mathbf{F}(L)\mathbf{D}\varepsilon_t^2 = \mathbf{w} + (1-\mathbf{B}(L))v_t$$

Where the matrix $\mathbf{D}$ has diagonal elements $(1-L)^{dj}$.

A further step in the modeling process brings in the cointegration concept developed by Granger (1987) and Engle (1987). Put simply, the idea is that linear combinations of non-stationary variables may be stationary or low-order integrated. For instance, while neither spot or futures prices are stationary processes, their difference typically is. The concept was further extended by Robinson and Marinuci (1989), Cheung and Lai (1993) and Baillie and Bollerslev (1994) to describe situations in which either or both of the parent processes and sub-processes may be fractionally integrated, which is the case with volatility series. The investment implication of this is that divergences in fractionally cointegrated volatility processes may be more stable than the processes themselves. Cointegrated baskets of volatility processes potentially offer opportunities for statistical arbitrage at relatively low risk. Cointegration analysis using the Engle-Granger testing procedure, or the more sophisticated methodology due to Johansen, facilitate the identification of cointegrated volatility baskets and so reduce the dimensionality of the investment problem. Further dimensionality reduction can be achieved using standard multivariate data exploratory techniques such as cluster or factor analysis. The end result is a reduced investment set of stocks whose volatility processes are linked in a specific way, enabling the construction of volatility portfolios with attractive risk-return characteristics.

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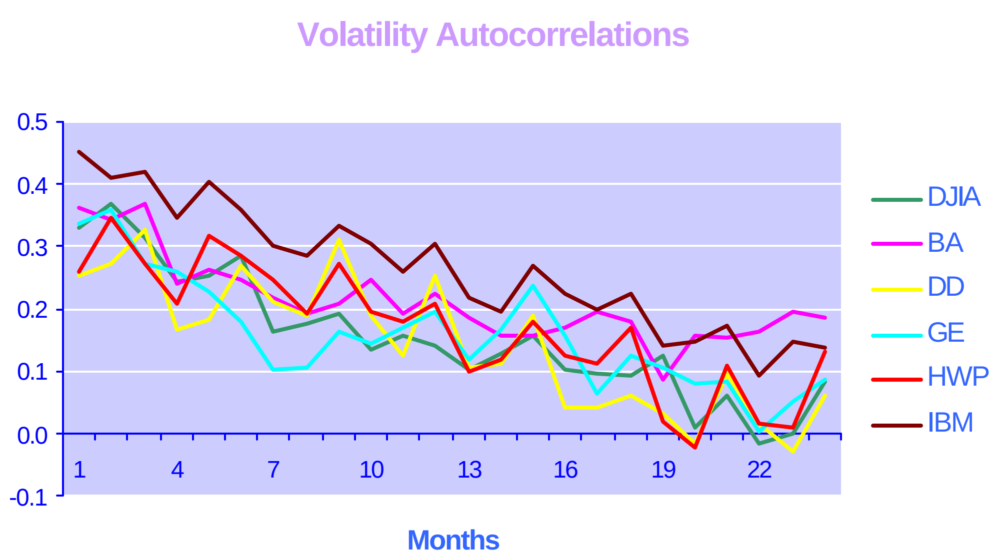

Figure 2

REVIEW OF RECENT RESEARCH

"If the two volatility processes are highly correlated, perhaps due to a common information arrival process, it may be possible to undertake risk arbitrage strategies, buying straddles in the lower volatility market and selling in the higher volatility market. Strategies of this kind will tend to drive the volatilities together, but will be risky if the two markets are not cointegrated. The question of cointegration consequently assumes some importance in terms of risk evaluation."

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Celso Brunetti and Christopher Gilbert,
Bivariate FIGARCH and fractional cointegration,
Journal of Empirical Finance 7 (2000) 509-530

Brunetti and Gilbert tackle the issue of modeling cointegrated volatility processes, in effect combining the univariate fractional (FIGARCH) volatility technology with multivariate GARCH models. The paper demonstrates the feasibility of estimating and testing cointegrated bivariate FIGARCH models and applies the techniques to the volatility processes of the NYMEX and IPE crude oil markets.

At the start of their paper the authors describe two possible hypotheses regarding the volatility processes. The first is that the two processes, while independent, are driven by a common information arrival process. If information relates to the energy market as a whole rather than to any particular sector one would expect similar price movements in both markets and this might result in similar levels of integration in the two processes, but not fractional integration. The other possibility is that changes in volatility in one market spills over into the second. The causal links may be unidirectional or bidirectional.

If the two volatility processes are highly correlated, perhaps due to a common information arrival process, it may be possible to undertake risk arbitrage strategies, buying straddles in the lower volatility market and selling in the higher volatility market. Strategies of this kind will tend to drive the volatilities together, but will be risky if the two markets are not cointegrated. The question of cointegration consequently assumes some importance in terms of risk evaluation.

Brunetti and Gilbert apply a constant correlation FIGARCH model of the form:

$$\Phi(L)\begin{pmatrix}(1-L)^{d_1} & 0\\ 0 & (1-L)^{d_2}\end{pmatrix}\varepsilon_t^2=\omega+(\mathbf{I}-\mathbf{B}(L))\mathbf{v}_t$$

In this form it is straightforward to test the hypothesis $d_1 = d_2$.

Testing for fractional cointegration in volatility processes raises additional issues. First, volatility is not directly observed and so typically inference is based on the squared residuals of some

(Continued on page 7)

DEFINITIONS OF LONG MEMORY

There are many possible definitions of 'long memory'. Given a discrete time series process y_t with autocorrelation function ρ_j at lag j, then according to McCleod and Hipel (1978), the process possesses long memory if the

quantity $\lim_{n\to\infty}\sum_{j=-\infty}^{\infty}|\rho_j|$ is non-finite.

Equivalently the spectral density f(w) will be unbounded at low frequencies. A stable and invertible ARMA process has autocorrelations which are geometrically bounded, i.e.

$$|\rho_k| \leq cm^{-k}$$

for large k, where 0 < m < 1 and hence is a short memory process.

Alternatively the memory of a process y_t can be expressed in terms of its partial sum:

$$S_T=\sum_{t=1}^{T} y_t$$

The process can be fined as having short memory if the variance of the process

$s^2 = \lim_{T\to\infty} E(S_T^2/T)$ exists and is nonzero

and $[1/\sigma T^{1/2}]S_{[rT]}$ converges in distribution to geometric Brownian motion for all r in [0, 1].

A wider definition of long memory is to include any process which has an autocovariance function for large lags k such that:

$$\gamma_k \approx \Theta(k)k^{2H-2}$$

where $\Theta(k)$ is any slowly varying function at infinity. Helson and Sarason (1967) show that for any process for which H > 0 and autocovariance function given by the above is long memory.

Richard Baillie, *Long Memory processes and Fractional Integration*, Journal of Econometrics 73 (1996)

LONG TERM MEMORY IN FINANCIAL MERKETS

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A sample portfolio of this kind is illustrated in our Volatility Report research product (see web site at www.volatility-report.com). This is a simple volatility portfolio comprising long and short option positions in five large-cap stocks (plus the SP500 index) with fractionally cointegrated volatility processes. Allocations are computed monthly on the basis of our multivariate FIGARCH model forecasts and optimised using a proprietary genetic algorithm procedure. The portfolio is not delta-hedged (although it can be), but rather overlaid with a Platinum Hedge using the Crashmetrics methodology developed by Paul Wilmott and Philip Hua. The hedge is integrated into the portfolio design at the time of construction. The result is a very stable, low volatility market-neutral and crash-protected portfolio averaging returns of around 2.5% a month and volatility of below 14% per annum.

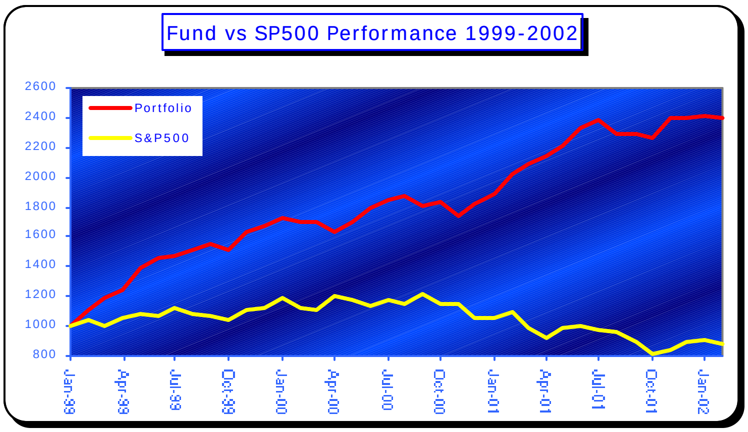

Figure 3

VOLATILITY DIRECTION PREDICTION

We use FIGARCH models to construct 1-period ahead forecasts of equity volatility using a proprietary volatility index. Direction prediction accuracy is assessed using a rolling 12-month moving average. As the Figure below illustrates, directional forecasting ability fluctuates, overt time, but a long term average in the region of 70% - 75% is typical for most stocks. This is both statistically and economically significant.

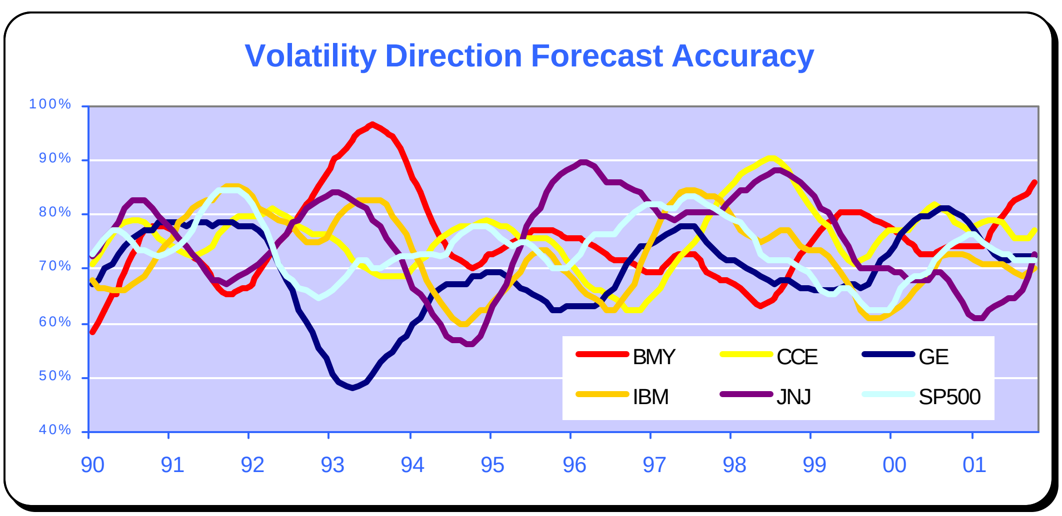

Figure 4

Protecting Portfolios Against Extreme Market Conditions.

During extreme markets the normal relationship between financial assets is lost, market movements exceed normal day-to-day amounts and assets become more closely correlated. At the time it is needed most, diversification is no longer possible and popular risk management techniques such as Value-at-Risk break down.

Xtremis is a proprietary risk management methodology that provides protection against extreme adverse market movements and monitors the risk to clients' fixed income or equity portfolios on an ongoing basis. The methodology is based on the CrashMetrics® and Platinum Hedging concepts invented by Paul Wilmott and Philip Hua. Market crash simulations using proprietary models are used for advising clients on their exposure and risk protection is constructed using our unique Platinum Hedging concept.

Xtremis offers a number of advantages:

- Portfolio protection is optimized to minimize worst-case losses
- Hedging is static to minimize cost to the client
- Dynamic hedging is not typically used. This avoids problems associated with illiquidity during crashes
- Static hedging reduces model error
- Unstable parameters such as volatility and correlation are not required
- Constraints on positions can be included (local quantities, such as deltas, and global quantities)

REVIEW OF RECENT RESEARCH

(Continued from page 5)

parametric process. Secondly, since skedastic processes in finance are generally found to be stationary, the procedure used to estimate the cointegrating combination cannot presuppose nonstationarity. In the context of the authors' bivariate framework, fractional cointegration is meaningful only if $d_1 = d_2$.

It is natural to consider direct tests for fractional cointegration. In performing such tests the authors consider both squared returns and absolute returns as proxy measure of volatility. The Engle-Granger procedure for estimating cointegrating vectors is available only for non-stationary series and is therefore not applicable in this case. The authors opt for a simpler procedure in which they assume a unit cointegration vector. This makes sense since it seems plausible that volatilities in the two markets should move together uniformly. With a know cointegrating vector, testing for fractional cointegration becomes straightforward since standard univariate procedures can be applied to the cointegrating vector. Consequently the analysis follows the following stages:

1. Estimation of fractional orders $d_1 = d_2$ using standard univariate techniques. The authors do this using ARFIMA models of absolute and squared returns, and also from univariate FIGARCH models.
2. Hypothesis testing of $d_1 = d_2$.
3. If there is a common degree of integration, test for fractional cointegration using the ARFIMA model of squared returns imposing a unit cointegrating vector.
4. If there is evidence of fractional integration, estimate a an error correcting bivariate FIGARCH model, again imposing a unit cointegrating vector.

In stage 1, model selection tests suggest concentration on the ARFIMA (1,d,1) model, which the authors estimate using the exact MLE derived by Sowell (1992). The long-memory parameter estimates from both ARFIMA and FIGARCH models indicates that both absolute and squared returns are long memory and stationary processes, with a common fractional integration parameter. To test the hypothesis the authors move the bivariate framework and apply the constant correlation bivariate FIGARCH model of Boller-

(Continued on page 8)

AUTOCORRELATIONS IN NYMEX ABSOLUTE RETURNS

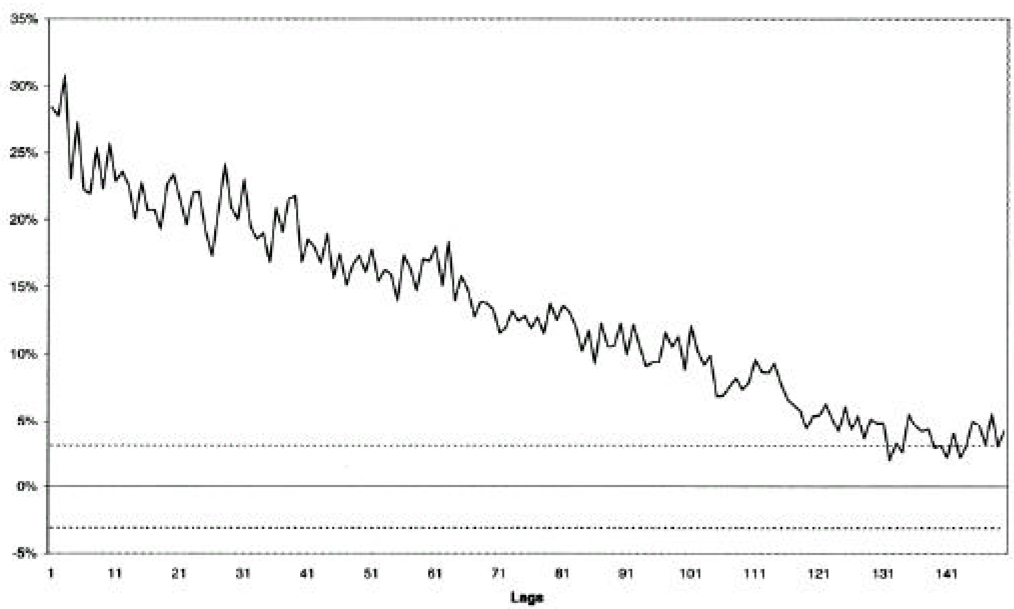

Figure 5

REVIEW OF RECENT RESEARCH

"... the two markets are causally linked and that IPE volatility reacts far more strongly to shocks in the NYMEX volatility process than the other way around. This is in line with the conventional wisdom that the NYMEX is the dominant market."

"The volatility processes on two crude oil markets are shown to be highly persistent, with a common degree of fractional integration, and are fractionally cointegrated.

Fractional cointegration implies that statistical arbitrage between the two markets, for example by straddle trading across the markets, is associated with a relatively low degree of risk."

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slev et al. A likelihood test fails to reject the null hypothesis of common fractional cointegration. The estimated order of integration of the parent series is d = 0.40 and is significantly different from zero. The results also indicate that the volatility processes of the two crude oil markets are fractionally cointegrated with parameter d' = 0.22 < d. This implies that the difference in the two volatility processes is still a long memory process, but exhibits a lower order of fractional integration than the two parent series. Further model parameter tests reveal that the two markets are causally linked and that IPE volatility reacts far more strongly to shocks in the NYMEX volatility process than the other way around. This is in line with the conventional wisdom that the NYMEX is the dominant market.

Conclusion

This paper demonstrates the feasibility of estimating and testing fractional cointegrated bivariate FIGARCH models , at least in the natural case where attention is restricted to a unit cointegrating vector. The volatility processes on two crude oil markets are shown to be highly persistent, with a common degree of fractional integration, and are fractionally cointegrated. Fractional cointegration implies that statistical arbitrage between the two markets, for example by straddle trading across the markets, is associated with a relatively low degree of risk.

- END -

OPTION PRICING WITH LONG MEMORY

Bollerslev and Mikkelsen (1999) find that the market prices of exchange traded options on the SP500 index, with maturities from nine months to three years, are described more accurately by a long memory pricing model. Hence some option prices reflect the long memory phenomenon in volatility, although the authors find that significant biases remain unexplained. LEAPS contracts are investigated by Bakshi et al. (2000).

Taylor (2000) develops FIEGARCH models for option pricing and compares theoretical and market prices of options on the SP100 index. The FIEGARCH specification is of the form:

$$\ln(h_t) = \alpha + (1-\phi L)^{-1}(1-L)^{-d}(1+\psi L)g(z_{t-1}),$$

$$g(z_t) = \theta z_t + \gamma(|z_t| - C),$$

where α, ϕ, d, ψ respectively denote the location, autoregressive, differencing and moving average parameters of the log conditional variance $\ln(h_t)$. The iid residuals $g(z_t)$ depend on a symmetric response parameter γ and an asymmetric response parameter θ that enables the conditional variances to depend on the sign of z_t. Taylor compares the theoretical Black-Scholes prices of hypothetical SP100 index options with the those derived from an estimated FIEGARCH model. The implied volatility term structure varies significantly for short vs. long memory models and it is common for short and long memory implied volatilities to differ by 1% or more. However, this may not be economically significant in equity options markets.

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