Forecasting and Trading Volatility in the S&P 500 Index – An Empirical Test of Options Market Efficiency

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Background

While tests of the efficiency of underlying asset markets has been the focus of a very considerable body of empirical research, it is only comparatively recently that attention has turned to examining the behavior and efficiency of asset volatility markets. Views on asset volatility are typically expressed in options markets, with the S&P 500 index options being amongst the most liquid: around 100,000 contracts trade daily, with around 4,000,000 contracts in open interest. Hull and White (1987) show that, when volatility is constant, the Black-Scholes implied volatility of an at-the-money option approximately equals the expected future volatility over the life of the option. Consequently, assuming options markets are efficient, the option implied volatility is likely to be an unbiased predictor of future market volatility. To the extent that options markets are efficient, implied volatility should be a superior predictor of future volatility than any forecasting model based on widely known theory and using historical volatility data.

There have been a number of studies examining the relationship between implied and future realized volatility, with very mixed findings with regard to the central question of market efficiency. Camina and Figlewski (1993) find that the implied volatility of S&P 100 index options contain no information about future volatility, suggesting that the options market may be inefficient. However, Christensen and Prabhala (1998) point out that the use of overlapping periods to estimate historical volatility introduces serious autocorrelation bias because the volatility estimates for two consecutive periods share almost all of the same information. Fleming (1998) corrects for serial dependence and goes on to find that the implied volatility from S&P100 index options produces superior forecasts of future realized volatility compared with forecasts made using historical volatility.

Taylor and Xu (1997) examine a parallel question in the foreign exchange markets, comparing the forecasting ability of implied and historical volatility for the Deutschemark/Dollar exchange rate. They find that when daily observations are used to construct the data series implied volatility forecasts are superior. However, when intra-day 5-minute returns are used, historical volatility outperforms implied volatility in forecasting realized volatility. In a more extensive study, Pong, Shakelton, Taylor and Xu (2003) compare the performance of implied volatility forecast against a number of short- and long-memory ARMA and GARCH models for Pound, D-Mark and Yen exchange rates against the US-Dollar. Their tests indicate that model forecasts have incremental information not found in implied volatilities, for forecast horizons of up to one week. By contrast, implied volatilities are found to incorporate most of the relevant historical information when the horizon extends to one month or longer. On the other hand, Blair, Poon and Taylor (2001) claim almost all the useful predictive information is in option prices when forecasting S&P 100 index volatility one or more days into the future.

In general, the use of high-frequency data has led to a dramatic improvement in both measuring and forecasting volatility. Anderson et al (2001) use a fractionally integrated autoregressive process to capture the long memory effects in volatility and forecast realized or integrated volatility, defined as the summation of high-frequency squared returns. Martens and Zein (2002) find that high-frequency forecasts do have incremental information over that contained in implied volatilities for futures and options on futures for the S&P 500 index.

Harvey and Whaley (1992) take a different approach to assessing option market efficiency. They test the profitability of a trading rule based on forecasting implied volatility, concluding in favor of option market efficiency after allowing for transaction costs. Noh and Engle (1994), by contrast, find that a simple strategy involving trading straddles on the S&P 500 index based on GARCH model volatility forecasts can make significant profits after transaction costs.

The purpose of this study is to examine the question of option market efficiency for the S&P 500 index from two perspectives. The first objective is to discover whether forecasting models using high-frequency data and incorporating both long- and short-term memory effects is capable of outperforming implied volatility in forecasting realized volatility. Secondly, an analysis is made of the potential for generating abnormal profits using a simple straddle trading strategy, based on model-driven volatility forecasts, taking into account both transaction and (delta) hedging costs.

Data and Sampling Procedure

Tick by tick data for the S&P 500 cash index from February 1983 to December 2003 was collected from multiple data feeds via a commercial data vendor and subjected to scrubbing methodologies to detect and repair bad ticks. The data were then aggregated at five minute intervals and used to compute approximately 5,000 observations of daily realized variance as per Andersen, Bollerslev, Diebold and Ebens, 2000. The realized variance data series was Winsorized at six standard deviations to remove outliers and then subject to log transformation to provide the base time series used for modeling purposes. While the log variance series is approximately Normally distributed, it does exhibit positive skewness (0.23) and excess kurtosis (3.29) when compared to the theoretical distribution. These small but statistically significant departures from Normality are picked up by the standard Chi-squared, Anderson-darling and Kolmagorov-Smirnov tests, all of which reject the null hypothesis of Normality at the 5% level.

A further examination of the data shows evidence of a regime shift in the variance process.

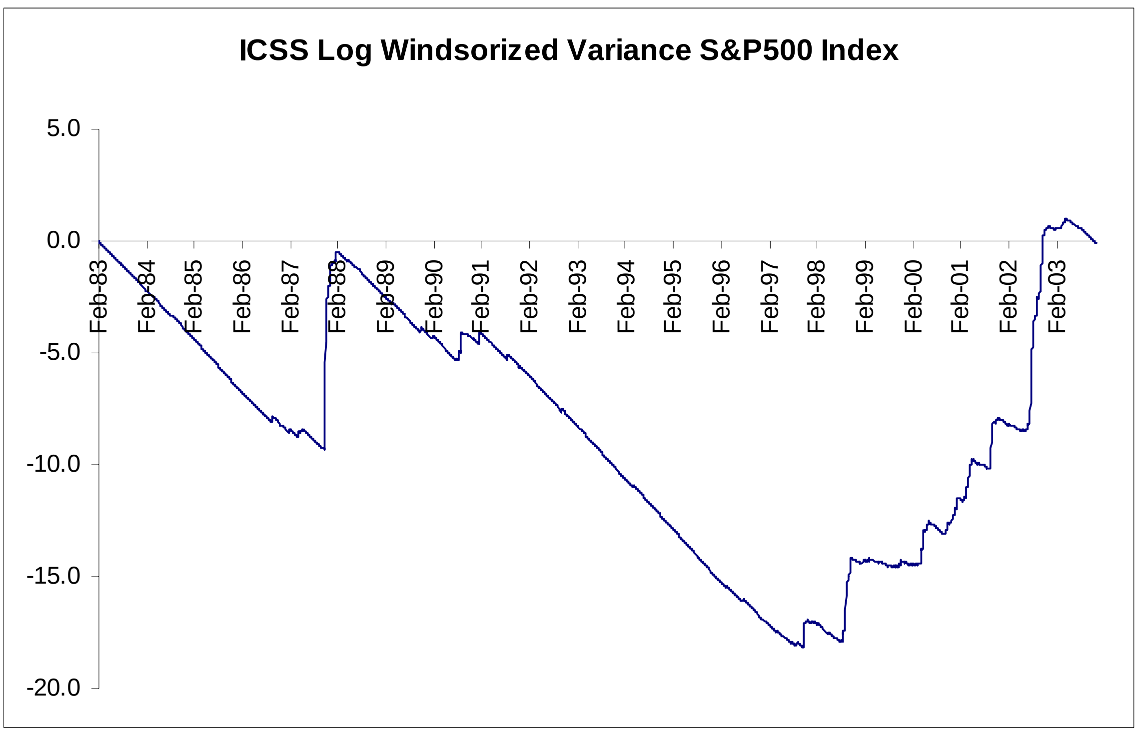


Figure 1: ICSS Test Statistic

The Iterative Cumulative Sum of Squares test statistic suggested by Aggarwal et al (2001) exceeds the 95% confidence limit, indicating a regime shift in the volatility process, in October 1987 and again in 1998 (see figure 1). For this reason the data series was truncated and only the data from 1988 onwards was used for model estimation. The truncated series is close to Normally distributed, having a skewness of 0.17 and kurtosis of 2.74 (see figures following).

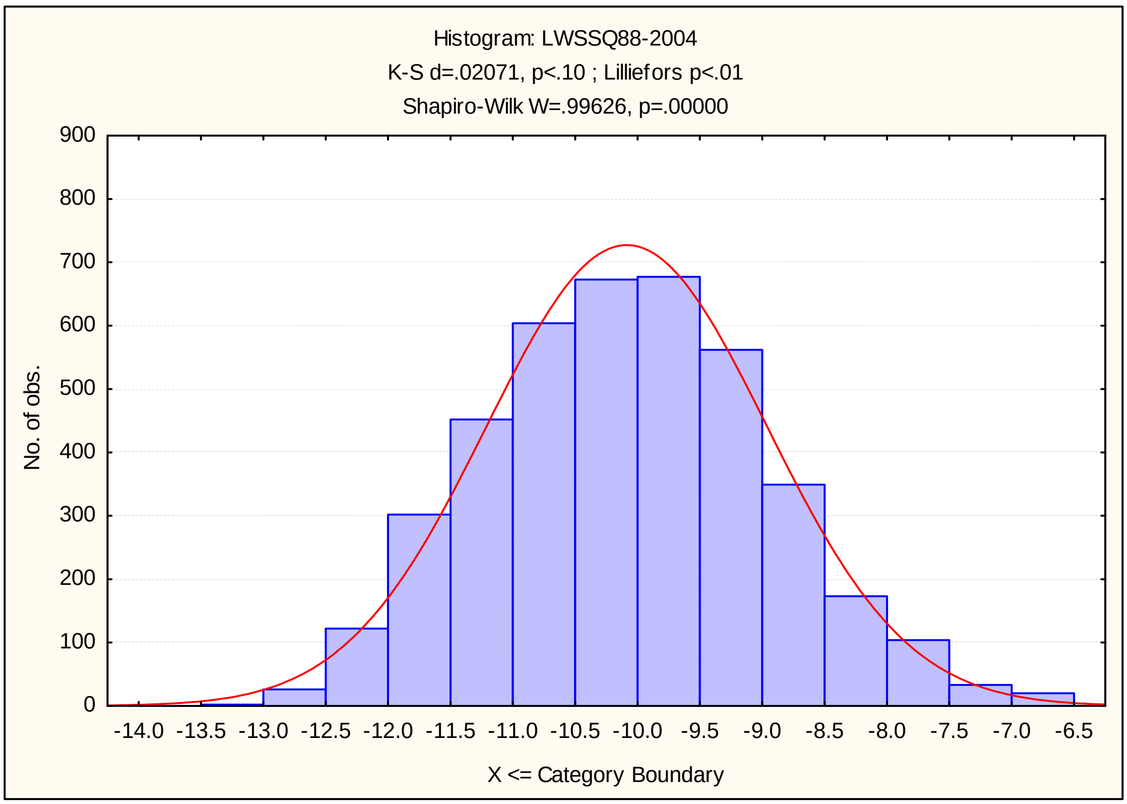


Figure 2: Log Realized Volatility Distribution

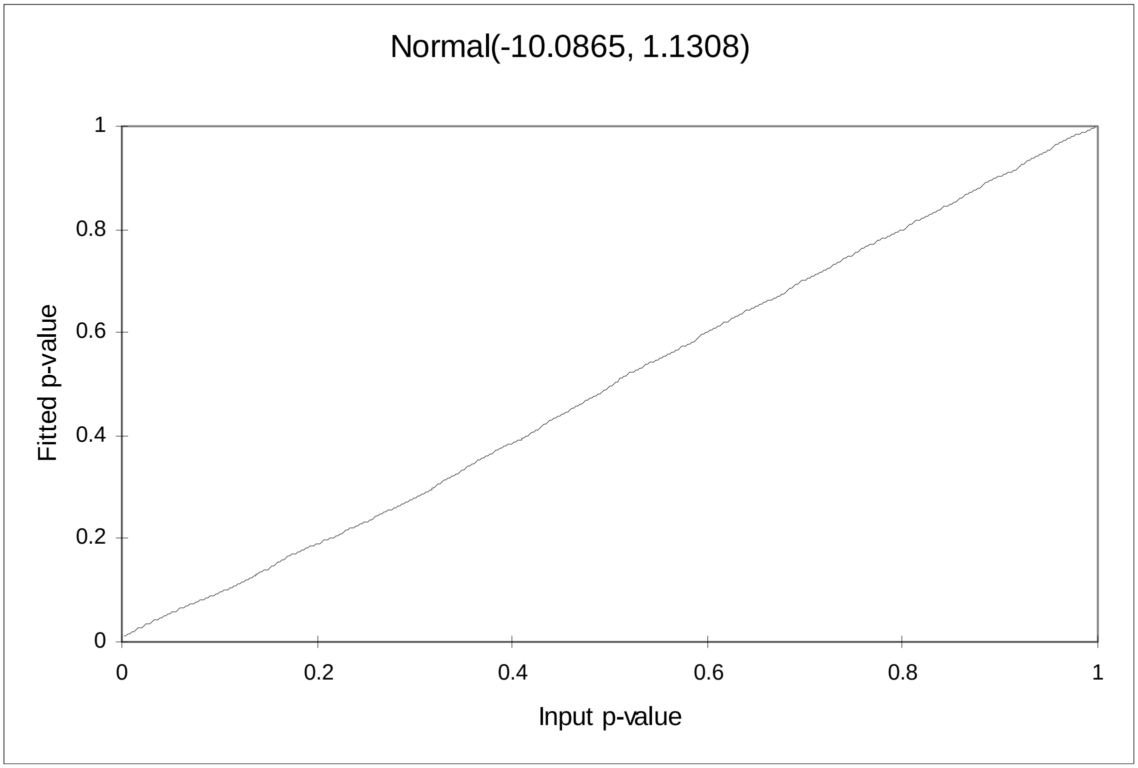


Figure 3: P=P Plot of Log Realized Volatility Distribution

The data series was partitioned to provide a sample used for model estimation, comprising 3,033 daily observations from 4-Jan-1988 to 31-Dec-1999. The out-of-sample series, comprising 1,004 daily log realized volatility observations from 3-Jan-2000 to 31-Dec-2003, was used for model testing as described below.

A further data source was used to provide daily prices and implied volatilities for options on the S&P 500 index from January 2000 to December 2003 and option expiration dates were identified for each month in the out-of-sample period. For each month, a trade date was selected approximately 18 trading days prior to expiration and the average of the implied volatilities of the at-the-money call and put options used as a proxy of the market's forecast of index volatility over the (approximately) three week period to expiration. This process yielded ex-ante market forecasts of S&P 500 index volatility for 48 non-overlapping periods between January 2000 and December 2003. Implied volatility estimates are derived from the Black Scholes option pricing formula:

$$C = SN(d_1) - Xe^{-rt} N(d_2)$$
$$P = Xe^{-rt} N(-d_2) - SN(-d_1)$$
$$d_1 = \frac{\ln(S/X) + (r + \sigma^2/2)t}{\sigma \sqrt{t}}$$
$$d_2 = d_1 - \sigma \sqrt{t}$$

Where S is the index price, assumed to follow a log-normal distribution with constant volatility σ , X is the option strike price, t is the time to maturity and r is the risk-free interest rate, with N() denoting the Gaussian density. The implied volatility is that which equates the observed market price of a given option with the model price for a call option C, or put option P.

The forecasting model, initially estimated to the end of 1999, was used to produce ex-ante forecasts of actual index volatility over each expiration period for comparison with implied volatility forecasts. The model was updated with data up to each successive expiration date for the current period before producing ex-ante volatility forecasts to the end of the next expiration period.

ARFIMA-GARCH Model

Let Y_t for t = 1, . . . T, denote the time series to be modeled. The model estimated is an ARFIMA-GARCH model of the form:

$$(1-L)^d \Phi(L)(Y_t - \gamma_0) = \gamma_{1t} + \theta(L)u_t$$

In which $u_t = h_t^{½} e_t$ where error terms e_t ~ iid N(0,1) and h_t is defined by:

$$h_t = \kappa + \alpha u_{t-1}^2 + \beta h_{t-1}$$

$\phi(L)$ and $\theta(L)$ are finite-order lag polynomials, such that, for example,

$$\theta(L) = 1 - \theta_1 L - \ldots - \theta_p L^P$$

The fractional difference operator term takes the following form:

$$(1-L)^d = \sum_{j=0}^{\infty} \pi_j L^j, \quad \pi_j = \frac{j-1-d}{j} \pi_{j-1}, \quad \pi_0 = 1$$

The model describes volatility as a long memory process, i.e. having long term serial autocorrelation, interrupted by periodic shocks from the kurtosis process, which itself evolves as an ARMA-in-squares process with Gaussian error terms.

Post Estimation Tests

Standard tests for serial dependence are the provided by the Q tests for levels (Box-Pierce 1970) and squares (McLeod-Li 1983) of the data. The latter is a test for nonlinear dependence in a serially uncorrelated (white noise) series. The default statistic is the Box-Pierce (1970) formula

$$Q = n\sum_{s=1}^{h}\rho_s^2$$

which is asymptotically chi-squared with $k-p-q$ degrees of freedom when applied to the residuals of an ARMA(p, q) model.

One-step ex-post forecasts are obtained by fitting the model using data up to time T, and then computing the usual fitted equation and residuals for periods T + 1 to T + F, such that all right- hand side variables are treated as known. The main purpose of this option is to test model stability. Two test statistics are computed,

$$\text{Forecast Test I} = \frac{\sum_{t=T+1}^{T+F}\hat{u}_t^2}{Var_T(\hat{u}_t)}$$

where the denominator is the usual residual variance from the sample period, and

$$\text{Forecast Test II} = \frac{F^{-1}\sum_{t=T+1}^{T+F}\hat{u}_t^2 - T^{-1}\sum_{t=1}^{T}\hat{u}_t^2}{\sqrt{F^{-1}Var_F(\hat{u}_t^2) + T^{-1}Var_T(\hat{u}_t^2)}}$$

Test I is an asymptotically valid version of Chow's prediction test, distributed as Chi2(F) under the null hypothesis of model stability, also assuming the disturbances are Gaussian. Test II is the usual difference-of-means test on the residual and forecast error variances. It is asymptotically N(0,1) under the stability hypothesis, assuming 4th moments exist, where ‘asymptotic’ is interpreted as min(F,T) $\rightarrow\infty$. That is, these tests are appropriate to ‘small’ and ‘large’ forecast periods, respectively.

Results

In-Sample Model Estimation

Parameter estimates for the initial sample (to 30-Dec-1999) are shown in the table below. All are significant at the 5% confidence level and confirm the presence of important long memory effects in the volatility process. The size of the GARCH AR coefficients exceed indicates that shocks to the volatility process introduced through the fourth moment are fairly persistent, having a half life of 1.14 days.

ARFIMA d	MA1	Intercept	GARCH AR1(1,1)	GARCH MA1(1,1)	GARCH Intercept
0.46343	0.33878	-9.98129	0.64538	0.58729	0.73917

Table 1: ARFIMA-GARCH Model Parameter Estimates

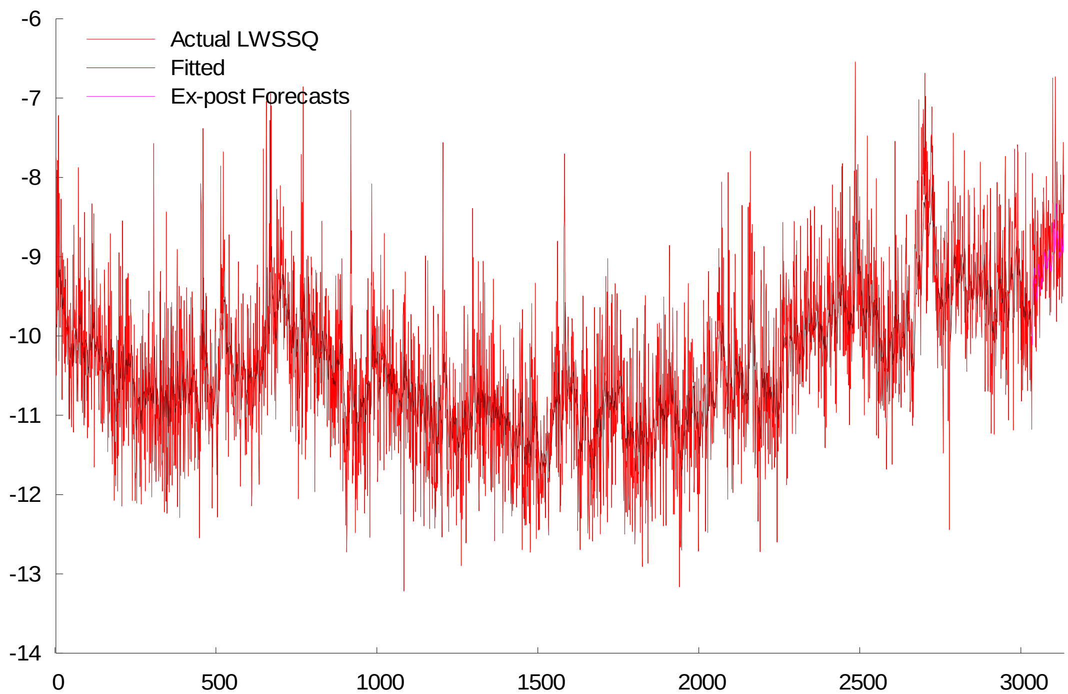


Figure 3: Actual and Forecast Log Volatility

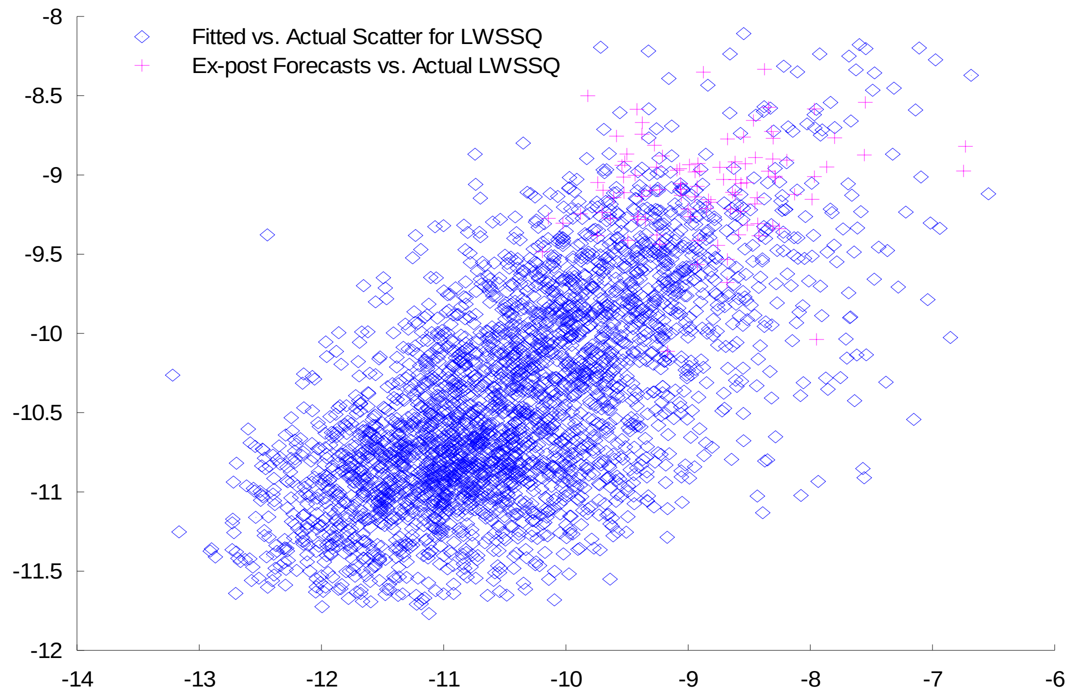


Figure 4: Actual vs. Forecast Log Volatility

There is some evidence of non-normality in the error process as indicated by the small residual skewness and kurtosis values and confirmed by the significant value of the Jaque-Bara test statistic (see Table 2). As the Q-Q plot confirms, this is primarily due to excess kurtosis ("fat tails") in the error process (Figure 6). However, Box-Pierce tests at 40-lags indicate no significant patterning in the residual autocorrelations, nor any indications of residual GARCH effects and this is confirmed by Lagrange Multiplier and Conditional Moment tests (see Table 3).

R Squared	Residual SD	Residual Skewness	Residual Kurtosis	Jarque-Bera Stat.	Box-Pierce(40)	Box-Pierce (Sq)(40)
0.4075	0.7984	0.3386	3.4547	81.317	47.1468	30.4714
				{0}	{0.203}	{0.861}

Table 2: Residuals Test Statistics

Autocorrelation (LM):	ChiSq(40) = 45.5831{0.251}
Autocorrelation (CM):	ChiSq(40) = 47.2716{0.199}
Neglected ARCH (LM):	ChiSq(40) = 8.3297{0.999}
Neglected ARCH (CM):	ChiSq(40) = 36.1712{0.643}

Table 3: Lagrange Multiplier and Conditional Moment Tests

A battery of diagnostic tests confirm that the process is correctly identified as a fractionally integrated process rather than a stationary, or first-difference stationary process (see Table 4).

Tests of I(0)/I(1) (Parzen kernel with bandwidth 4)
Lo's RS test = 7.87784{<0.005}
KPSS test = 12.9966{<0.01}
Phillips-Perron test = -32.2676{<0.01}
Robinson's d = 0.428306

Table 4: Fractional Integration Tests

Stability tests conducted on a sample of 100 ex-post forecasts suggest no instability in the model specification (see Table 5).

Forecast Test 1:	ChiSq(100) = 101.967 {0.426}
Forecast Test 2:	N(0,1) = 0.1417 {0.887}

Table 5: Stability Tests

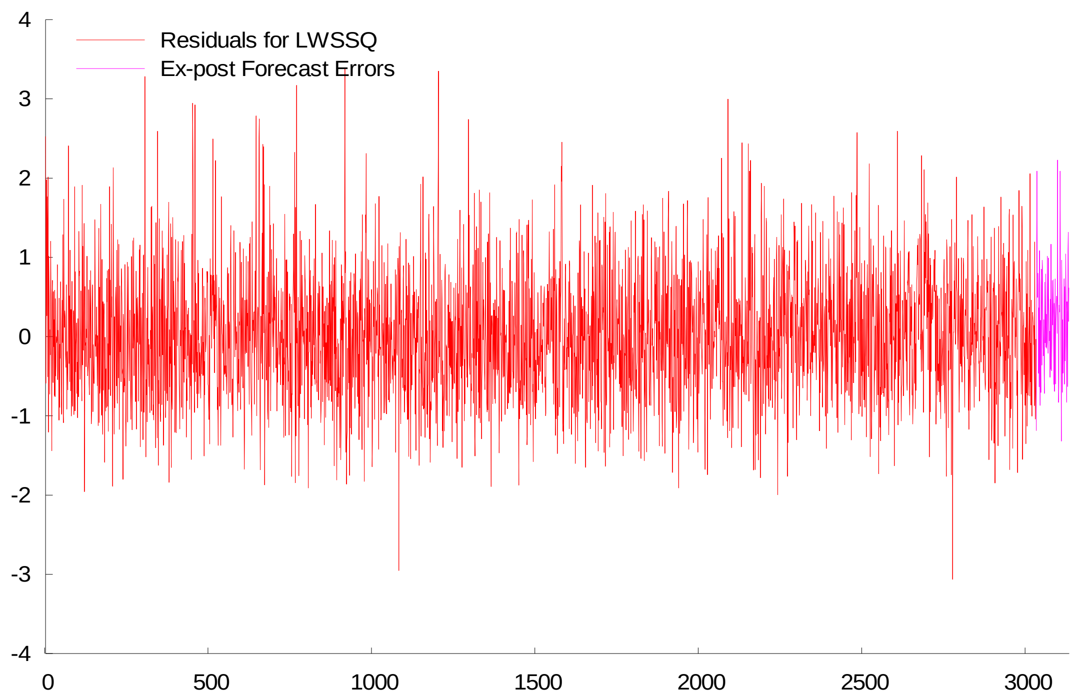


Figure 5: Residuals

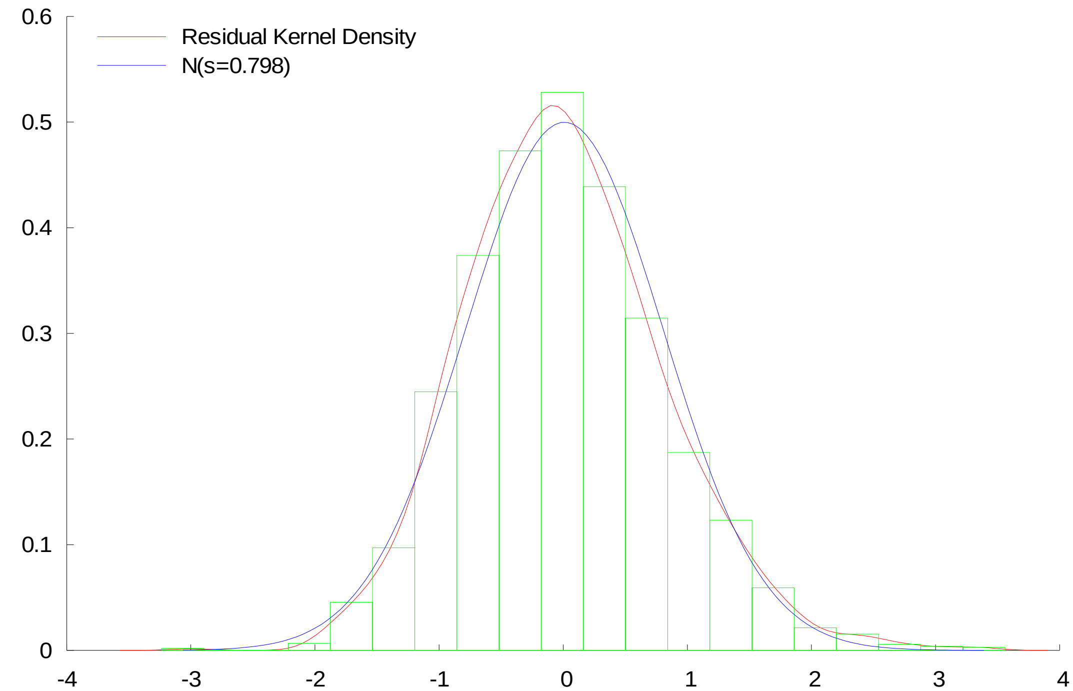


Figure 6: Residuals Distribution

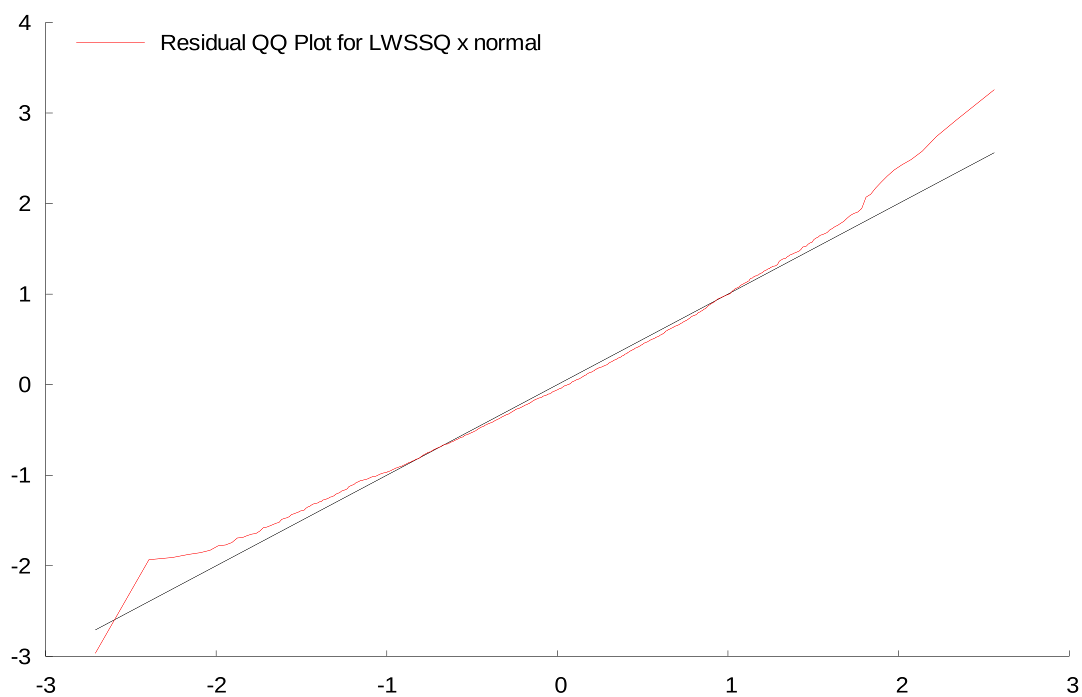


Figure 7: Residuals Distribution Q-Q Plot

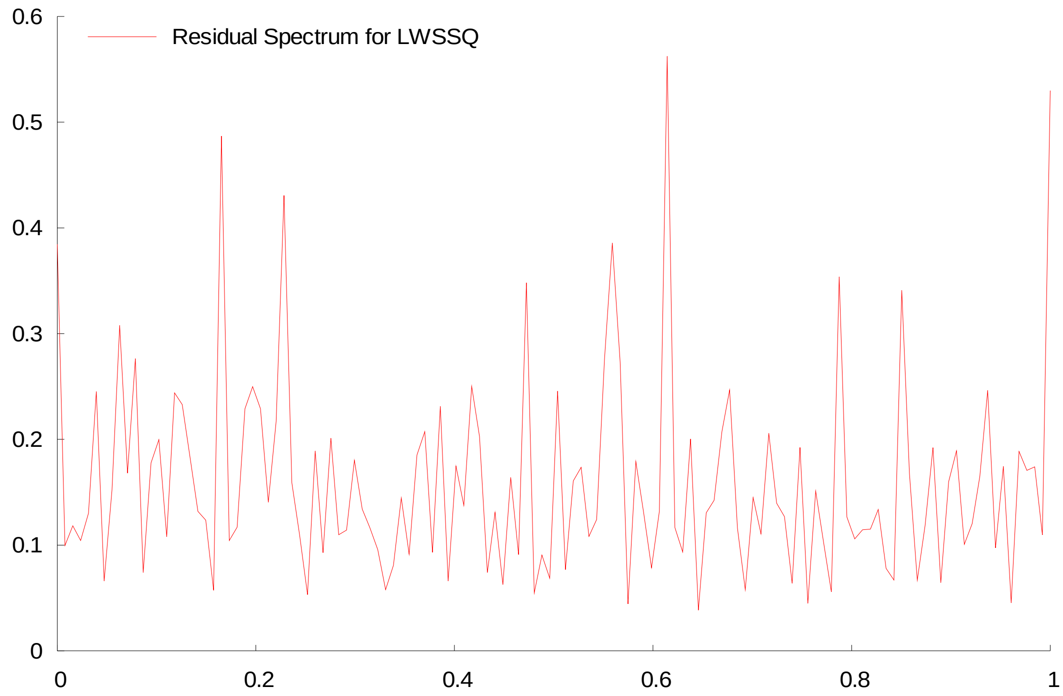


Figure 8: Residual Spectrum

Out-of-Sample Recursive Estimation and Volatility Forecasting

In the second stage of the analysis the model is recursively re-estimated over the entire out-of-sample period from Jan-Dec 2003 and ex-ante, daily log volatility forecasts produced to the end of each expiration period. The daily forecasts are then aggregated to produce ex-ante forecasts for volatility over each entire expiration cycle. Model forecasts are then compared to market forecasts of realized volatility-to-expiration, as expressed in the implied volatilities of call and put options on the S&P 500 index. In this analysis only the closest to the money strikes are used to estimate implied volatility, as the presence of a substantial volatility skew in the out-of-the-money put options would otherwise inflate the market estimate of volatility.

Results of the recursive estimation procedure are summarized in the figures in the Appendix. All of the model parameter estimates shown signs of drift over the out-of-sample period, as the figure for the ARFIMA-d parameter estimates illustrates (see below). However, portmanteau test statistic estimates show stability over the entire period and are statistically insignificant for both error and error-squared autocorrelations. Recursive estimate of residual skewness and kurtosis are likewise very stable.

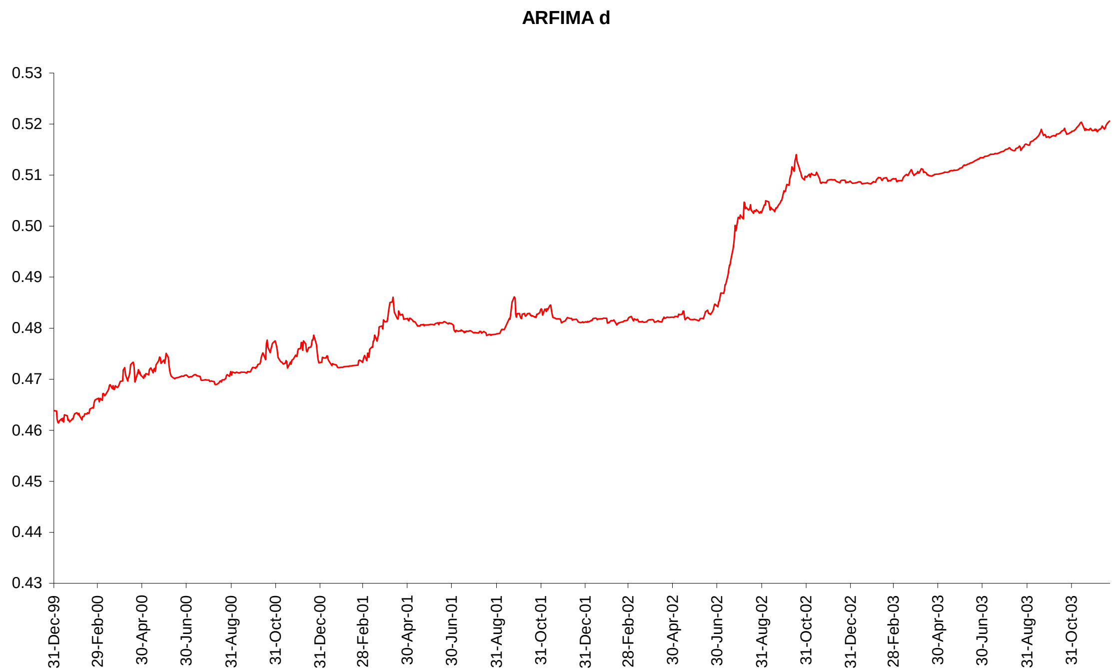


Figure 9: Recursive Estimates of ARFIMA-d Parameter

Turning now to an examination of the forecasts produced by the model we find that, as shown in Table 6, the correlation between realized and implied volatility is significantly higher than that between realized and forecast volatility, while the standard deviation of implied volatility estimates more closely matches that of realized volatility than does the standard deviation of forecast estimates.

	Realized Volatility	Implied Volatility	Forecast Volatility
N	48	48	48
Mean	16.31%	22.41%	16.07%
SD	5.16%	5.33%	3.43%
Skewness	1.06	1.27	0.87
Kurtosis	1.31	1.88	0.23
Range	23.43%	24.78%	13.37%
Min	8.58%	14.62%	11.40%
Max	32.02%	39.40%	24.78%
MAPE		43.2%	17.8%
MSE		23.4%	6.7%
Corr.		78.6%	68.4%
Sign		40.4%	61.7%
# Hits		19	29
Prob		87.85%	3.95%
Theil's U		1.46	0.82

Table 6: Forecast Test Statistics

In other words implied volatility appears to do a better job of anticipating the variation in future volatility (or kurtosis). This explains why the results of regression analysis tend to favor implied volatility over forecast volatility as a (linear) predictor of future volatility. An encompassing regression indicates that there is no additional predictive power to be derived from the addition of volatility forecasts based on historical data, once implied volatility is included (see Table 7)

	Coefficients	*SE*	*t Stat*	*P-value*
Intercept	-0.00776	0.0228	-0.34076	0.734871
Implied	0.758165	0.1806	4.198554	0.000125
Forecast	0.005751	0.28	0.020539	0.983704

Table 7: Encompassing Regression

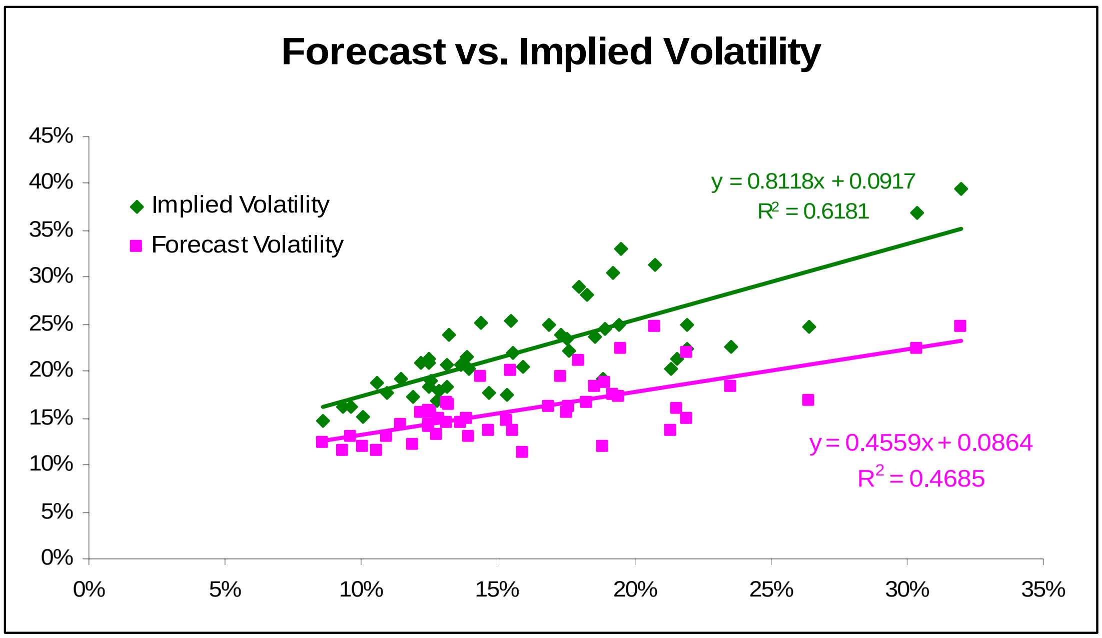


Figure 10: Forecast vs. Implied Volatility

These findings mirror those of other studies, which tend to conclude that volatility markets are more or less efficient.

However, a comparison of the actual (realized), implied and forecast volatility reveals a consistent pattern of bias: implied volatility projections tend to over-estimate future realized volatility, by an average of 610 basis points over the out-of-sample period. Volatility forecasts tend to underestimate realized volatility, but only by an average of 24 basis points. Further, the mean absolute percentage error (MAPE) of the volatility forecasts is less than half that of implied volatility estimates, while the mean square error (MSE) is less than a third.

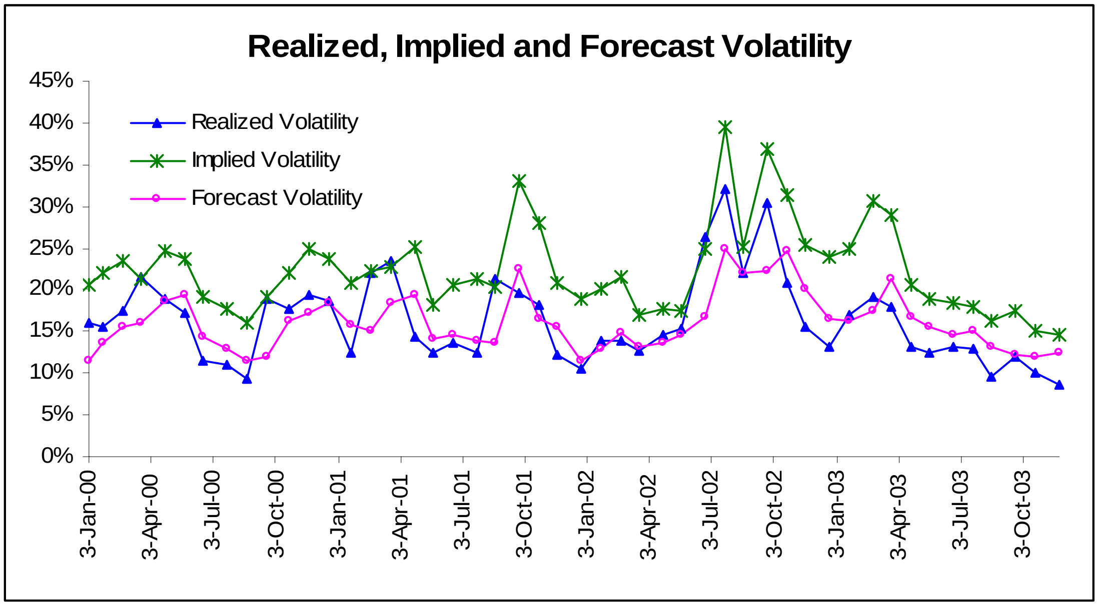


Figure 11: Out of Sample Realized, Implied and Forecast Volatility

Exploring the relative predictive abilities more deeply, we find that (unsurprisingly, given the tendency towards upward bias) implied volatility provides very poor predictive ability vis-à-vis the direction of future realized volatility. Of the 48 out-of-sample period, implied volatility correctly anticipated the change in direction in realized volatility in only 19 cases (40.43%), whereas volatility forecast correctly anticipated the change of direction in 29 periods (61.70%), which is significant at the 4% level. One conjecture is that this superior sign prediction capability derives from the ability of the model to capture important long-memory effects. The latter may not feature appropriately in the ad-hoc day-to-day volatility estimates of market traders, as it must surely be very difficult for a trader in the pit to assign appropriate weightings for historical volatility in the asset as far back two years or more, even supposing the data is available to him. The predictive superiority of the model is confirmed by the Theil's-U test statistic, which at 0.82 demonstrates a substantial improvement over the naïve predictor. Implied volatility, by contrast, performs much worse than a naïve predictor throughout the out-of-sample period (Theil's-u = 1.46).

These finding have important implications as far as market efficiency is concerned, as hence the potential for making abnormal returns from volatility trading. Firstly, the ability to predict the general level of volatility is considered more important than the ability to explain its variation: an average overpricing of index options by 610 volatility basis points is very likely to facilitate the generation of abnormal returns. Secondly, the ability to predict the direction of a market, rather than the magnitude of the change, is in and of itself likely to be valuable. Market timing strategies that achieve a "hit" rate of 56% - 57% are generally considered by market practitioners to afford a sufficient edge to be worth trading in most markets. Here the edge in terms of market timing ability is substantially greater.

Volatility Trading Strategy

The potential for making abnormal profits from volatility trading is tested by means of the following simple procedure. We gather daily data for near-to-the money options on the S&P 500 index for each day in the out-of-sample period, including both the price and the option delta at market close. We then simulate the purchase or sale of at-the-money straddles, depending on whether the forecast volatility to expiration exceeds or is exceeded by the average implied volatility of the two option contracts. The resulting long or short straddle position is held to maturity and delta-hedged at the end of each trading day (using the net of the closing deltas of the two option contracts). It can be assumed that delta hedging is accomplished by trading SPYDrs, contracts which trade at the American Stock Exchange at 1/10 the size of the S&P 500 index. Hedge contracts are bought (sold) at the reported end-of-day offer (bid) price. We factor in trading costs at a rate of $1 per round turn for each option contract and at $0.005 for each SPYder. These costs are somewhat higher than would be incurred by a sizeable fund trading on the available ECN's (electronic exchanges). No allowance is made for market impact, which however should be relatively minor in these highly liquid instruments up to notional capital amounts of $500M or so.

The results of this rather elementary trading strategy are presented in the table and figure below:

		Winner	Losers
# Periods	48	30	18
		62.5%	37.5%
Avg, Return	1.56%	4.10%	-2.68%
Largest		17.52%	-8.35%
St. dev.	4.78%	4.07%	2.13%
Compound	98.10%		
Ann. Compound	18.64%		
Ann. Volatility	16.55%	14.08%	7.39%
Sharpe	0.88		
Beta	0.00		
Alpha	22.64%		
Correl.	0.01		
Max Drawdown	-19.26%		
# Periods	7	(18 Mar 02 - 21 Oct 02)	
Recovery Time	10		

Table 8: Strategy Performance Analysis

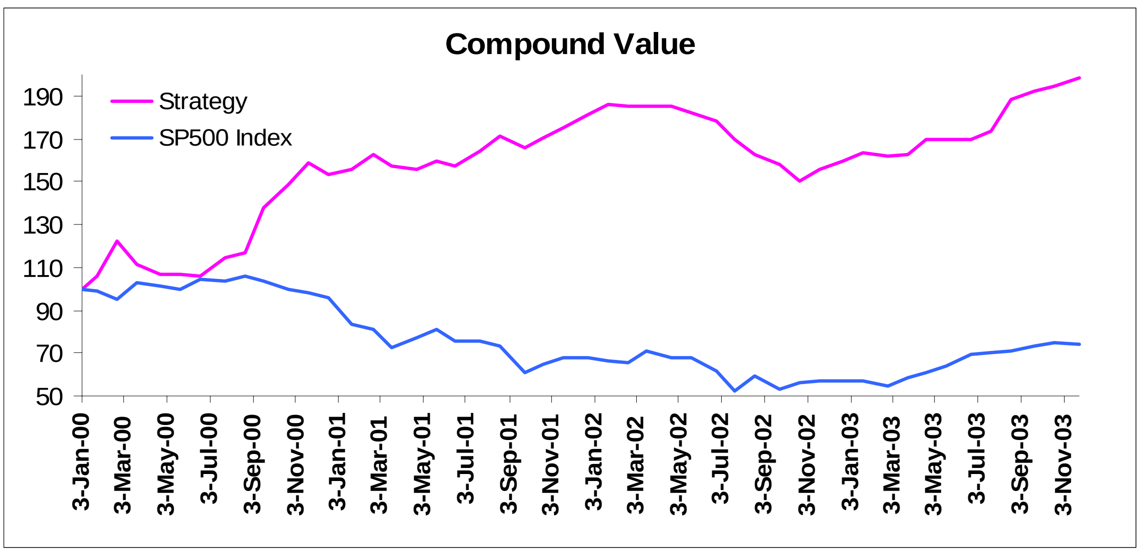


Figure 12: Compound Returns or Simple Volatility Trading Strategy

Of the 48 option expiration periods, 30 (62.5%) turn out to be profitable for the strategy, a proportion which closely mirrors that of the percentage direction prediction accuracy (see Table 6). The average return per option expiration period is

1.56%, which equates to a net annual compound return of 18.64% over the 4-year test period, during which the annual return on the S&P index itself was -7.24%. The strategy's risk-adjusted rate of return (Sharpe ratio) of 0.88 is higher than the long term average of the majority of investment strategies and, unlike for many of them, strategy returns are uncorrelated with the underlying index. With an annual alpha of 22.64%, this simple strategy appears to offer clear empirical evidence of the potential for generating abnormal returns using volatility forecasts based on high-frequency observations of historical volatility alone.

This is not to say, however, that the strategy is an investable proposition. The assumption that the portfolio is to be delta-hedged only at market close is probably unrealistic, as it ignores the sizeable delta-swings that often arise from intra-day market movements. Again, there is a lengthy period of seven months from March to October 2002 in which the strategy suffers a drawdown in excess of 19%. Many investors would regard as unacceptable an investment proposition which could result in losses on that scale over half the trading year. An examination of the evolution of realized volatility in Figure 8 reveals the nature of the problem: over the period from March to October 02 the model, while often correctly predicting the direction of volatility movements, significantly under-forecasts future realized volatility. This is the result of a weakness previously identified in the model – its underestimation of the kurtosis of the returns process and hence of the magnitude of volatility spikes. This could in principle be corrected by fine tuning the model – either by using a non-Gaussian distribution (such as, for instance a Student-t distribution) to model the error process, or by including an explicit kurtosis term in the model itself.

Conclusion

Like other similar empirical studies this research points to the conclusion that the inclusion of historical realized volatility adds little to the explanatory power of implied volatility forecasts. However, one important finding that appears to have been overlooked by other researchers is that forecasting models using high-frequency historical data may have an edge over implied volatility forecasts in predicting the *direction* of future realized volatility. The ability to time the market by correctly predicting its direction approximately 62% of the time appears to offer the potential to generate abnormal returns by a simple strategy of buying and selling at-the-money straddles and delta-hedging the resulting positions on a daily basis through to expiration, even after allowing for realistic transaction and hedging costs.

References

Aggarwal, R., Inclan, C., Leal, R., 1999, Volatility in Emerging Stock Markets, Journal of Financial and Quantitative Analysis, Forthcoming.

Andersen, T.G., 2000, Some reflections on analysis of high-frequency data, *Journal of Business & Economic Statistics* 18, 146-153.

Andersen, T.G., Bollerslev, T., Diebold, F.X., and Labys, P. (1999), "The Distribution of Exchange Rate Volatility," Wharton Financial Institutions Center Working Paper 99-08 and NBER Working Paper 6961.

Andersen T.B., Bollerslev, Diebold and Labys, 2001. Modeling and forecasting realized volatility. Working paper Northwestern University

Bakshi G., Cao C., and Chen Z., 1997, Empirical performance of alternative option pricing models, *Journal of Finance* 52, 2003-2049.

Breidt, J.F., Crato, N., de Lima, P., 1998. The detection and estimation of long memory in stochastic volatility. Journal of Economics 83, 325–348.

Canian, L. and Figlewski, S., 1993, The information content of implied volatility, *Review of Financial Studies* 6, 659-681.

Christensen, B.J., and Prabhala, N.R., 1998, The relationship between implied and realized volatility, *Journal of Financial Economics* 50, 125-150.

Dumas, B., Fleming, J., and Whaley, R.E., 1998, Implied Volatility Functions: Empirical Tests, *The Journal of Finance* 6, 2059-2107.

Fleming, J., 1998, The quality of market volatility forecasts implied by S&P 100 index option prices, *Journal of Empirical Finance* 5, 317-345.

Geweke, J., Porter-Hudak, S., 1983, The Estimation and Application of Long Memory Time Series Models, Journal of Time Series Analysis, 4, 15-39.

Granger, C.W.J., Joyeux, R., 1980. An introduction to long memory time series models and fractional differencing. Journal of Time Series Analysis 1, 15–29.

Harvey, C.R., Whaley, R.E., 1992, Market volatility prediction and the efficiency of the S&P 100 index option market. Journal of Financial Economics 31, 43-73.

Heston, S.L., 1993, A closed-form solution for options with stochastic volatility with applications to bond and currency options, *Review of Financial Studies* 6, 327-343.

Hull, J., White, A., 1987, The pricing of options on assets with stochastic volatilities, *Journal of Finance* 42, 281-300.

Martens, M. and Zein, J., 2002, Predicting financial volatility: High-frequency time-series forecasts vis-à-vis implied volatility, forthcoming.

Nandi, S., 1998, How important is the correlation between returns and volatility in a stochastic volatility model? Empirical evidence from pricing and hedging in the S&P 500 index options market, *Journal of Banking and Finance* 22, 589-610.

Noh, J., Engel, R.F., Kane, A., 1994. Forecasting volatility and option prices of the S&P 500 index. Journal of Derivatives 2(1), 17-30.

Parkinson, M., 1980, The extreme value method for estimating the variance of the rate of return, *Journal of Business* 53, 61-68.

Poteshman, A.M., 2000, Forecasting future volatility from option prices, working paper, University of Illinois at Urbana-Champaign.

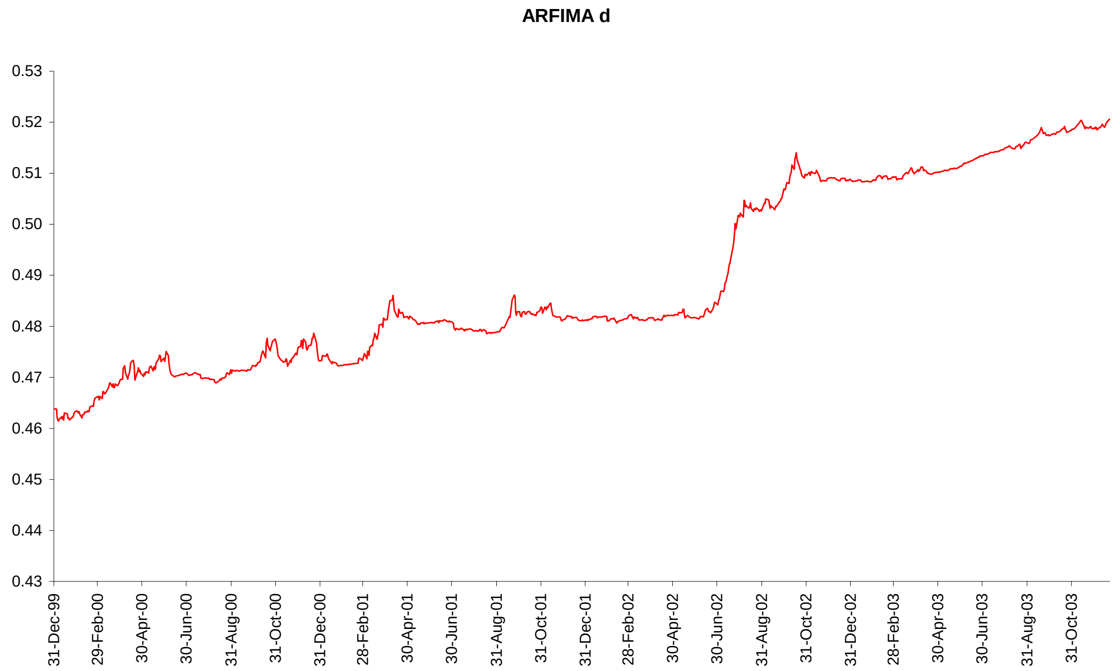
Pong, S, Shackleton, M. B., Taylor, S. J. and Xu, X., 2003, Forecasting currency volatility: a comparison of implied volatilities and AR(FI)MA models, forthcoming, Journal of Banking and Finance.

Sowell, F., 1992. Maximum likelihood estimation of stationary univariate fractionally integrated time series models. Journal of Econometrics 53, 165–188.

Taylor, S.J., and Xu X., 1997, The incremental volatility information in one million foreign exchange quotations, *Journal of Empirical Finance* 4, 317-340.

Yang, D., and Zhang Q., 2000, Drift independent volatility estimation based on high, low, open and close prices, *Journal of Business* 73, 477-491.

Appendix



ARFIMA d
0.53
0.52
0.51
0.50
0.49
0.48
0.47
0.46
0.45
0.44
0.43
31-Dec-99
29-Feb-00
30-Apr-00
30-Jun-00
31-Aug-00
31-Oct-00
31-Dec-00
28-Feb-01
30-Apr-01
30-Jun-01
31-Aug-01
31-Oct-01
31-Dec-01
28-Feb-02
30-Apr-02
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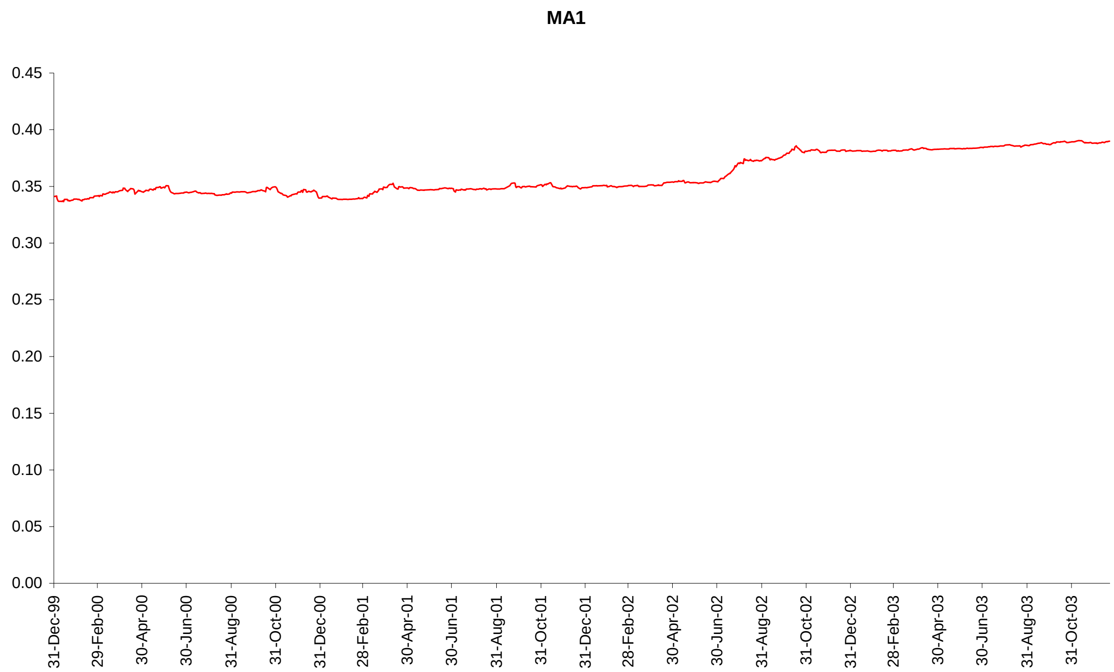
31-Aug-03
31-Oct-03


MA1
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0.40
0.35
0.30
0.25
0.20
0.15
0.10
0.05
0.00
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29-Feb-00
30-Apr-00
30-Jun-00
31-Aug-00
31-Oct-00
31-Dec-00
28-Feb-01
30-Apr-01
30-Jun-01
31-Aug-01
31-Oct-01
31-Dec-01
28-Feb-02
30-Apr-02
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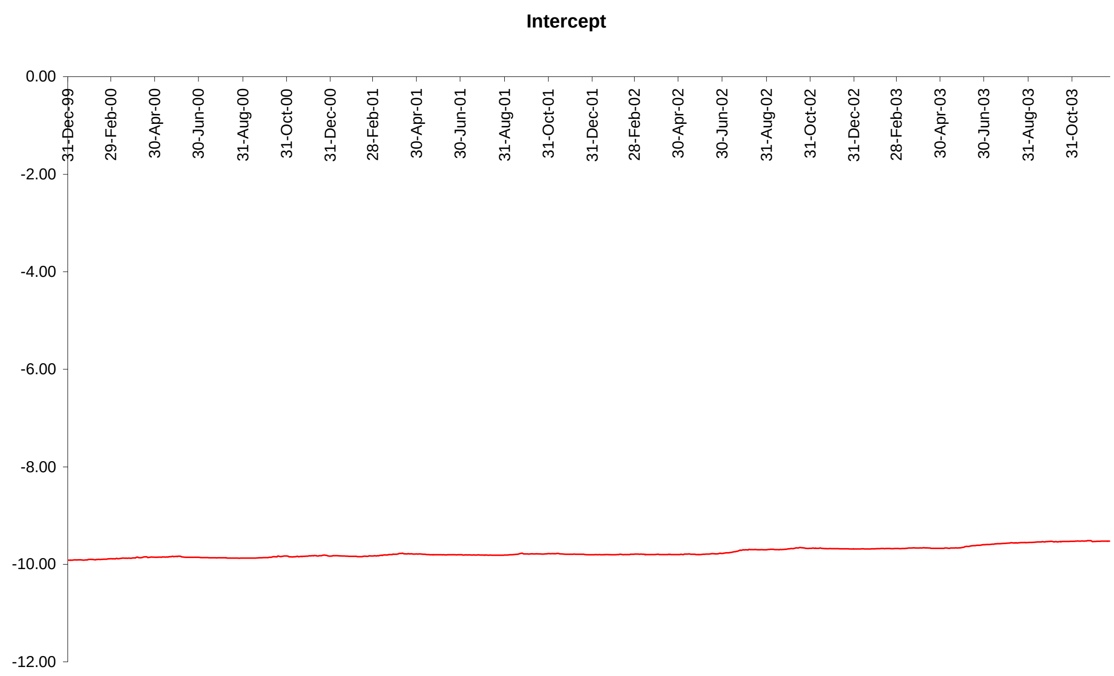
31-Oct-03


Intercept
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-4.00
-6.00
-8.00
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-12.00
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29-Feb-00
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30-Jun-00
31-Aug-00
31-Oct-00
31-Dec-00
28-Feb-01
30-Apr-01
30-Jun-01
31-Aug-01
31-Oct-01
31-Dec-01
28-Feb-02
30-Apr-02
30-Jun-02
31-Aug-02
31-Oct-02
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28-Feb-03
30-Apr-03
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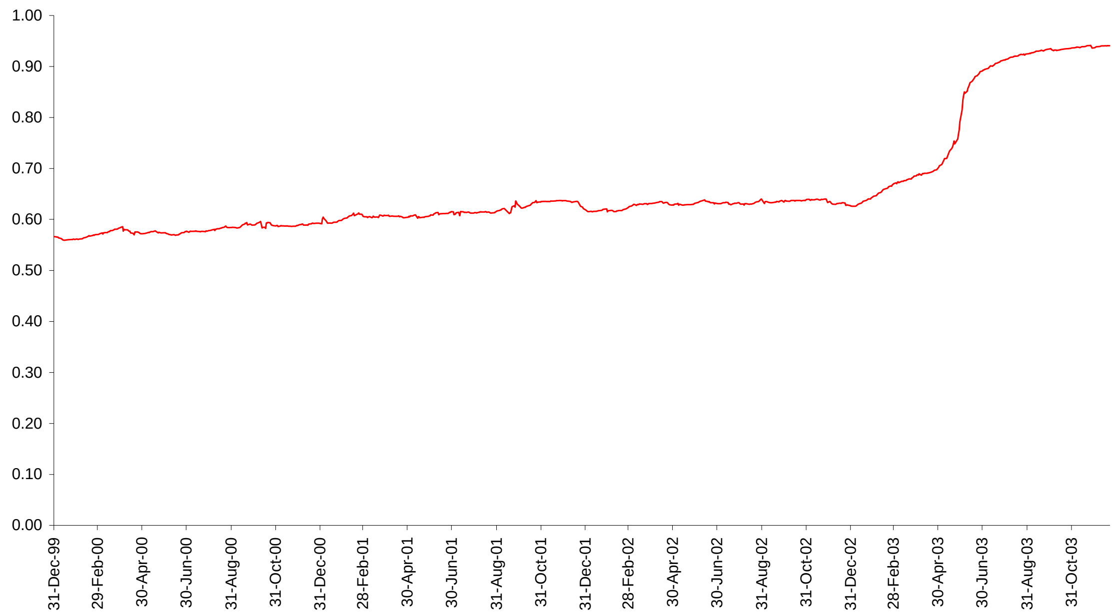
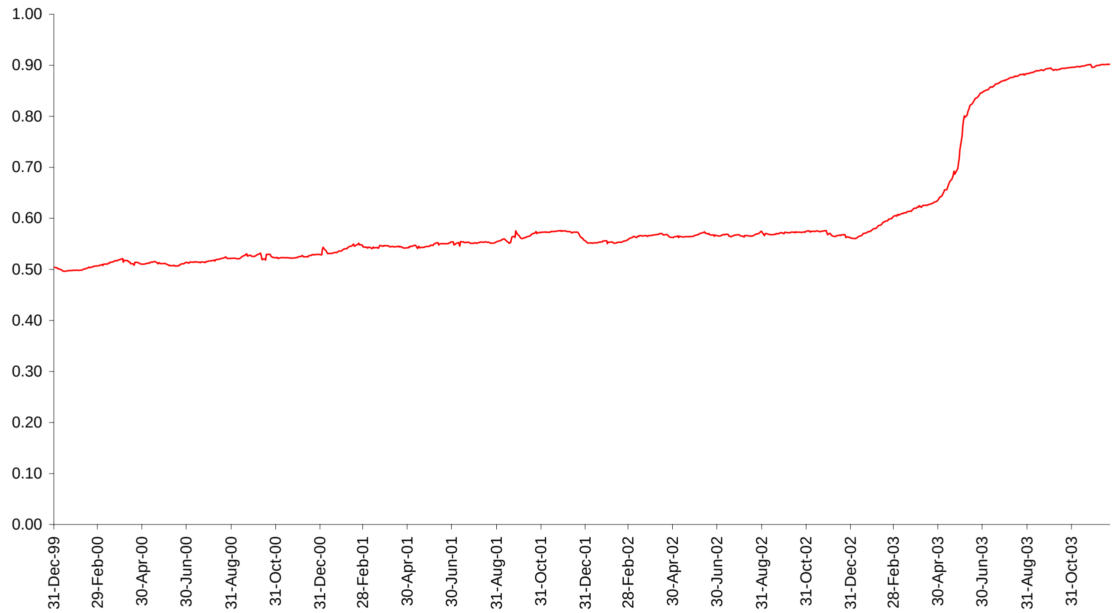
31-Aug-03
31-Oct-03

GARCH AR1(1,1)



GARCH MA1(1,1)



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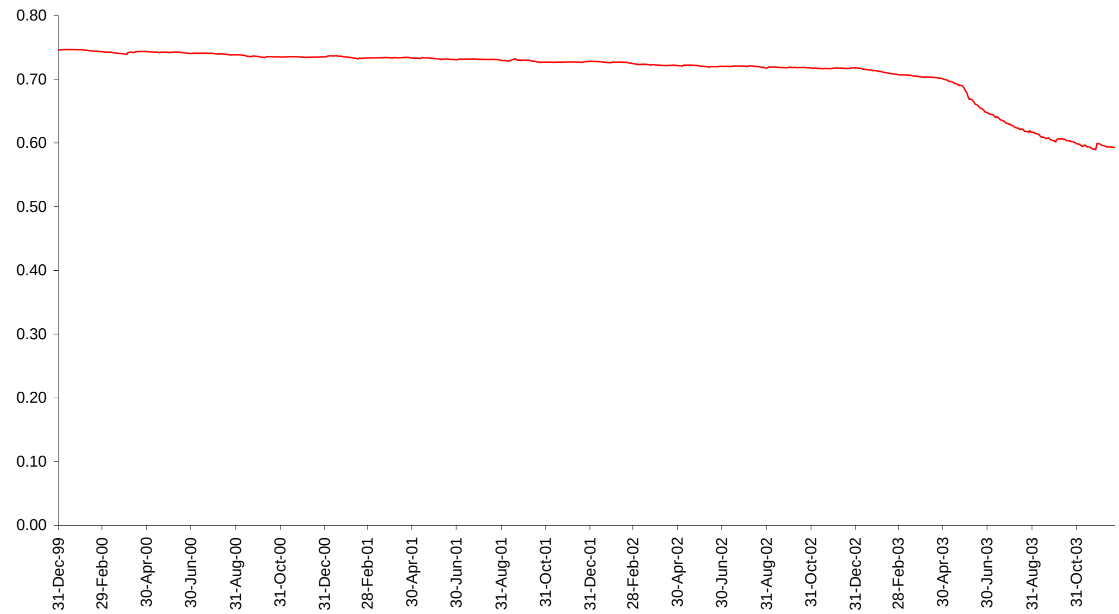
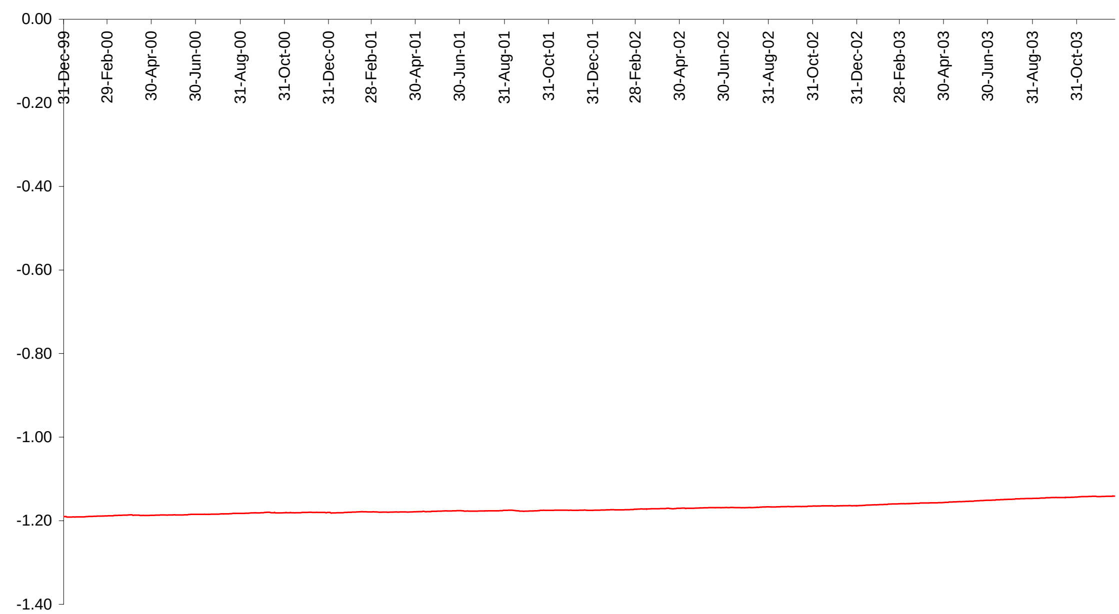
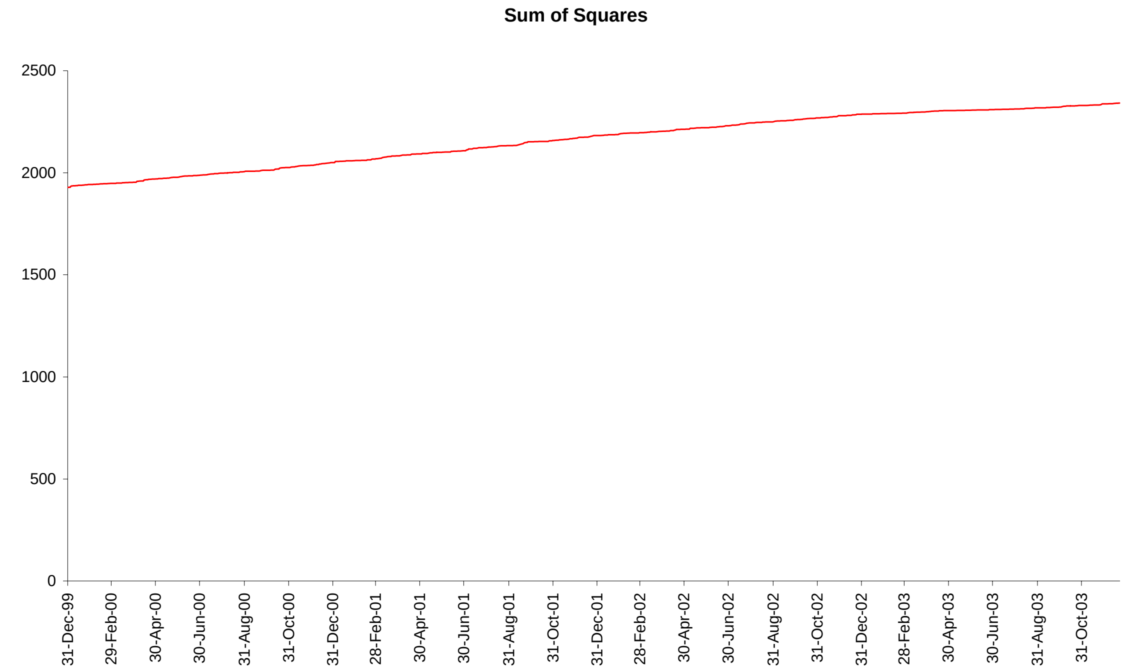


Criterion/N

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2000
1500
1000
500
0
31-Dec-99
29-Feb-00
30-Apr-00
30-Jun-00
31-Aug-00
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31-Dec-00
28-Feb-01
30-Apr-01
30-Jun-01
31-Aug-01
31-Oct-01
31-Dec-01
28-Feb-02
30-Apr-02
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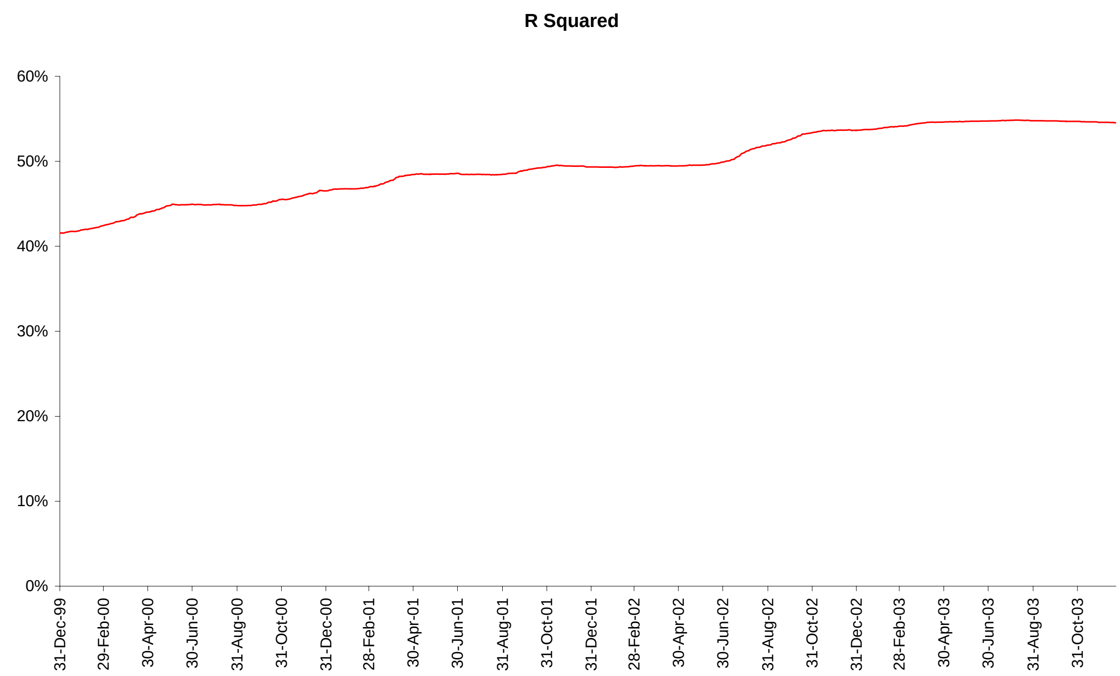
R Squared
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50%
40%
30%
20%
10%
0%
31-Dec-99
29-Feb-00
30-Apr-00
30-Jun-00
31-Aug-00
31-Oct-00
31-Dec-00
28-Feb-01
30-Apr-01
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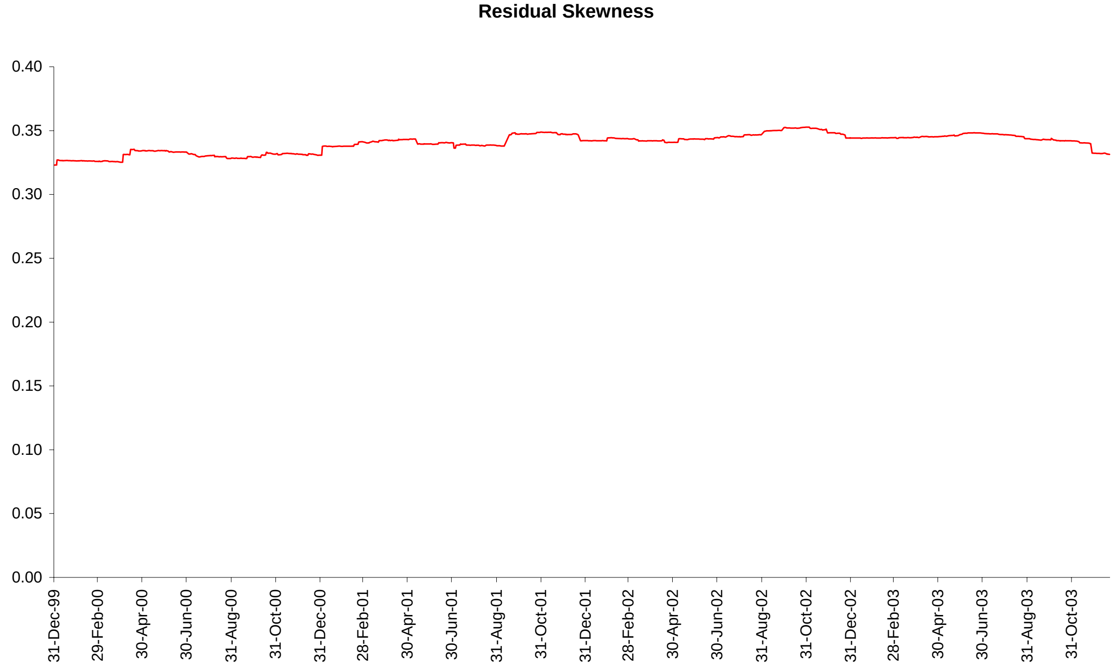
Residual Skewness
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0.35
0.30
0.25
0.20
0.15
0.10
0.05
0.00
31-Dec-99
29-Feb-00
30-Apr-00
30-Jun-00
31-Aug-00
31-Oct-00
31-Dec-00
28-Feb-01
30-Apr-01
30-Jun-01
31-Aug-01
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31-Aug-03
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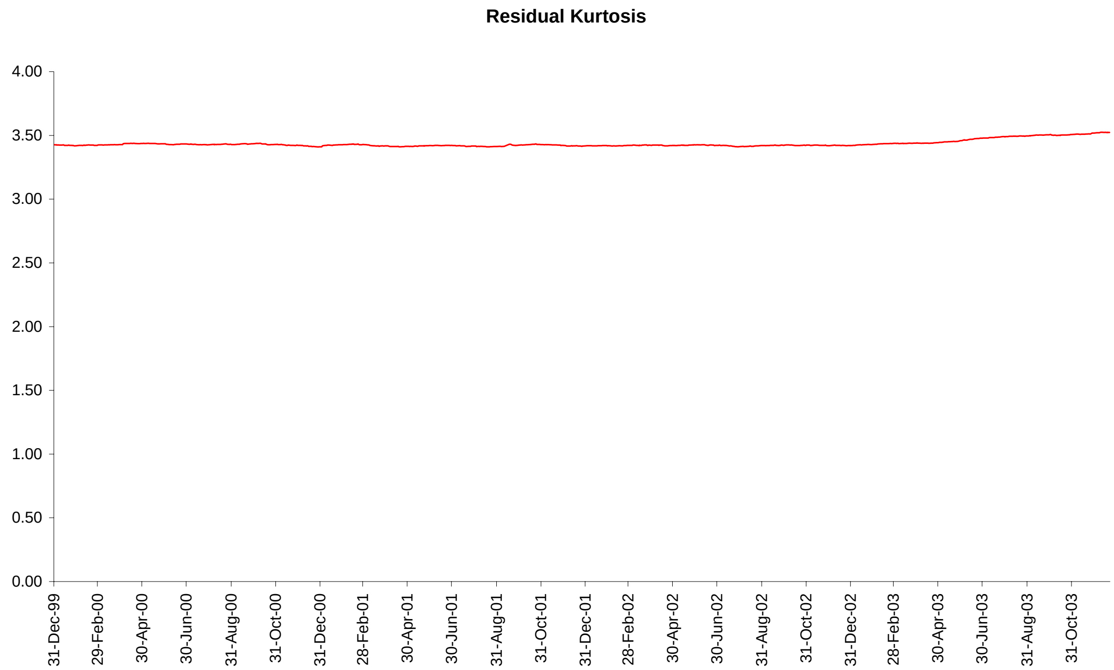
Residual Kurtosis
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3.50
3.00
2.50
2.00
1.50
1.00
0.50
0.00
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29-Feb-00
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30-Jun-00
31-Aug-00
31-Oct-00
31-Dec-00
28-Feb-01
30-Apr-01
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31-Oct-01
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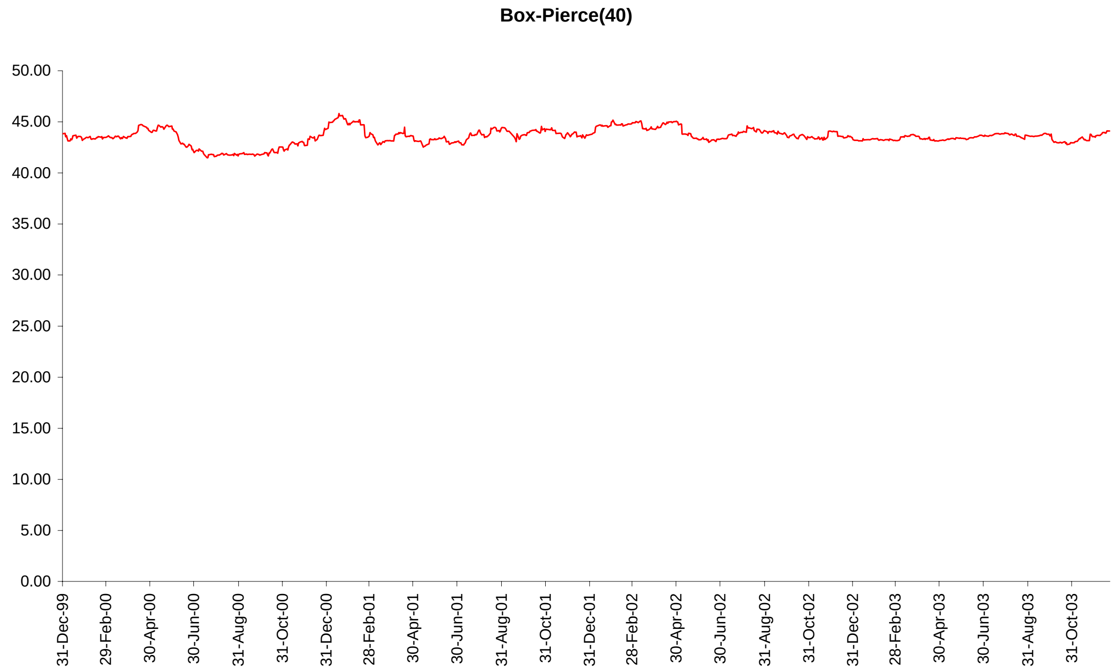
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20.00
15.00
10.00
5.00
0.00
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30-Apr-00
30-Jun-00
31-Aug-00
31-Oct-00
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28-Feb-01
30-Apr-01
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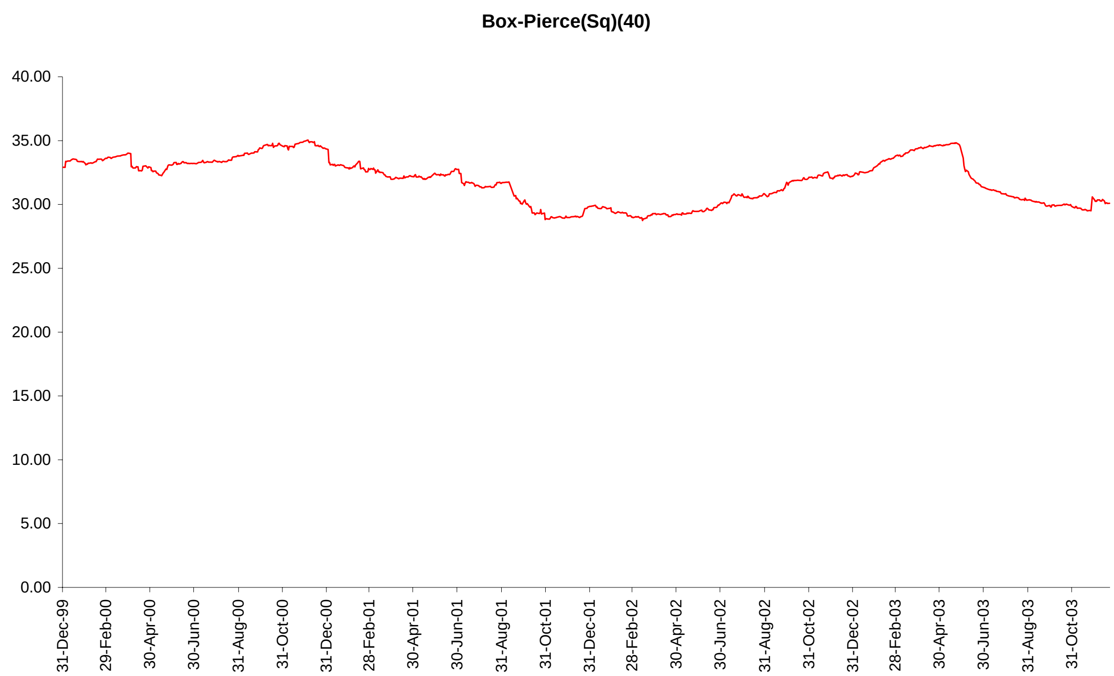
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5.00
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31-Aug-00
31-Oct-00
31-Dec-00
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